

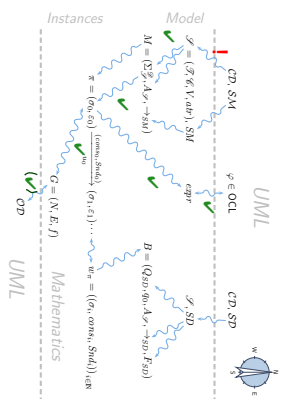
Software Design, Modelling and Analysis in UML

Lecture 05: Class Diagrams I

2012-11-07

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Course Map

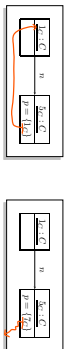


Contents & Goals

- Last Lecture:**
- OCL Semantics
 - Object Diagrams
- This Lecture:**
- Educational Objectives: Capabilities for following tasks/questions:
 - What is a class diagram?
 - For what purposes are class diagrams useful?
 - Could you please map this class diagram to a signature?
 - Could you please map this signature to a class diagram?
 - Content:
 - Object Diagrams Cont'd.
 - Study UML syntax.
 - Prepare (extend) definition of signature.
 - Map class diagram to (extended) signature.
 - Stereotypes – for documentation.

Closed Object Diagrams vs. Dangling References

Find the 10 differences! (Both diagrams shall be complete.)



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Definition. Let σ be a system state. We say attribute $v \in V_{0,1}$ has a **dangling reference** in object $o \in \text{dom}(\sigma)$ if and only if the attribute's value comprises an object which is not alive in σ , i.e. if $\sigma(v)(o) \notin \text{dom}(\sigma)$.

We call σ **closed** if and only if no attribute has a dangling reference in any object alive in σ .

Closed Object Diagrams vs. Dangling References

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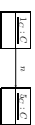
Observation: Let C be the (!) complete object diagram of a **closed** system state σ . Then the nodes in C are labelled with \mathcal{S} -typed attribute/value pairs only.

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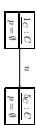
Special Notation

$\mathcal{S} = (\{In\}, \{C\}, \{n, p : C_1\}, \{C \mapsto \{n, p\}\})$.

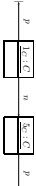
• Instead of



we want to write



or



to explicitly indicate that attribute $p : C_1$ has value \emptyset (also for $p : C_{0,1}$).

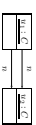
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Alternativ

We slightly deviate from the standard (for reasons):

• In the course, $C_{0,1}$ and C_1 -typed attributes **only** have **sets as values**. UML also considers multisets, that is, they can have



(This is not an object diagram in the sense of our definition because of the requirement on the edges E . Extension is straightforward but tedious.)

• We **allow** to give the valuation of $C_{0,1}$ - or C_1 -typed attributes in the **values compartment**.

• Allows us to indicate that a certain r is not referring to another object.

• Allows us to represent “dangling references”, i.e. references to objects which are not alive in the current system state.

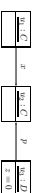
• We introduce a graphical representation of \emptyset values.

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The Other Way Round

• If we **only** have a picture as below, we typically assume that, it's **meant** to be an object diagram wrt. some signature and structure.



• In the example, we can conclude (by “good will”) that the author is referring to some signature $\mathcal{S} = (\mathcal{S}, \mathcal{K}, V, \text{attr})$ with at least

- $\{C, D\} \subseteq \mathcal{C}$
 - $T \subseteq \mathcal{T}$
 - $\{x\} \subseteq \text{attr}(C)$
 - $\{p\} \subseteq \text{attr}(C)$
 - $\{A, B\} \subseteq \text{val}(D)$
- and a structure with
- $\{k_x\} \subseteq \mathcal{D}(C)$
 - $\{k_p\} \subseteq \mathcal{D}(C)$
 - $0 \in \text{val}(D)$

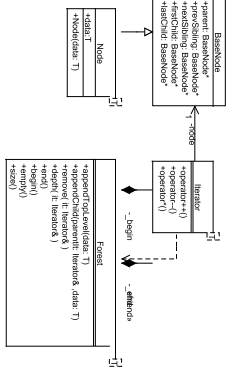
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Example: Object Diagrams for Documentation

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OCL Satisfaction Relation

In the following, \mathcal{S} denotes a signature and \mathcal{D} a structure of \mathcal{S} .

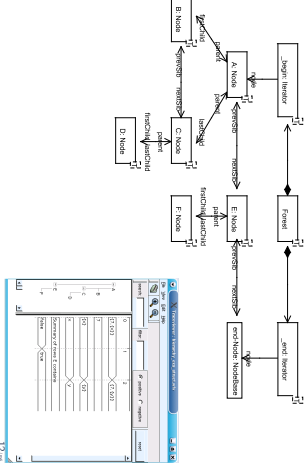
Definition (Satisfaction Relation).

Let φ be an OCL constraint over \mathcal{S} and $\sigma \in \Sigma_{\mathcal{D}}^{\mathcal{S}}$ a system state.

- We write $\sigma \models \varphi$ (“ σ satisfies φ ”) if and only if $I[\varphi](\sigma, \emptyset) = true$.
- $\sigma \not\models \varphi$ if and only if $I[\varphi](\sigma, \emptyset) = false$.

Note: In general we can't conclude from $\neg(\sigma \models \varphi)$ to $\sigma \not\models \varphi$ or vice versa.

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OCL Consistency

Object Diagrams and OCL

Let G be an object diagram of signature \mathcal{S} wrt. structure \mathcal{D} .

Let $expr$ be an OCL expression over \mathcal{S} .

We say G satisfies $expr$, denoted by $G \models expr$, if and only if $\forall \sigma \in G^{-1} : \sigma \models expr$.

If G is complete, we can also talk about “ \models ”. (Otherwise, to avoid confusion, avoid “ \models ”: G^{-1} could comprise system states in which $expr$ evaluates to true, false, and \perp .)

Example: (complete — what if not complete wrt. object/attribute/both?)

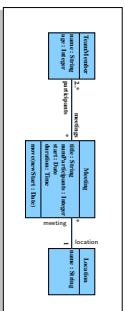
- context $C' \text{ inv : } n \rightarrow isEmpty() \text{ is false}$
- context $C' \text{ inv : } !n \rightarrow isEmpty() \text{ is true}$
- context $D \text{ inv : } n \neq 0$

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Definition (Consistency). A set $Inv = \{c_1, \dots, c_n\}$ of OCL constraints over \mathcal{S} is called **consistent** (or **satisfiable**) if and only if there exists a system state of \mathcal{S} wrt. \mathcal{S} which satisfies all of them, i.e. if

$$\exists \sigma \in \Sigma_{\mathcal{S}} : \sigma \models c_1 \wedge \dots \wedge \sigma \models c_n$$

and **inconsistent** (or **unrealizable**) otherwise.



Deciding OCL Consistency

- Whether a set of OCL constraints is satisfiable or not is **in general not as obvious** as in the made-up example.
 - **Wanted:** A procedure which decides the OCL satisfiability problem.
 - **Unfortunately:** in general **undecidable**.
- Otherwise we could, for instance, solve **diophantine equations**
- $$c_1 x_1^{k_1} + \dots + c_m x_m^{k_m} = d.$$

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- $$c_1 x_1^{k_1} + \dots + c_m x_m^{k_m} = d.$$
- Encoding in OCL:
- $$\text{allInstances} \rightarrow \text{exists}(w : C \mid c_1 * w_1 x_1^{k_1} + \dots + c_m * w_m x_m^{k_m} = d).$$
- [Cobat and Christó, 2008]
- **And now?** Options:
 - Constrain OCL, use a **less rich** fragment of OCL.
 - Revert to **finite domains** — basic types vs. number of objects.

$\mathcal{S} = (\mathcal{S}, \mathcal{C}, V, \text{attr})$ where

- (basic) types \mathcal{S} and classes \mathcal{C} , (both finite),
- typed attributes V , τ from \mathcal{S} or \mathcal{C}_{A1} or \mathcal{C}_1 , $C \in \mathcal{C}$,
- $\text{attr} : \mathcal{C} \rightarrow 2^V$ mapping classes to attributes.

Too abstract to represent class diagram, e.g. no "place" to put class stereotypes or attribute visibility.

So: **Extend** definition for classes and attributes: Just as attributes already have types, we will assume that

- classes have (among other things) **stereotypes** and
- attributes have (in addition to a type and other things) a **visibility**.

From now on, we assume that each class $C \in \mathcal{C}$ has:

- a finite (possibly empty) set S_C of **stereotypes**,
- a boolean flag $a \in \mathbb{B}$ indicating whether C is **abstract**,
- a boolean flag $t \in \mathbb{B}$ indicating whether C is **active**.

We use S_C to denote the set $\bigcup_{C \in \mathcal{C}} S_C$ of stereotypes in \mathcal{S} . (Alternatively, we could add a set S as 5-th component to \mathcal{S} to provide the stereotypes (names of stereotypes) to choose from. But: too unimportant to care.)

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Convention:

- We write $(C, S_C, a, t) \in \mathcal{C}$ when we want to refer to all aspects of C .
- If the new aspects are irrelevant (for a given context), we simply write $C \in \mathcal{C}$, i.e. old definitions are still valid.

- From now on, we assume that each attribute $v \in V$ has (in addition to the type):

- a **visibility**

$$\xi \in \left\{ \underbrace{\text{public}}_{\text{vis}}, \underbrace{\text{private}}_{\text{vis}}, \underbrace{\text{protected}}_{\text{vis}}, \underbrace{\text{package}}_{\text{vis}} \right\}$$

- an **initial value** exp_0 , given as a word from **language for initial values**, e.g. OCL expressions. (If using Java as **action language** (later) Java expressions would be fine.)
 - a finite (possibly empty) set of **properties** P_v .
- We define P_v analogously to stereotypes.

Extended Attributes

- From now on, we assume that each attribute $v \in V$ has (in addition to the type):
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(If using Java as **action language** (later) Java expressions would be fine.)
 - a finite (possibly empty) set of **properties** P_v .
- We define R'_ξ analogously to stereotypes.

Convention:

- We write $(v : \tau, \xi, \text{expr}_0, P_v) \in V$ when we want to refer to all aspects of v .
- Write only $v : \tau$ or v if details are irrelevant.

And?

- **Note:**
All definitions we have up to now **principally still apply** as they are stated in terms of, e.g., $C \in \mathcal{C}$ — which still has a meaning with the extended view.
For instance, system states and object diagrams remain mostly unchanged.
- The other **way round**: **most** of the newly added aspects **don't contribute** to the constitution of system states or object diagrams.

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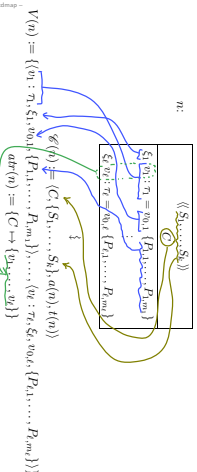
- Then what **are** they useful for..?
- First of all, to represent class diagrams.
- And then we'll see.



What If Things Are Missing?

- For instance, what about the box above?
- v has **no visibility**, **no initial value**, and (strictly speaking) **no properties**.

Mapping UML CDs to Extended Signatures



where "abstract" is determined by the front: $t(n) = \begin{cases} \text{true} & \text{if } n = \boxed{\square} \text{ or } n = \boxed{C, I, A} \\ \text{false} & \text{otherwise} \end{cases}$

where "active" is determined by the frame: $t(n) = \begin{cases} \text{true} & \text{if } n = \boxed{\square} \text{ or } n = \boxed{C} \\ \text{false} & \text{otherwise} \end{cases}$



What If Things Are Missing?

- For instance, what about the box above?
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It depends.

- What does the standard say? [OMG, 2007a, 121]
- **Presentation Options.** The type, visibility, default, multiplicity, property string may be suppressed from being displayed, even if there are values in the model.
- **Visibility:** There is no "no visibility" — an attribute has a visibility in the (extended) signature. Some (and we) assume **public** as default, but conventions may vary.
- **Initial value:** some assume it is given by domain (such as "itermost value", but what is "itermost" of Z^2). Some (and we) understand **non-deterministic initialization**.
- **Properties:** probably safe to assume \emptyset if not given at all.

From Class Boxes to Extended Signatures

A class box n induces an (extended) signature class as follows:

$V(n) := \{ (n; \tau_1, S_1, \nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8, \nu_9, \nu_{10}) \}$

$\mathcal{G}(n) := (C_1, S_1, \dots, S_9, \alpha(n), t(n))$

$act(n) := \{ C \rightarrow \{ \nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8, \nu_9, \nu_{10} \} \}$

From Class Diagrams to Extended Signatures

- We view a **class diagram** CD as a graph with nodes $\{n_1, \dots, n_N\}$ (each "class rectangle" is a node).
- $\mathcal{G}(CD) := \bigcup_{i=1}^N \mathcal{G}(n_i) = \{ \mathcal{G}(n_i) \mid 1 \leq i \leq N \}$
- $V(CD) := \bigcup_{i=1}^N V(n_i)$
- $act(CD) := \bigcup_{i=1}^N act(n_i)$
- In a **UML model**, we can have **finitely many** class diagrams

$\mathcal{G}\mathcal{D} = \{ CD_1, \dots, CD_k \}$

which induce the following signature:

$\mathcal{G}(\mathcal{G}\mathcal{D}) = \left(\mathcal{G} \cdot \bigcup_{i=1}^k \mathcal{G}(CD_i), \bigcup_{i=1}^k V(CD_i), \bigcup_{i=1}^k act(CD_i) \right)$

(Assuming \mathcal{D} given, in "reality", we can introduce types in class diagrams, the class diagram then contributes to \mathcal{D} .)

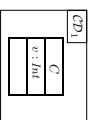
Is the Mapping a Function?

- Is $\mathcal{S}(\mathcal{C}, \mathcal{D})$ well-defined?

Two possible sources for problems:

(1) A class C may appear in multiple class diagrams:

(i)



(ii)



Simply forbid the case (i) — easy syntactical check on diagram.

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Is the Mapping a Function?

(2) An attribute v may appear in multiple classes:



Two approaches:

- Require unique attribute names.
- This requirement can easily be established (implicitly, behind the scenes) by viewing v as an abbreviation for

$C::v$ or $D::v$

depending on the context. ($C::v: bool$ and $D::v: int$ are unique.)

- Subtle, formalist's approach: observe that $(v: bool, \dots)$ and $(v: int, \dots)$ are different things in V . But we don't follow that path. . .

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Class Diagram Semantics

Semantics

- The semantics of a set of class diagrams \mathcal{C}, \mathcal{D} first of all is the induced (extended) signature $\mathcal{S}(\mathcal{C}, \mathcal{D})$.
- The signature gives rise to a set of system states given a structure \mathcal{S} .
- Do we need to redefine/extend \mathcal{S} ?

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- (Would be different if we considered the definition of enumeration types in class diagrams. Then the domain of an enumeration type τ , i.e. the set $\mathcal{D}(\tau)$, would be determined by the class diagram, and not free for choice.)

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- What is the effect on $\Sigma_{\mathcal{S}}^{\mathcal{C}, \mathcal{D}}$?

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- The semantics of a set of **class diagrams** \mathcal{C} first of all is the induced (extended) signature $\mathcal{S}(\mathcal{C})$.
- The **signature** gives rise to a set of **system states** given a **structure** \mathcal{S} .
- Do we need to redefine/extend \mathcal{S} ? **No**.
(Would be different if we considered the definition of enumeration types in class diagrams. Then the domain of an enumeration type τ , i.e. the set $\mathcal{S}(\tau)$, would be determined by the class diagram, and not free for choice.)

• What is the effect on $\Sigma_{\mathcal{S}}$? **Little**.

For now, we only **remove** abstract class instances, i.e.

$$\sigma : \mathcal{S}(\mathcal{C}) \mapsto (V \mapsto (\mathcal{S}(\mathcal{C}) \cup \mathcal{S}(\mathcal{C}_1)))$$

is now **only** called **system state** if and only if, for all $(C; S_C, I, t) \in \mathcal{C}$,

$$\text{dom}(\sigma) \cap \mathcal{S}(C) = \emptyset.$$

With $\alpha = 0$ as default “abstractness”, the earlier definitions apply directly. We’ll revisit this when discussing inheritance.

- **Classes**:
- **Active**: not represented in σ .
Later: relevant for behaviour, i.e. how system states evolve over time.
- **Stereotypes**: in a minute.
- **Attributes**:
- **Initial value**: not represented in σ .
Later: provides an initial value as effect of “creation action”.
- **Visibility**: not represented in σ .
Later: viewed as additional **typing information** for well-formedness of system transformers; and with inheritance.
- **Properties**: such as readability, ordered, composite (**Deprecated** in the standard)
- **readably** — **later** treated similar to visibility.
- **ordered** — too fine for our representation.
- **composite** — cf. lecture on associations.

Stereotypes as Labels or Tags

- So, a class is

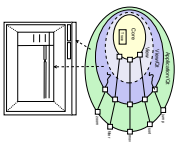
$$(C; S_C, \alpha, t)$$

with α the abstractness flag, t activeness flag, and S_C a set of **stereotypes**.

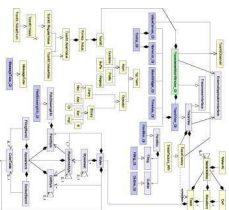
- What are Stereotypes?
 - **Not** represented in system states.
 - **Not** contributing to typing rules.
(cf. later lecture on type theory for UML)

- [Osterreich, 2006]:
View stereotypes as (additional) “labelling” (“tags”) or as “grouping”.
Useful for documentation and MDA.
- **Documentation**: e.g. layers of an architecture.
- Sometimes, packages (cf. the standard) are sufficient and “right”.
- **Model Driven Architecture (MDA)**: later.

Example: Stereotypes for Documentation



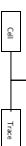
- Example: Tiling Diagram Viewer [Schramm et al., 2001]
- Architecture of four layers:
 - con. data layer
 - abstract view layer
 - toolkit-specific view layer/widget
 - application using widget
- Stereotype “_uml” layer “_uml” colour



Stereotypes as Inheritance

- Another view (due to whom?) distinguish
- **Technical Inheritance**
If the **target platform**, such as the programming language for the implementation of the blueprint, is object-oriented, assume a 1-on-1 relation between inheritance in the model and on the target platform.
- **Conceptual Inheritance**
Only meaningful with a **common idea** of what stereotypes stand for. For instance, one could label each class with the team that is responsible for realising it. Or with licensing information (e.g. LGPL and proprietary).
Or one could have labels understood by code generators (cf. lecture on MOSE)

- **Contains**:
Inheritance is often referred to as the “is a”-relation. Sharing a stereotype also expresses “being something”.
- We can always (ab-)use UML-inheritance for the conceptual case, e.g.



Type Theory

Recall: In lecture 03, we introduced OCL expressions with **types**, for instance:

```

expr ::= w
      | true | false           : Bool
      | 0 | -1 | 1 | ...      : Int
      | expr1 + expr2       : Int × Int → Int
      | size(expr1)         : Set(τ) → Int
  
```

Wanted: A procedure to tell **well-typed**, such as (w : Bool)

from **not well-typed**, such as, size(w)

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Approach: Derivation System, that is, a finite set of derivation rules.

We then say *expr* is **well-typed** if and only if we can derive

$$A, C \vdash \text{expr} : \tau \quad (\text{read: "expression expr has type } \tau \text{"})$$

for some OCL type τ , i.e. $\tau \in T_U \cup T_C \cup \{\text{Set}(\tau_0) \mid \tau_0 \in T_U \cup T_C\}$, $C \in \mathcal{C}$. 42%

A Type System for OCL

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We will give a finite set of **type rules** (a **type system**) of the form

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These rules will establish well-typedness statements (**type sentences**) of three different **qualities**:

- (i) Universal well-typedness:

$$\vdash \text{expr} : \tau$$
- (ii) Well-typedness in a **type environment** A :

$$A \vdash \text{expr} : \tau$$

(for logical variables)
- (iii) Well-typedness in type environment A and **context** D :

$$A, D \vdash \text{expr} : \tau$$

(for visibility)

Constants and Operations

- If $expr$ is a **boolean constant**, then $expr$ is of type $Bool$:
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- If $expr$ is an **integer constant**, then $expr$ is of type Int :
 $(INT) \quad \vdash N : Int \quad N \in \{0, 1, -1, \dots\}$

Constants and Operations

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Constants and Operations Example

$(BOOL)$	$\vdash B : Bool$	$B \in \{true, false\}$
(INT)	$\vdash N : Int$	$N \in \{0, 1, -1, \dots\}$
$(FUN0)$	$\vdash \omega(expr_1, \dots, expr_n) : \tau$	$\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau$ $n \geq 1, \omega \notin \text{dir}(G)$

Example:

- not true
- true + 3

Type Environment

- Problem:** Whether

$$w + 3$$

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- Approach:** Type Environments

Definition. A type environment is a (possibly empty) finite sequence of type declarations:
 The set of type environments for a given set IV of logical variables and types T is defined by the grammar

$$A ::= \emptyset \mid A, w : \tau$$

where $w \in IV, \tau \in T$.

Clear: We use this definition for the set of OCL logical variables IV and the types $T = T_B \cup T_G \cup \{\text{Str}(t_0) \mid t_0 \in T_B \cup T_G\}$.

Environment Introduction and Logical Variables

- If $expr$ is of type τ , then it is of type τ in any type environment:

$$(EnvIntro) \frac{\vdash expr : \tau}{A \vdash expr : \tau}$$

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$$(Rule) \frac{A \vdash expr_1 : \tau_1 \dots A \vdash expr_n : \tau_n, \quad \omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, \quad n \geq 1, \omega \notin \text{atr}(\emptyset)}{A \vdash \omega(expr_1, \dots, expr_n) : \tau}$$

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- If $expr$ is a logical variable such that $w : \tau$ occurs in A , then we say w is of type τ .

$$(Var) \frac{w : \tau \in A}{A \vdash w : \tau}$$

Type Environment Example

$$\begin{array}{l} (EnvIntro) \quad \frac{\vdash expr : \tau}{A \vdash expr : \tau} \\ (Rule) \quad \frac{A \vdash expr_1 : \tau_1 \dots A \vdash expr_n : \tau_n, \quad \omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, \quad n \geq 1, \omega \notin \text{atr}(\emptyset)}{A \vdash \omega(expr_1, \dots, expr_n) : \tau} \\ (Var) \quad \frac{w : \tau \in A}{A \vdash w : \tau} \end{array}$$

Example:

- $w + 3, A = w : Int$

All Instances and Attributes in Type Environment

- If $expr$ refers to all instances of class C , then it is of type $\text{Set}(C)$.

$$(AllInst) \frac{}{\vdash \text{allInstancesOf} : \text{Set}(C)}$$

All Instances and Attributes in Type Environment

- If $expr$ refers to all instances of class C , then it is of type $\text{Set}(C)$.

$$(AllInst) \frac{}{\vdash \text{allInstancesOf} : \text{Set}(C)}$$

- If $expr$ is an attribute access of an attribute of type τ for an object of C as denoted by $expr_1$, then the premise is that $expr_1$ is of type τ_C :

$$(Attr0) \frac{A \vdash expr_1 : \tau_C}{A \vdash v(expr_1) : \tau} \quad v : \tau \in \text{atr}(C), \tau \in \mathcal{D}$$

$$(Attr0^1) \frac{A \vdash expr_1 : \tau_C}{A \vdash r_1(expr_1) : \tau_D}, \quad r_1 : D_{0,1} \in \text{atr}(C)$$

$$(Attr0^2) \frac{A \vdash expr_1 : \tau_C}{A \vdash r_2(expr_1) : \text{Set}(\tau_D)}, \quad r_2 : D_0 \in \text{atr}(C)$$

Attributes in Type Environment Example

$$\begin{array}{l}
 (Attr) \quad \frac{A \vdash expr_1 : \tau_1}{A \vdash \tau_1 \in attr(C)} \quad \tau \in \mathcal{S} \\
 (Attr_0^1) \quad \frac{A \vdash expr_1 : \tau_1}{A \vdash \tau_1 \in attr(C)} \quad \tau_1, D_{in} \in attr(C) \\
 (Attr_2) \quad \frac{A \vdash expr_1 : \tau_1 \quad A \vdash expr_2 : \tau_2}{A \vdash \tau_1 \in attr(C)} \quad \tau_2, D_2 \in attr(C)
 \end{array}$$



- $sdf : \tau_C \vdash sdf\ x$
- $sdf : \tau_C \vdash sdf\ x.x$
- $sdf : \tau_C \vdash sdf\ x.y$
- $sdf : \tau_D \vdash sdf\ x$

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Iterate

- If $expr$ is an **iterate expression**, then
- the iterator variable has to be type consistent with the base set, and
- initial and update expressions have to be consistent with the result variable.

$$\begin{array}{l}
 (Iter) \quad \frac{A \vdash expr_1 \rightarrow iterate(u_1 : \tau_1; u_2 : \tau_2 = expr_2 \mid expr_3) : \tau_2}{A \vdash expr_1 \rightarrow iterate(u_1 : \tau_1 \oplus (u_2 : \tau_2))}
 \end{array}$$

where $A' = A \oplus (u_1 : \tau_1) \oplus (u_2 : \tau_2)$.

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Iterate Example

$$\begin{array}{l}
 (Attr) \quad \frac{A \vdash allstates : Set(\tau_C)}{A \vdash \tau_C \in attr(C)} \quad (Attr) \quad \frac{A \vdash expr_1 : \tau_1}{A \vdash \tau_1 \in attr(C)} \\
 (Iter) \quad \frac{A \vdash expr_1 \rightarrow iterate(u_1 : \tau_1; u_2 : \tau_2 = expr_2 \mid expr_3) : \tau_2}{A \vdash \tau_1 \in attr(C)} \quad \tau_1, u_2 : \tau_2 = expr_2 \mid expr_3 : \tau_2
 \end{array}$$

where $A' = A \oplus (u_1 : \tau_1) \oplus (u_2 : \tau_2)$

Example $(\mathcal{S} = \{\{In\}, \{C\}, \{x : In\}, \{C \rightarrow \{x\}\}\})$

$$\begin{array}{l}
 allstates : \rightarrow iterate(sdf : C \rightarrow Bool \rightarrow true \mid u : Bool \rightarrow false) : Bool \\
 allstates : \rightarrow iterate(sdf : C \rightarrow Bool \rightarrow true \mid u : Bool \rightarrow false) : Bool \\
 context\ sdf : C \rightarrow Bool \rightarrow true \\
 context\ inv : x = 0
 \end{array}$$

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First Recapitulation

- I **only** defined for well-typed expressions.
- **What can hinder** something, which looks like a well-typed OCL expression, from being a well-typed OCL expression...?

- Plain syntax error
- Syntax error error
- Type error

context $C \text{ inv} : y = 0$

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