Software Design, Modelling and Analysis in UML

Lecture 13: Core State Machines IV

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Contents & Goals

Last Lecture:

- System configuration
- Transformer

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this State Machine mean? What happens if I inject this event?
  - Can you please model the following behaviour.
  - What is: Signal, Event, Ether, Transformer, Step, RTC.
- **Content:**
  - Transformer cont'd
  - Examples for transformer
  - Run-to-completion Step
  - Putting It All Together

System Configuration, Ether, Transformer

System Configuration

Definition. Let $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$ be a signature with signals, $\mathscr{D}_0$ a structure of $\mathscr{S}_0$, $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}_0$ and $\mathscr{D}_0$. Furthermore assume there is one core state machine M_C per class $C \in \mathscr{C}$.

A system configuration over $\mathscr{S}_0$, $\mathscr{D}_0$, and Eth is a pair

a type name for the set of states in C's state machine

$$(\sigma, \varepsilon) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth$$

If $Bool \notin \mathscr{T}_0$ then add it here, and have $\mathscr{D}(Bool) = \mathbb{B}$

where

- $\mathscr{S} = (\mathscr{T}_0 \,\dot\cup\, \{S_{M_C} \mid C \in \mathscr{C}\}, \quad \mathscr{C}_0,$

initial state of C's state machine

$$V_0 \,\dot\cup\, \{\langle stable : Bool, -, \mathit{true}, \emptyset\rangle\}$$
$$\dot\cup\, \{\langle st_C : S_{M_C}, +, s_0, \emptyset\rangle \mid C \in \mathscr{C}\}$$
$$\dot\cup\, \{\langle params_E : E_{0,1}, +, \emptyset, \emptyset\rangle \mid E \in \mathscr{E}_0\},$$

each object can refer to signal instances (at most one at a time) in order to access signal attributes

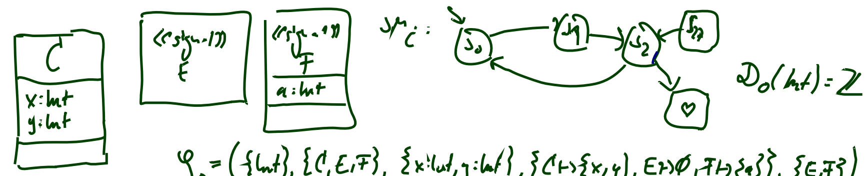
$\in 2^{\mathscr{D}(C)}$ if $r : C_{0,1}$ or $r : C_*$

$$\{C \mapsto atr_0(C)$$
$$\cup\, \{stable, st_C\} \cup \{params_E \mid E \in \mathscr{E}_0\} \mid C \in \mathscr{C}\}, \quad \mathscr{E}_0)$$

- $\mathscr{D} = \mathscr{D}_0 \,\dot\cup\, \{S_{M_C} \mapsto S(M_C) \mid C \in \mathscr{C}\}$, and

states of state machine M_C of C

- $\sigma(u)(r) \cap \mathscr{D}(\mathscr{E}_0) = \emptyset$ for each $u \in \mathrm{dom}(\sigma)$ and $r \in V_{0_{0,1,*}}$ (e.g. $r : C'_{0,1}$)



$\mathcal{D}_0(Int) = \mathbb{Z}$

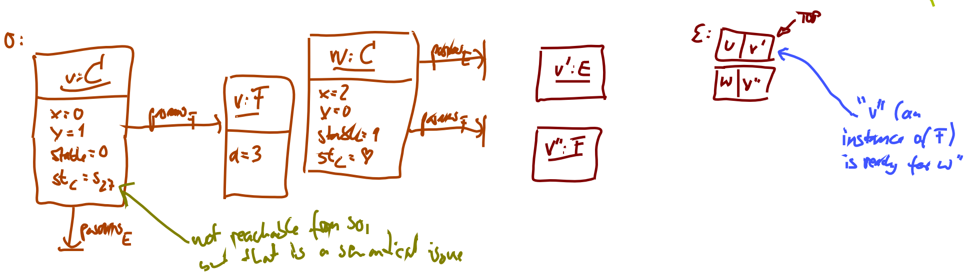
$\mathcal{S}_0 = (\{Int\}, \{C, E, F\}, \{x:Int, y:Int, a:Int\}, \{C \mapsto \{x,y\}, E \mapsto \emptyset, F \mapsto \{a\}\}, \{E, F\})$

$\mathcal{S} = (\{Int, S_{M_C}, Bool\}, \{C, E, F\}, \{x, y, a : Int, stable : Bool, st_C : S_{M_C}, params_E : E_{0,1}, params_F : F_{0,1}\},$
$\{C \mapsto \{x, y, stable, st_C, params_E, params_F\}, E \mapsto \emptyset, F \mapsto \{a\}\}, \{E, F\})$

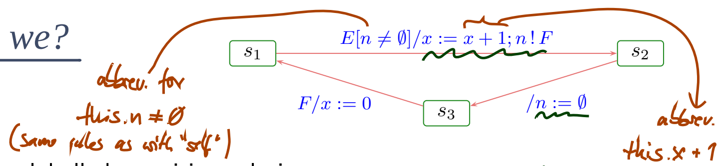
$\mathcal{D}(Int) = \mathcal{D}_0(Int)$

$\mathcal{D}(S_{M_C}) = \{s_0, s_1, s_2, \heartsuit, s_{23}\}$

if Eth is shared FIFO queue



Where are we?



- **Wanted**: a labelled transition relation

here

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$$

on system configuration, labelled with the **consumed** and **sent** events, (σ', ε') being the result (or effect) of **one object** u_x taking a transition of **its** state machine from the current state mach. state $\sigma(u_x)(st_C)$.

- **Have**: system configuration (σ, ε) comprising current state machine state and stability flag for each object, and the ether.

- **Plan**:
  (i) Introduce **transformer** as the semantics of action annotions.
  Intuitively, (σ', ε') is the effect of applying the transformer of the taken transition.
  (ii) Explain how to choose transitions depending on ε and when to stop taking transitions — the **run-to-completion "algorithm"**.

Transformer

because of non-determinism

Definition.
Let $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ the set of system configurations over some $\mathscr{S}_0$, $\mathscr{D}_0$, Eth.

We call a relation

the object "executing" the action — sys. config after

$$t \subseteq \mathscr{D}(\mathscr{C}) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth) \times (\Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth)$$

a (system configuration) **transformer**. system configuration before

- In the following, we assume that each application of a transformer t to some system configuration (σ, ε) for object u_x is associated with a set of **observations**

sender — $\cup\{\bot\}$ — events without identity — special symbols for create/destroy

$$Obs_t[u_x](\sigma, \varepsilon) \in 2^{\mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E}) \times Evs(\mathscr{E} \,\dot{\cup}\, \{*, +\}, \mathscr{D}) \times \mathscr{D}(\mathscr{C})}.$$

signal instance — receiver or destination

- An observation $(u_{src}, u_e, (E, \vec{d}), u_{dst}) \in Obs_t[u_x](\sigma, \varepsilon)$ represents the information that, as a "side effect" of u_x executing t, an event (!) $(E, \vec{d})$ has been sent from u_{src} to u_{dst}.

  Special cases: creation/destruction.

In the following, we consider:

$Act_{\mathscr{S}} ::= \{skip\}$
$\quad \cup \{update(expr_1, v, expr_2) \mid expr_1, expr_2 \in OCLExpr, v \in V\}$
$\quad \cup \{send(expr_1, E, expr_2) \mid expr_1, expr_2 \in OCLExpr, E \in \mathscr{E}\}$
$\quad \cup \{create(C, expr, v) \mid expr \in OCLExpr, C \in \mathscr{C}, v \in V\}$
$\quad \cup \{destroy(expr) \mid expr \in OCLExpr\}$

$Expr_{\mathscr{S}}$: OCL expressions over $\mathscr{S}$

Transformer: Skip

abstract syntax	**concrete syntax**
`skip`	skip
intuitive semantics	
do nothing	
well-typedness	
./.	
semantics	
$t[u_x](\sigma, \varepsilon) = \{(\sigma, \varepsilon)\}$	
observables	
$Obs_{\texttt{skip}}[u_x](\sigma, \varepsilon) = \emptyset$	
(error) conditions	

Transformer: Update

abstract syntax	**concrete syntax**
$\texttt{update}(expr_1, v, expr_2)$	$expr_1 . v := expr_2$
intuitive semantics	
Update attribute v in the object denoted by $expr_1$ to the value denoted by $expr_2$.	
well-typedness	
$expr_1 : \tau_C$ and $v : \tau \in atr(C)$; $expr_2 : \tau$; $expr_1, expr_2$ obey visibility and navigability	
semantics	
$t_{\texttt{update}(expr_1, v, expr_2)}[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon)\}$ where $\sigma' = \sigma[u \mapsto \sigma(u)[v \mapsto I[\![expr_2]\!](\sigma, \beta)]]$ with $u = I[\![expr_1]\!](\sigma, \beta)$, $\beta = \{\mathsf{this} \mapsto u_x\}$.	
observables	
$Obs_{\texttt{update}(expr_1, v, expr_2)}[u_x] = \emptyset$	
(error) conditions	
Not defined if $I[\![expr_1]\!](\sigma, \beta)$ or $I[\![expr_2]\!](\sigma, \beta)$ not defined.	

Update Transformer Example

other example: $\underbrace{\text{this.n}}_{expr_1}.\underbrace{x}_{v} := x+1;$

$\mathcal{SM}_C$:

$s_1 \xrightarrow{\overbrace{/x := x+1}^{}} s_2$

$\underbrace{\text{this}}_{expr_1}.x := \underbrace{x+1}_{expr_2}$

$$\texttt{update}(expr_1, v, expr_2)$$

$$t_{\texttt{update}(expr_1, v, expr_2)}[u_x](\sigma, \varepsilon) = (\sigma[u \mapsto \sigma(u)[v \mapsto I[\![expr_2]\!](\sigma, \beta)]], \varepsilon),$$
$$u = I[\![expr_1]\!](\sigma, \beta)$$

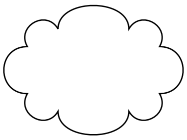
σ:

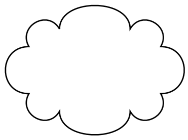
$\underline{u_1 : C}$
$x = 4$
$y = 0$

$\underline{u_1 : C}$
$x = 5$
$y = 0$

$:\sigma'$

ε:





$:\varepsilon'$

Transformer: Send

abstract syntax

$\texttt{send}(E(expr_1, ..., expr_n), expr_{dst})$

concrete syntax

$expr_{dst} \; ! \; E(expr_1, ..., expr_n)$

intuitive semantics

Object $u_x : C$ sends event E to object $expr_{dst}$, i.e. create a fresh signal instance, fill in its attributes, and place it in the ether.

well-typedness

don't send to signal instance

$expr_{dst} : \tau_D$, $C, D \in \mathscr{C} \setminus \mathscr{E}$; $E \in \mathscr{E}$;

$atr(E) = \{v_1 : \tau_1, \ldots, v_n : \tau_n\}$; $expr_i : \tau_i$, $1 \le i \le n$;

all expressions obey visibility and navigability in C

semantics

$$t_{\texttt{send}(E(expr_1, ..., expr_n), expr_{dst})}[u_x](\sigma, \varepsilon) \ni (\sigma', \varepsilon')$$

the new signal instance

where $\sigma' = \sigma \,\dot\cup\, \{u \mapsto \{v_i \mapsto d_i \mid 1 \le i \le n\}\}$; $\varepsilon' = \varepsilon \oplus (u_{dst}, u)$;
if $u_{dst} = I[\![expr_{dst}]\!](\sigma, \beta) \in \text{dom}(\sigma)$; $d_i = I[\![expr_i]\!](\sigma, \beta)$ for $1 \le i \le n$;
$u \in \mathscr{D}(E)$ a fresh identity, i.e. $u \notin \text{dom}(\sigma)$,

our choice – we could also consider it to be an error

and where $(\sigma', \varepsilon') = (\sigma, \varepsilon)$ if $u_{dst} \notin \text{dom}(\sigma)$; $\beta = \{\text{this} \mapsto u_x\}$.

observables

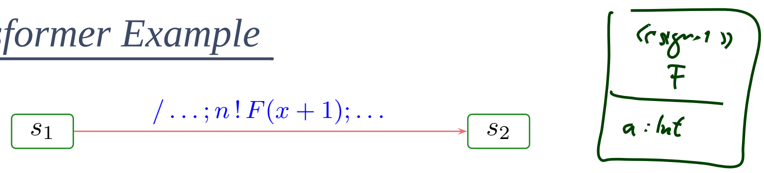
$$Obs_{\texttt{send}}[u_x] = \{(u_x, u, \underbrace{(E, d_1, \ldots, d_n)}_{\text{an event}}, u_{dst})\}$$

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined for any $expr \in \{expr_{dst}, expr_1, \ldots, expr_n\}$

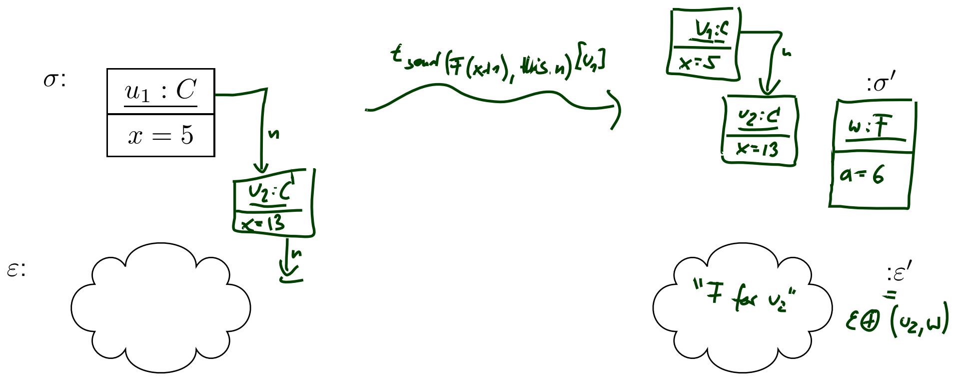
Send Transformer Example

$\mathcal{SM}_C$:



$$\mathtt{send}(E(expr_1, ..., expr_n), expr_{dst})$$

$$t_{\mathtt{send}(\cancel{expr_{src}}, E(expr_1,...,expr_n), expr_{dst})}[u_x](\sigma, \varepsilon) = \ldots$$



Transformer: Create

(*) so NOT: x := (new C).x + (new C).y

if needed:
tmp1 := new C;
tmp2 := new C;
x := tmp1.x + tmp2.y
tmp1 := NULL;
tmp2 := NULL;

abstract syntax

$\mathtt{create}(C, expr, v)$

concrete syntax

expr.v := new C

intuitive semantics

Create an object of class C and assign it to attribute v of the object denoted by expression $expr$.

well-typedness

$expr : \tau_D$, $v \in atr(D)$, $atr(C) = \{\langle v_1 : \tau_1, expr_i^0 \rangle \mid 1 \le i \le n\}$

semantics

...

observables

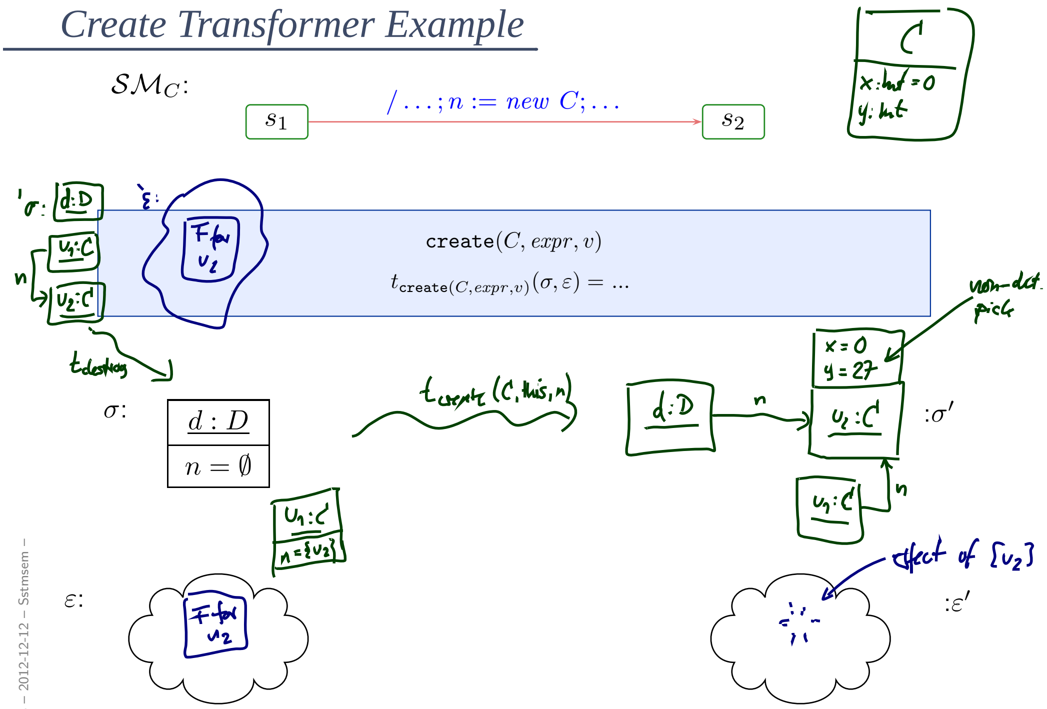
...

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined.

- We use an "and assign"-action for simplicity — it doesn't add or remove expressive power, but moving creation to the expression language raises all kinds of other problems such as order of evaluation (and thus creation). (*)
- Also for simplicity: no parameters to construction ($\sim$ parameters of constructor). Adding them is straightforward (but somewhat tedious).

Create Transformer Example



How To Choose New Identities?

- **Re-use**: choose any identity that is not alive **now**, i.e. not in $\mathrm{dom}(\sigma)$.
  - Doesn't depend on history.
  - May "undangle" dangling references – may happen on some platforms.
- **Fresh**: choose any identity that has not been alive **ever**, i.e. not in $\mathrm{dom}(\sigma)$ and any predecessor in current run.
  - Depends on history.
  - Dangling references remain dangling – could mask "dirty" effects of platform.

Transformer: Create

abstract syntax | **concrete syntax**

create$(C, expr, v)$

intuitive semantics

Create an object of class C and assign it to attribute v of the object denoted by expression $expr$.

well-typedness

$expr : \tau_D,\ v \in atr(D),\ atr(C) = \{\langle v_1 : \tau_1, expr_i^0\rangle \mid 1 \le i \le n\}$, $C \notin \mathscr{E}$

semantics

updating v in u_0 in σ

$$((\sigma, \varepsilon), (\sigma', \varepsilon')) \in t$$

add new object to σ

iff $\sigma' = \sigma[u_0 \mapsto \sigma(u_0)[v \mapsto u]] \cup \{u \mapsto \{v_i \mapsto d_i \mid 1 \le i \le n\}\}$,

cleanup operation

$\varepsilon' = [u](\varepsilon)$; $u \in \mathscr{D}(C)$ fresh, i.e. $u \notin \mathrm{dom}(\sigma)$;

$u_0 = I[\![expr]\!](\sigma, \beta)$; $d_i = I[\![expr_i^0]\!](\sigma, \beta)$ if $expr_i^0 \neq$ '' and arbitrary value from $\mathscr{D}(\tau_i)$ otherwise; $\beta = \{this \mapsto u_x\}$.

observables

$$Obs_{\texttt{create}}[u_x] = \{(u_x, \perp, (*, \emptyset), u)\}$$

(error) conditions

$I[\![expr]\!](\sigma)$ not defined.

Transformer: Destroy

abstract syntax | **concrete syntax**

destroy$(expr)$ | delete expr

intuitive semantics

Destroy the object denoted by expression $expr$.

well-typedness

$expr : \tau_C,\ C \in \mathscr{C}$

semantics

$\ldots$

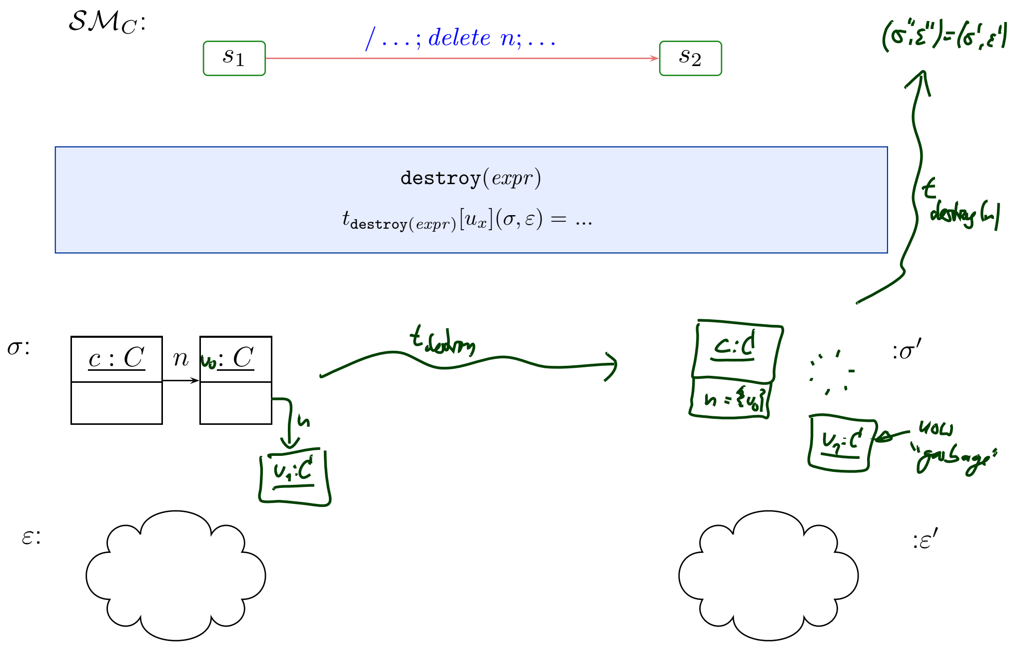
observables

$$Obs_{\texttt{destroy}}[u_x] = \{(u_x, \perp, (+, \emptyset), u)\}$$

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined.

Destroy Transformer Example



What to Do With the Remaining Objects?

Assume object u_0 is destroyed...

- object u_1 may still refer to it via association r:
  - allow dangling references?
  - or remove u_0 from $\sigma(u_1)(r)$?
- object u_0 may have been the last one linking to object u_2:
  - leave u_2 alone?
  - or remove u_2 also?
- Plus: (temporal extensions of) OCL may have dangling references.

Our choice: Dangling references and no garbage collection!
This is in line with "expect the worst", because there are target platforms which don't provide garbage collection — and models shall (in general) be correct without assumptions on target platform.

But: the more "dirty" effects we see in the model, the more expensive it often is to analyse. Valid proposal for simple analysis: monotone frame semantics, no destruction at all.

Transformer: Destroy

abstract syntax **concrete syntax**

$\mathtt{destroy}(expr)$

intuitive semantics

Destroy the object denoted by expression $expr$.

well-typedness

$$expr : \tau_C,\ C \in \mathscr{C}$$

semantics

$$t[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon)\}$$

function restriction

where $\sigma' = \sigma|_{\mathrm{dom}(\sigma)\setminus\{u\}}$ with $u = I[\![expr]\!](\sigma, \beta)$.

observables

$$Obs_{\mathtt{destroy}}[u_x] = \{(u_x, \bot, (+, \emptyset), u)\}$$

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined.

Sequential Composition of Transformers

- **Sequential composition** $t_1 \circ t_2$ of transformers t_1 and t_2 is canonically defined as

$$(t_2 \circ t_1)[u_x](\sigma, \varepsilon) = t_2[u_x](t_1[u_x](\sigma, \varepsilon))$$

with observation

$$Obs_{(t_2 \circ t_1)}[u_x](\sigma, \varepsilon) = Obs_{t_1}[u_x](\sigma, \varepsilon) \cup Obs_{t_2}[u_x](t_1(\sigma, \varepsilon)).$$

- **Clear**: not defined if one the two intermediate "micro steps" is not defined.

$x := x+1;\ n.y := 27;\ n!F$

$t_{update}(t_{update}(t_{send}[\cdot](\sigma, \varepsilon)))$

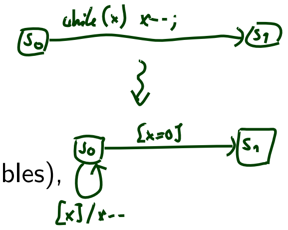
Transformers And Denotational Semantics

Observation: our transformers are in principle the **denotational semantics** of the actions/action sequences. The trivial case, to be precise.

Note: with the previous examples, we can capture

- empty statements, skips,
- assignments,
- conditionals (by normalisation and auxiliary variables),
- create/destroy,

but not **possibly diverging loops**.



Our (Simple) Approach: if the action language is, e.g. Java,
then (**syntactically**) forbid loops and calls of recursive functions.

Other Approach: use full blown denotational semantics.

No show-stopper, because loops in the action annotation can be converted into transition cycles in the state machine.

Run-to-completion Step

Transition Relation, Computation

Definition. Let A be a set of **actions** and S a (not necessarily finite) set of of **states**.

We call

$$\rightarrow \subseteq S \times A \times S$$

a (labelled) **transition relation**.

Let $S_0 \subseteq S$ be a set of **initial states**. A sequence

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

with $s_i \in S$, $a_i \in A$ is called **computation** of the **labelled transition system** $(S, \rightarrow, S_0)$ if and only if

- **initiation**: $s_0 \in S_0$
- **consecution**: $(s_i, a_i, s_{i+1}) \in \rightarrow$ for $i \in \mathbb{N}_0$.

Note: for simplicity, we only consider infinite runs.

Active vs. Passive Classes/Objects

- **Note**: From now on, assume that all classes are **active** for simplicity.

  We'll later briefly discuss the Rhapsody framework which proposes a way how to integrate non-active objects.

- **Note**: The following RTC "algorithm" follows [Harel and Gery, 1997] (i.e. the one realised by the Rhapsody code generation) where the standard is ambiguous or leaves choices.

From Core State Machines to LTS

Definition. Let $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \mathscr{E})$ be a signature with signals (all classes **active**), $\mathscr{D}_0$ a structure of $\mathscr{S}_0$, and $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}_0$ and $\mathscr{D}_0$. Assume there is one core state machine M_C per class $C \in \mathscr{C}$.

We say, the state machines induce the following labelled transition relation on states $S := (\Sigma_{\mathscr{S}}^{\mathscr{D}} \,\dot\cup\, \{\#\} \times Eth)$ with actions $A := \left(2^{\mathscr{D}(\mathscr{C})\times(\mathscr{D}(\mathscr{E})\,\dot\cup\,\{\perp\})\times Evs(\mathscr{E},\mathscr{D})\times\mathscr{D}(\mathscr{C})}\right)^2$:

$\mathscr{D}(\mathscr{C})\times$

- $(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$
  if and only if
  (i) an event with destination u is discarded,
  (ii) an event is dispatched to u, i.e. stable object processes an event, or
  (iii) run-to-completion processing by u commences,
  i.e. object u is not stable and continues to process an event,
  (iv) the environment interacts with object u,
- $s \xrightarrow{(cons, \emptyset)} \#$ "error"
  if and only if
  (v) $s = \#$ and $cons = \emptyset$, or an error condition occurs during consumption of $cons$.

(i) Discarding An Event

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$$

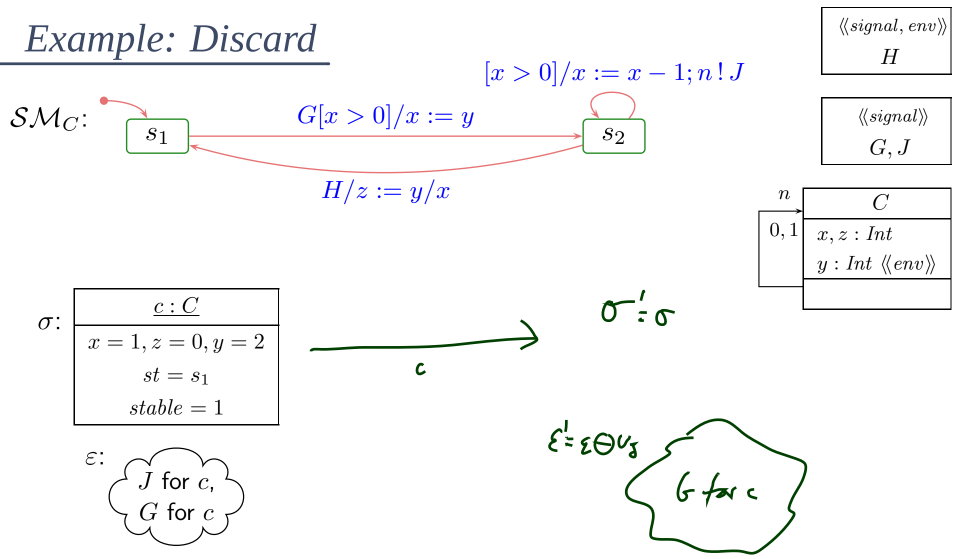
if

- an E-event (instance of signal E) is ready in ε for object u of a class $\mathscr{C}$, i.e. if
$$u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(C) \wedge \exists\, u_E \in \mathscr{D}(\mathscr{E}) : u_E \in ready(\varepsilon, u)$$
- u is stable and in state machine state s, i.e. $\sigma(u)(stable) = 1$ and $\sigma(u)(st) = s$,
- but there is no corresponding transition enabled (all transitions incident with current state of u either have other triggers or the guard is not satisfied)
$$\forall\, (s, F, expr, act, s') \in \longrightarrow (\mathcal{SM}_C) : F \neq E \vee I[\![expr]\!](\tilde\sigma) = 0$$
! with $\tilde\sigma$: see slide 30

and

- the system configuration doesn't change, i.e. $\sigma' = \sigma$
- the event u_E is removed from the ether, i.e.
$$\varepsilon' = \varepsilon \ominus u_E,$$
- consumption of u_E is observed, i.e.
$$cons = \{(\overset{u_E}{u}, (E, \sigma(u_E)))\}, Snd = \emptyset.$$

Example: Discard



- $\exists\, u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(C)$
  $\exists\, u_E \in \mathscr{D}(\mathscr{E}) : u_E \in ready(\varepsilon, u)$
- $\forall\,(s, F, expr, act, s') \in \to (\mathcal{SM}_C)$:
  $F \neq E \vee I[\![expr]\!](\sigma) = 0$
- $\sigma(u)(stable) = 1,\ \sigma(u)(st) = s$,
- $\sigma' = \sigma,\ \varepsilon' = \varepsilon \ominus u_E$
- $cons = \{(u, (E, \sigma(u_E)))\},\ Snd = \emptyset$

(ii) Dispatch

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon') \text{ if}$$

- $u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(C) \wedge \exists\, u_E \in \mathscr{D}(\mathscr{E}) : u_E \in ready(\varepsilon, u)$
- u is stable and in state machine state s, i.e. $\sigma(u)(stable) = 1$, and $\sigma(u)(st) = s$,
- a transition is enabled, i.e.

  $$\exists\,(s, F, expr, act, s') \in \to (\mathcal{SM}_C) : F = E \wedge I[\![expr]\!](\tilde{\sigma}) = 1$$

  where $\tilde{\sigma} = \sigma[u.params_E \mapsto u_E]$.

and

- (σ', ε') results from applying t_{act} to (σ, ε) and removing u_E from the ether, i.e.

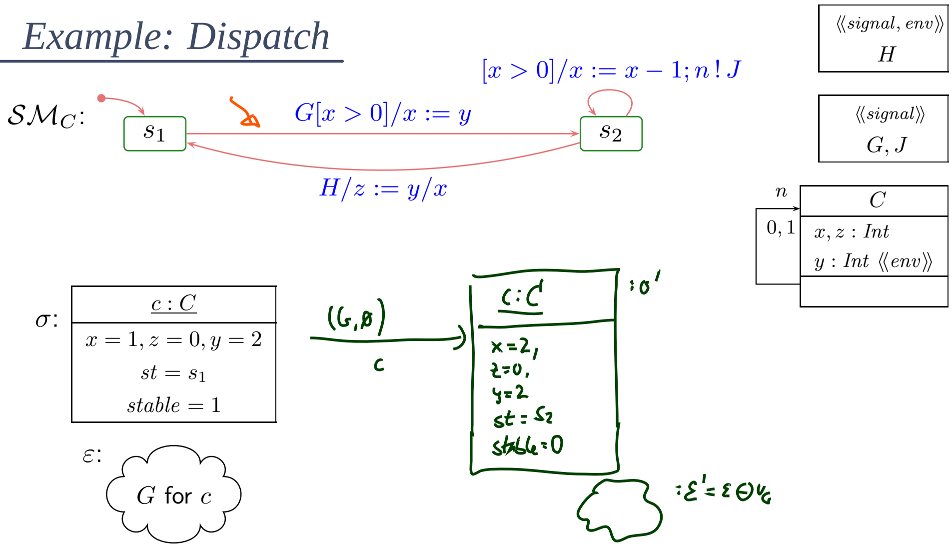
  $$(\sigma'', \varepsilon') = t_{act}(\tilde{\sigma}, \varepsilon \ominus u_E),$$
  $$\sigma' = (\sigma''[u.st \mapsto s', u.stable \mapsto b, u.params_E \mapsto \emptyset])|_{\mathscr{D}(\mathscr{C}) \setminus \{u_E\}}$$

  remove signal instance

  where b **depends**:
  - If u becomes stable in s', then $b = 1$. It **does** become stable if and only if there is no transition **without trigger** enabled for u in (σ', ε').
  - Otherwise $b = 0$.
- Consumption of u_E and the side effects of the action are observed, i.e.

  $$cons = \{(u, (E, \sigma(u_E)))\}, Snd = Obs_{t_{act}}(\tilde{\sigma}, \varepsilon \ominus u_E).$$

Example: Dispatch



- $\exists\, u \in \text{dom}(\sigma) \cap \mathscr{D}(C)$
  $\exists\, u_E \in \mathscr{D}(\mathscr{E}) : u_E \in \mathit{ready}(\varepsilon, u)$
- $\exists\, (s, F, \mathit{expr}, \mathit{act}, s') \in \to (\mathcal{SM}_C)$:
  $F = E \wedge I[\![\mathit{expr}]\!](\tilde{\sigma}) = 1$
- $\tilde{\sigma} = \sigma[u.\mathit{params}_E \mapsto u_E]$.
- $\sigma(u)(\mathit{stable}) = 1,\ \sigma(u)(\mathit{st}) = s,$
- $(\sigma'', \varepsilon') = t_{act}(\tilde{\sigma}, \varepsilon \ominus u_E)$
- $\sigma' = (\sigma''[u.st \mapsto s', u.\mathit{stable} \mapsto b, u.\mathit{params}_E \mapsto \emptyset])|_{\mathscr{D}(\mathscr{C}) \setminus \{u_E\}}$
- $\mathit{cons} = \{(u, (E, \sigma(u_E)))\},\ \mathit{Snd} = \mathit{Obs}_{t_{act}}(\tilde{\sigma}, \varepsilon \ominus u_E)$

(iii) Commence Run-to-Completion

$$(\sigma, \varepsilon) \xrightarrow[u]{(\mathit{cons}, \mathit{Snd})} (\sigma', \varepsilon')$$

if

- there is an unstable object u of a class $\mathscr{C}$, i.e.

$$u \in \text{dom}(\sigma) \cap \mathscr{D}(C) \wedge \sigma(u)(\mathit{stable}) = 0$$

- there is a transition without trigger enabled from the current state $s = \sigma(u)(st)$, i.e.

$$\exists\, (s, \_, \mathit{expr}, \mathit{act}, s') \in \to (\mathcal{SM}_C) : I[\![\mathit{expr}]\!](\sigma) = 1$$

and

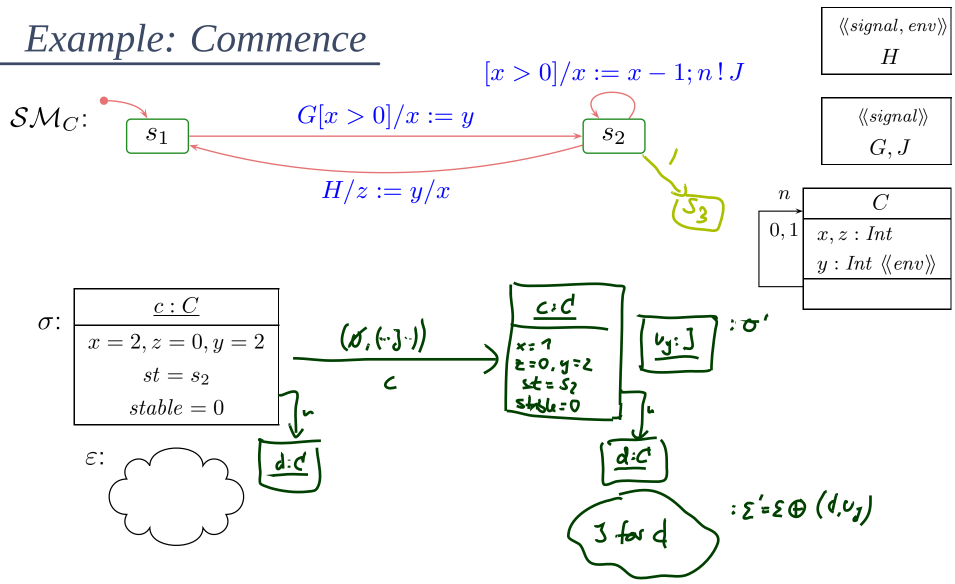
- (σ', ε') results from applying t_{act} to (σ, ε), i.e.

$$(\sigma'', \varepsilon') \in t_{act}[u](\sigma, \varepsilon), \quad \sigma' = \sigma''[u.st \mapsto s', u.\mathit{stable} \mapsto b]$$

  where b **depends** as before.
- Only the side effects of the action are observed, i.e.

$$\mathit{cons} = \emptyset, \mathit{Snd} = \mathit{Obs}_{t_{act}}(\sigma, \varepsilon).$$

Example: Commence



- $\exists\, u \in \text{dom}(\sigma) \cap \mathscr{D}(C) : \sigma(u)(stable) = 0$
- $\exists\, (s, \_, expr, act, s') \in\to (\mathcal{SM}_C) : I[\![expr]\!](\sigma) = 1$
- $\sigma(u)(stable) = 1,\ \sigma(u)(st) = s,$
- $(\sigma'', \varepsilon') = t_{act}(\sigma, \varepsilon)$, $\sigma' = \sigma''[u.st \mapsto s', u.stable \mapsto b]$
- $cons = \emptyset,\ Snd = Obs_{t_{act}}(\sigma, \varepsilon)$

(iv) Environment Interaction

Assume that a set $\mathscr{E}_{env} \subseteq \mathscr{E}$ is designated as **environment** signal/ ~~**events**~~ and a set of attributes $v_{env} \subseteq V$ is designated as **input attributes**.

Then

$$(\sigma, \varepsilon) \xrightarrow[env]{(cons, Snd)} (\sigma', \varepsilon')$$

if

- an environment event $E \in \mathscr{E}_{env}$ is spontaneously sent to an alive object $u \in \mathscr{D}(\sigma)$, i.e.

  $$\sigma' = \sigma\ \dot\cup\ \overbrace{\{u_E \mapsto \{v_i \mapsto d_i \mid 1 \leq i \leq n\}}^{\text{one new instance of } E},\quad \varepsilon' = \varepsilon \oplus u_E$$

  where $u_E \notin \text{dom}(\sigma)$ and $atr(E) = \{v_1, \ldots, v_n\}$.
- Sending of the event is observed, i.e. $cons = \emptyset$, $Snd = \{(env, E(\vec{d}))\}$.

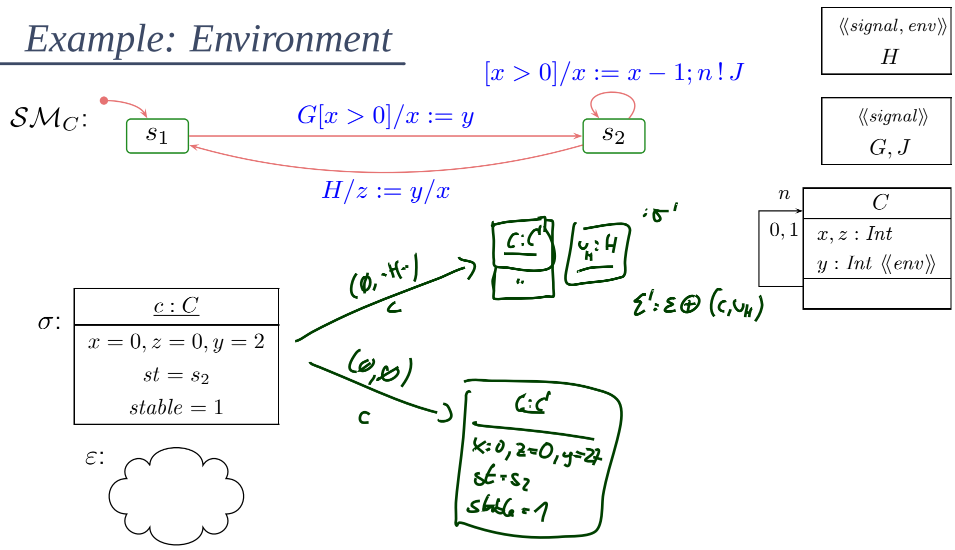
or

- Values of input attributes change freely non-det. in alive objects, i.e.

  $$\forall\, v \in V\ \forall\, u \in \text{dom}(\sigma) : \sigma'(u)(v) \neq \sigma(u)(v) \implies v \in V_{env}.$$

  and no objects appear or disappear, i.e. $\text{dom}(\sigma') = \text{dom}(\sigma)$.
- $\varepsilon' = \varepsilon$.

Example: Environment



- $\sigma' = \sigma \,\dot\cup\, \{u_E \mapsto \{v_i \mapsto d_i \mid 1 \leq i \leq n\}$
- $\varepsilon' = \varepsilon \oplus u_E$ where $u_E \notin \mathrm{dom}(\sigma)$ and $atr(E) = \{v_1, \dots, v_n\}$.
- $u \in \mathrm{dom}(\sigma)$
- $cons = \emptyset$, $Snd = \{(env, E(\vec{d}))\}$.

(v) Error Conditions

$$s \xrightarrow[u]{(cons,Snd)} \#$$

(i) (*)

if, in (ii) or (iii),

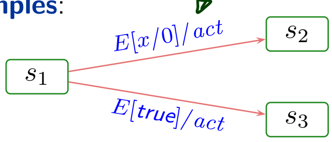
- $I[\![expr]\!]$ is not defined for σ, or
- t_{act} is not defined for (σ, ε), i.e. $t_{act}[u](\sigma,\varepsilon) = \emptyset$

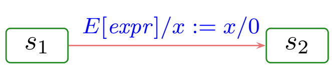
and

- consumption **is observed** according to (i) (*) (ii) or (iii), but $Snd = \emptyset$.

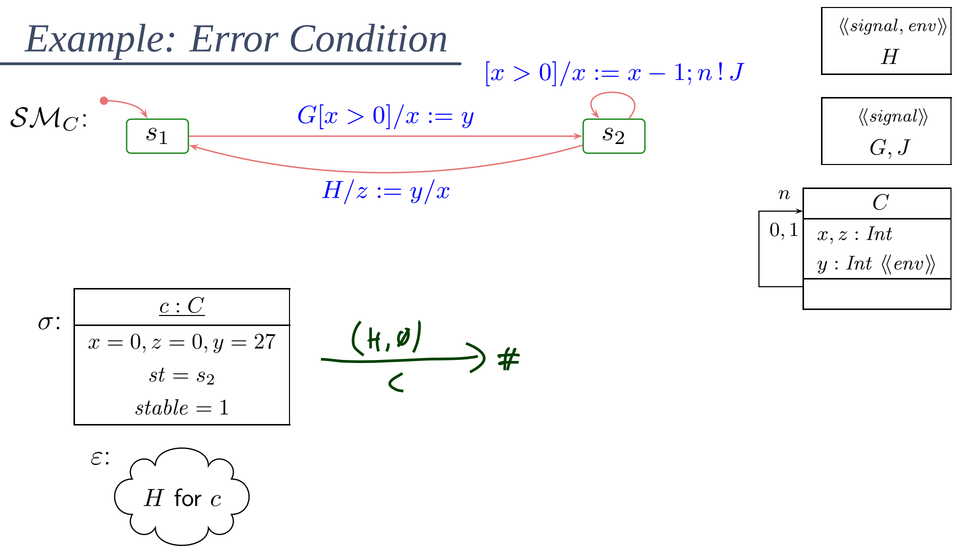
if we add . (*), this is belong to # with F only

Examples:

- 

- 

Example: Error Condition



- $I[\![expr]\!]$ not defined for σ, or
- t_{act} is not defined for (σ, ε)
- consumption according to (ii) or (iii)
- $Snd = \emptyset$

Notions of Steps: The Step

Note: we call one evolution $(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$ a **step**.

Thus in our setting, **a step directly corresponds** to

one object (namely u) takes **a single transition** between regular states.

(We have to extend the concept of "single transition" for hierarchical state machines.)

That is: We're going for an interleaving semantics without true parallelism.

Remark: With only methods (later), the notion of step is not so clear.
For example, consider

- c_1 calls `f()` at c_2, which calls `g()` at c_1 which in turn calls `h()` for c_2.
- Is the completion of `h()` a step?
- Or the completion of `f()`?
- Or doesn't it play a role?

It does play a role, because **constraints**/**invariants** are typically (= by convention) assumed to be evaluated at step boundaries, and sometimes the convention is meant to admit (temporary) violation in between steps.

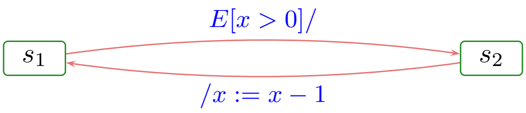
Notions of Steps: The Run-to-Completion Step

What is a **run-to-completion** step...?

- **Intuition**: a maximal sequence of steps, where the first step is a **dispatch** step and all later steps are **commence** steps.
- **Note**: one step corresponds to one transition in the state machine.

  A run-to-completion step is in general not syntacically definable — one transition may be taken multiple times during an RTC-step.

Example:



σ: $\underline{:C}$ | $x = 2$

ε: E for u

References

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[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.