Software Design, Modelling and Analysis in UML

Lecture 16: Hierarchical State Machines II

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Contents & Goals

Last Lecture:

- Hierarchical State Machines Syntax
- Initial and Final State

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this State Machine mean? What happens if I inject this event?
  - Can you please model the following behaviour.
  - What does this **hierarchical** State Machine mean? What **may happen** if I inject this event?
  - What is: AND-State, OR-State, pseudo-state, entry/exit/do, final state, . . .
- **Content:**
  - Composite State Semantics
  - The Rest

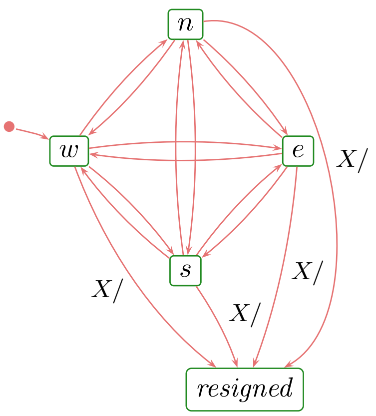
Composite States

(formalisation follows [Damm et al., 2003])

Composite States

- In a sense, composite states are about **abbreviation**, **structuring**, and **avoiding redundancy**.

- Idea: in Tron, for the Player's Statemachine, instead of



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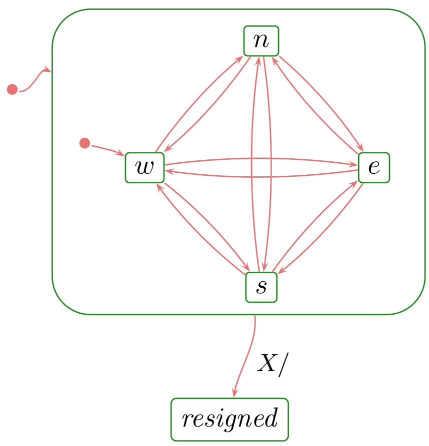
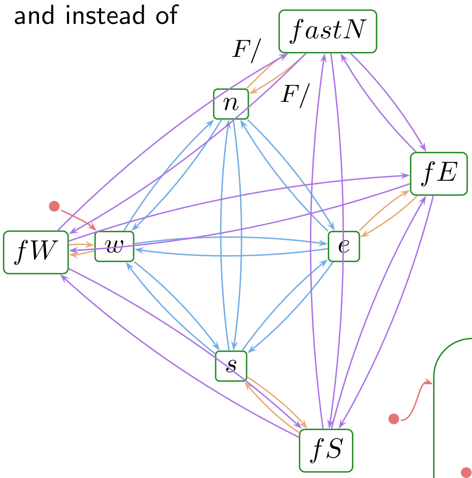


Composite States

and instead of



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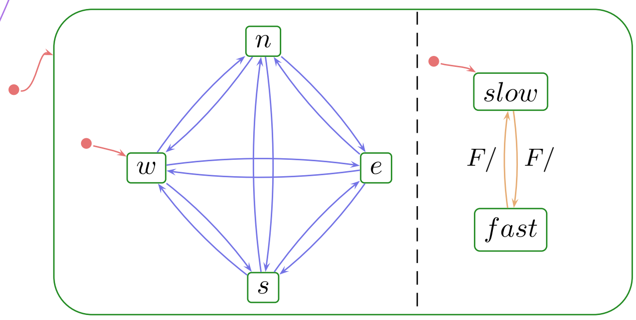
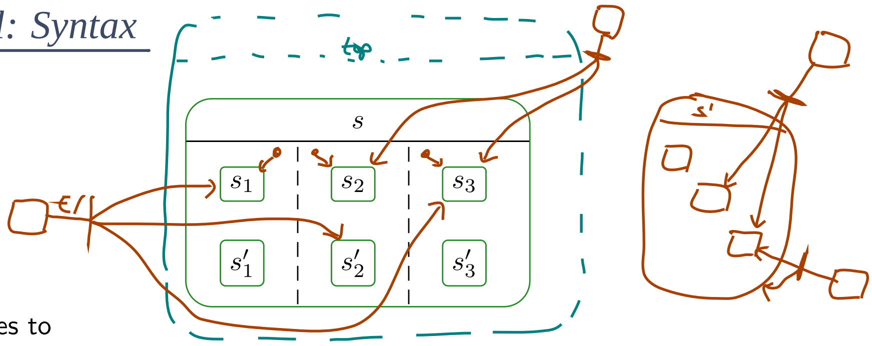


Recall: Syntax

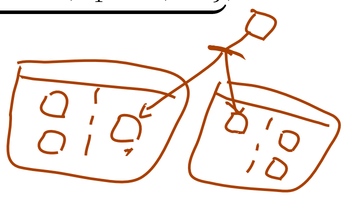


translates to

$$(\underbrace{\{(top, \mathsf{st}), (s, \mathsf{st}), (s_1, \mathsf{st})(s'_1, \mathsf{st})(s_2, \mathsf{st})(s'_2, \mathsf{st})(s_3, \mathsf{st})(s'_3, \mathsf{st})\}}_{S, kind},$$

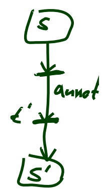
$$\underbrace{\{top \mapsto \{\{s\}\}, s \mapsto \{\{s_1, s'_1\}, \{s_2, s'_2\}, \{s_3, s'_3\}\}, s_1 \mapsto \emptyset, s'_1 \mapsto \emptyset, \dots\}}_{region},$$

$$\to, \psi, annot)$$



Syntax: Fork/Join

SPECIAL CASE:



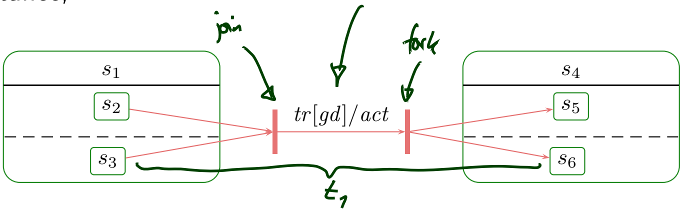
maps to $t' \mapsto (\{s\}, \{s'\})$

- For brevity, we always consider transitions with (possibly) multiple sources and targets, i.e.

$$\psi : (\rightarrow) \to (\underbrace{2^S \setminus \emptyset}_{\text{set of source}}) \times (\underbrace{2^S \setminus \emptyset}_{\text{set of destination states}})$$

- For instance,

here: annotation is between

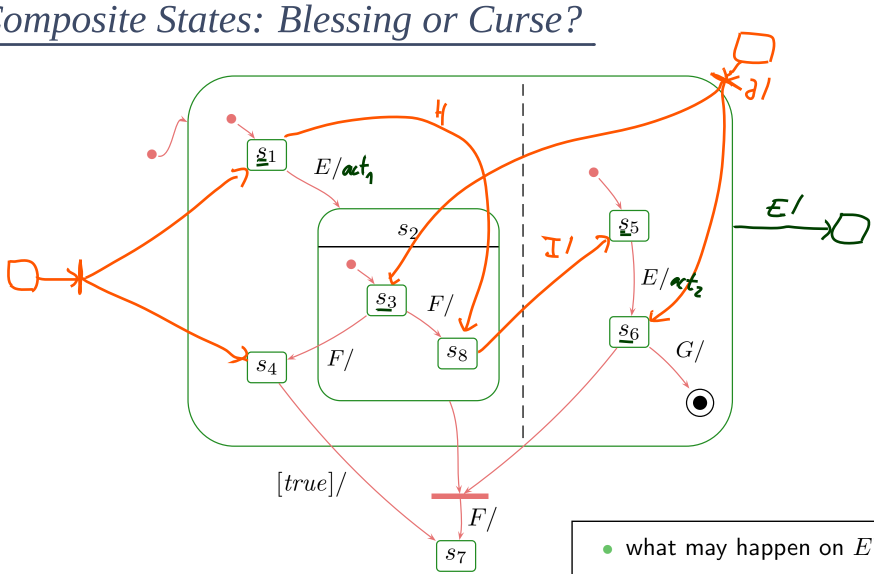


translates to

$$(S, \mathit{kind}, \mathit{region}, \underbrace{\{t_1\}}_{\to}, \underbrace{\{t_1 \mapsto (\{s_2, s_3\}, \{s_5, s_6\})\}}_{\psi}, \underbrace{\{t_1 \mapsto (tr, gd, act)\}}_{annot})$$

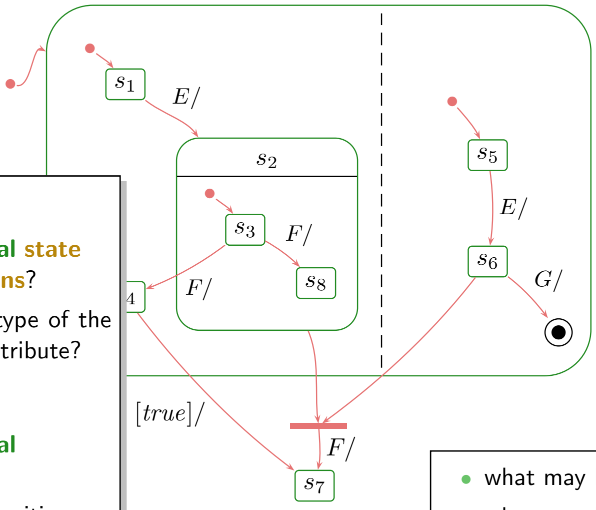
- Naming convention: $\psi(t) = (\underline{\mathit{source}}(t), \underline{\mathit{target}}(t))$.

Composite States: Blessing or Curse?



- what may happen on E?
- what may happen on E, F?
- can E, G kill the object?
- ...

Composite States: Blessing or Curse?



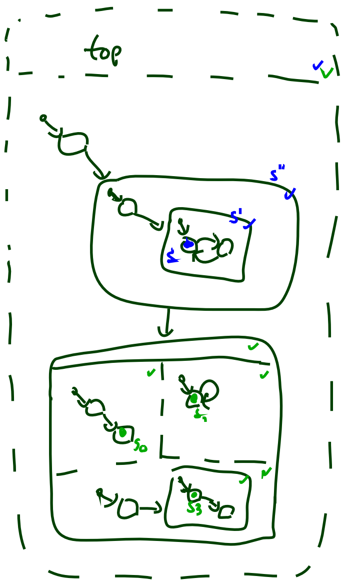
States:

- what are **legal state configurations**?
- what is the type of the implicit st attribute?

Transitions:

- what are **legal** transitions?
- when is a transition enabled?
- what effects do transitions have?

- what may happen on E?
- what may happen on E, F?
- can E, G kill the object?
- ...



$st = \{s\}$

$st = \{s, s', s'', top\}$

$st = \{s_0, s_1, s_2\}$

$st = \{s_0, s_1, s_2, \ldots, top\}$

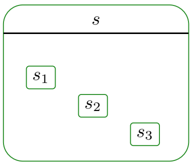
$st = \{s_0, s_2\}$ NO!

State Configuration

- The type of st is from now on **a set of** states, i.e. $st : 2^S$
- A set $S_1 \subseteq S$ is called (**legal**) **state configurations** if and only if
  - $top \in S_1$, and
  - for each state $s \in S_1$, for each non-empty region $\emptyset \neq R \in region(s)$, exactly one (non pseudo-state) child of s (from R) is in S_1, i.e.

$$|\{s_0 \in R \mid kind(s_0) \in \{st, fin\}\} \cap S_1| = 1.$$

- **Examples**:



$S_1 = \{s\}$ NOT LEGAL, top missing

$S_2 = \{top, s\}$ NOT LEGAL, missing child of s

$S_3 = \{top, s, s_1, s_3\}$ NOT LEGAL, too many children of s

$S_4 = \{top, s, s_1\}$ LEGAL

State Configuration

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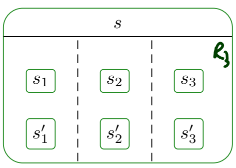

$S_1 = \{top, s_1, s_2', s_3\}$ NOT LEGAL, child of top is missing

$S_2 = \{top, s, s_1, s_1\}$ NOT LEGAL, child of s from R_3 missing

$S_3 = \{top, s, s_1, s_2, s_3\}$

A Partial Order on States

The substate- (or **child-**) relation **induces** a **partial order on states**:

- $top \leq s$, for all $s \in S$,
- $s \leq s'$, for all $s' \in child(s)$,
- transitive, reflexive, antisymmetric,
- $s' \leq s$ and $s'' \leq s$ implies $s' \leq s''$ or $s'' \leq s'$.

$s' \geq s$, $s'' \geq s$ $\qquad$ $s' = s'' \geq s$ $\qquad$ $s'' \geq s' \geq s$ $\qquad$ $s' \geq s'' \geq s$

A Partial Order on States

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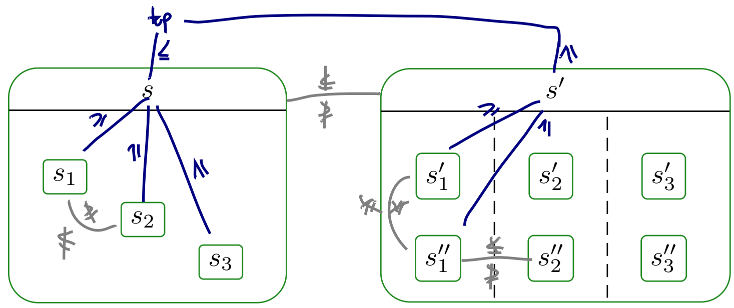


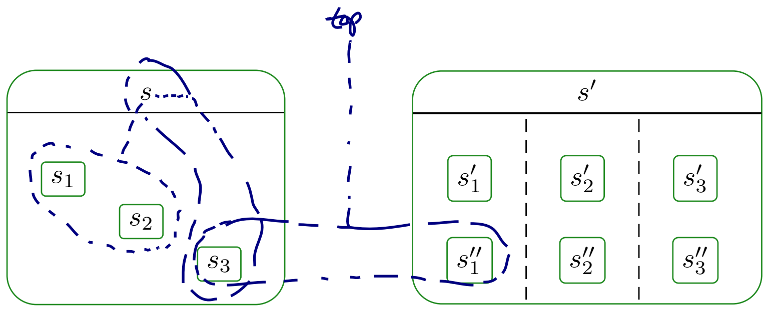
Least Common Ancestor and Ting

misleading name: closest, nearest, innermost common parent

- The **least common ancestor** is the function $lca : 2^S \setminus \{\emptyset\} \to S$ such that
  - The states in S_1 are (transitive) children of $lca(S_1)$, i.e.

$$lca(S_1) \leq s, \text{ for all} s \in S_1 \subseteq S,$$

  - $lca(S_1)$ is minimal, i.e. if $\hat{s} \leq s$ for all $s \in S_1$, then $\hat{s} \leq lca(S_1)$
- **Note**: $lca(S_1)$ exists for all $S_1 \subseteq S$ (last candidate: top).



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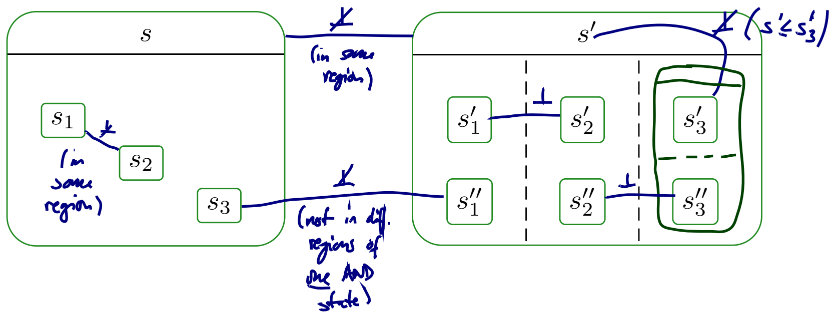
Least Common Ancestor and Ting

- Two states $s_1, s_2 \in S$ are called **orthogonal**, denoted $s_1 \perp s_2$, if and only if
  - they are unordered, i.e. $s_1 \not\leq s_2$ and $s_2 \not\leq s_1$, and
  - they "live" in different regions of an AND-state, i.e.

transitive closure of child

$$\exists\, s, region(s) = \{S_1, \dots, S_n\}\ \exists\, 1 \leq i \neq j \leq n : s_1 \in child^*(S_i) \wedge s_2 \in child^*(S_j),$$



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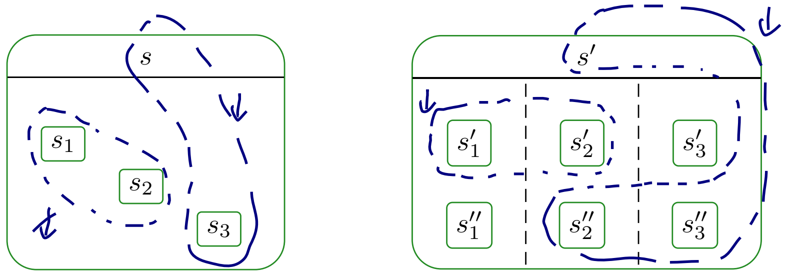
Least Common Ancestor and Ting

- A set of states $S_1 \subseteq S$ is called **consistent**, denoted by $\downarrow S_1$, if and only if for each $s, s' \in S_1$,
  - $s \leq s'$, or
  - $s' \leq s$, or
  - $s \perp s'$.

NOTE: $\forall S_1 \subseteq S \bullet S_1$ is a legal state config. iff S_1 is maximal consistent



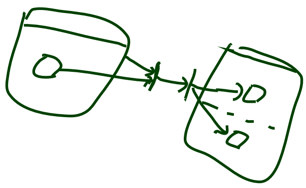
Legal Transitions

A hierarchical state-machine $(S, kind, region, \rightarrow, \psi, annot)$ is called **well-formed** if and only if for all transitions $t \in \rightarrow$,

(i) source and destination are consistent, i.e. $\downarrow source(t)$ and $\downarrow target(t)$,

(ii) source (and destination) states are pairwise orthogonal, i.e.
- forall $s \neq s' \in source(t)$ $(\in target(t))$, $s \perp s'$,

(iii) the top state is neither source nor destination, i.e.
- $top \notin source(t) \cup source(t)$.



- Recall: final states are not sources of transitions.

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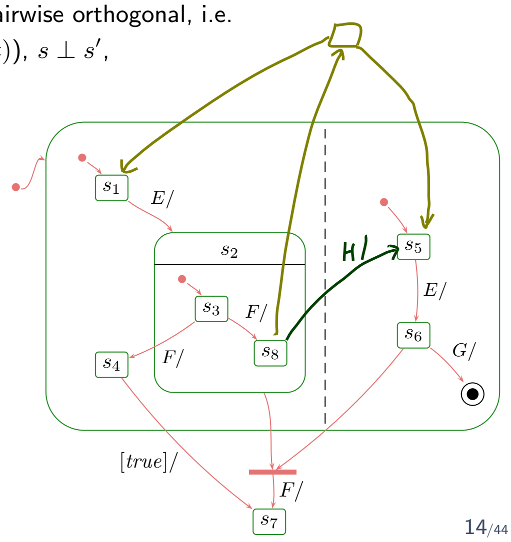
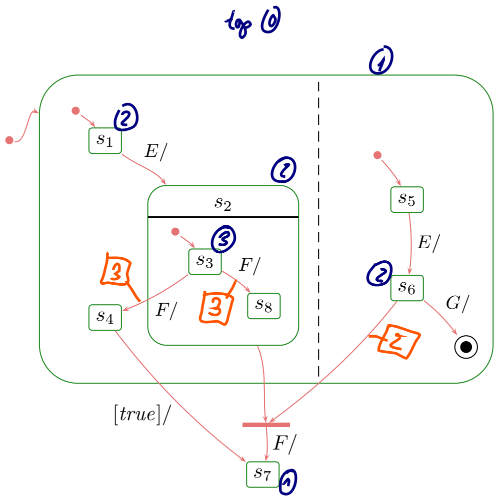
Example:



The Depth of States

- $depth(top) = 0$,
- $depth(s') = depth(s) + 1$, for all $s' \in child(s)$

Example:

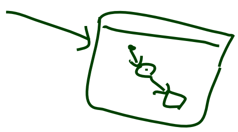


Enabledness in Hierarchical State-Machines

- The **scope** ("set of possibly affected states") of a transition t is the **least common region** of
$$source(t) \cup target(t).$$
- Two transitions t_1, t_2 are called **consistent** if and only if their scopes are orthogonal (i.e. states in scopes pairwise orthogonal).
- The **priority** of transition t is the depth of its innermost source state, i.e.
$$prio(t) := \max\{depth(s) \mid s \in source(t)\}$$
- A set of transitions $T \subseteq \rightarrow$ is **enabled** in an object u if and only if
  - T is consistent,
  - T is maximal wrt. priority,
  - all transitions in T share the same trigger,
  - all guards are satisfied by $\sigma(u)$, and
  - for all $t \in T$, the source states are active, i.e.
$$source(t) \subseteq \sigma(u)(st) \; (\subseteq S).$$

Transitions in Hierarchical State-Machines

- Let T be a set of transitions enabled in u.
- Then $(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$ if



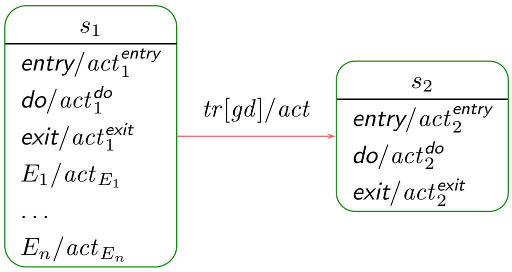
  - $\sigma'(u)(st)$ consists of the target states of T,

    i.e. for simple states the simple states themselves, for composite states the initial states,
  - σ', ε', $cons$, and Snd are the effect of firing each transition $t \in T$ **one by one**, **in any order**, i.e. for each $t \in T$,
    - the exit transformer of all affected states, highest depth first,
    - the transformer of t,
    - the entry transformer of all affected states, lowest depth first.

$\rightsquigarrow$ adjust (2.), (3.), (5.) accordingly.

Entry/Do/Exit Actions, Internal Transitions

Entry/Do/Exit Actions



- In general, with each state $s \in S$ there is associated
  - an **entry**, a **do**, and an **exit** action (default: skip)
  - a possibly empty set of trigger/action pairs called **internal transitions**, (default: empty). $E_1, \dots, E_n \in \mathscr{E}$, 'entry', 'do', 'exit' are reserved names!
- Recall: each action's supposed to have a transformer. Here: $t_{act_1^{entry}}$, $t_{act_1^{exit}}$, ...
- Taking the transition above then amounts to applying

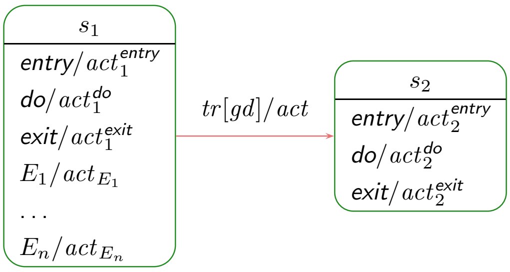
$$t_{act_{s_2}^{entry}} \circ t_{act} \circ t_{act_{s_1}^{exit}}$$

instead of only

$$t_{act}$$

$\rightsquigarrow$ adjust (2.), (3.) accordingly.

Internal Transitions

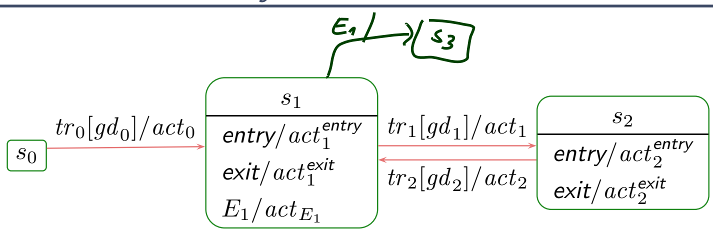


- For **internal transitions**, taking the one for E_1, for instance, still amounts to taking **only** $t_{act_{E_1}}$.
- Intuition: The state is neither left nor entered, so: no exit, no entry.

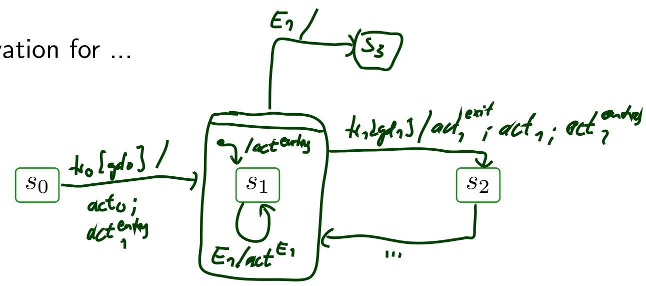
  $\rightsquigarrow$ adjust $(2.)$ accordingly.
- Note: internal transitions also start a run-to-completion step.
- Note: the standard seems not to clarify whether internal transitions have **priority** over regular transitions with the same trigger at the same state.

  Some code generators assume that internal transitions have priority!

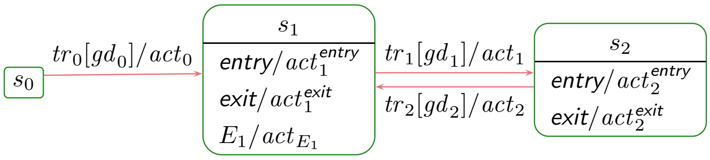
Alternative View: Entry/Exit/Internal as Abbreviations



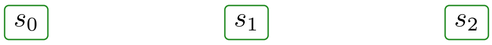
- ... as abbrevation for ...



Alternative View: Entry/Exit/Internal as Abbreviations

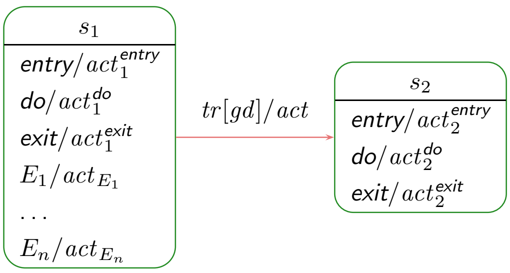


- ... as abbrevation for ...



- That is: Entry/Internal/Exit don't add expressive power to Core State Machines. If internal actions should have priority, s_1 can be embedded into an OR-state (see later).
- Abbreviation may avoid confusion in context of hierarchical states (see later).

Do Actions



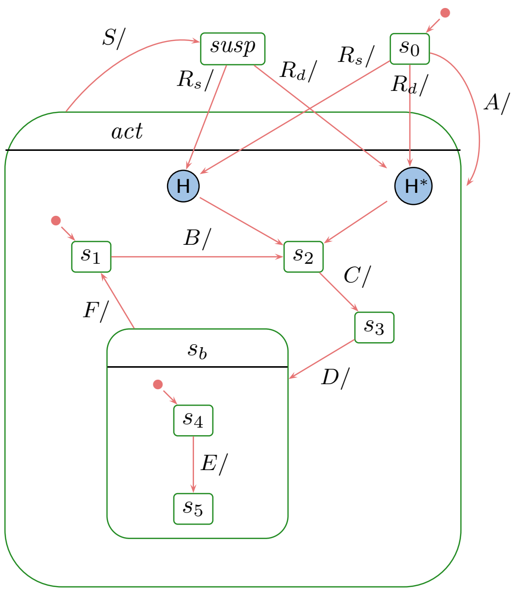
- **Intuition**: after entering a state, start its do-action.
- If the do-action terminates,
  - then the state is considered **completed**,
- otherwise,
  - if the state is left before termination, the do-action is stopped.

- Recall the overall UML State Machine philosophy:

  "An object is either idle or doing a run-to-completion step."
- Now, what is it exactly while the do action is executing...?

The Concept of History, and Other Pseudo-States

History and Deep History: By Example



What happens on...

- R_s?

  s_0, s_2
- R_d?

  s_0, s_2
- A, B, C, S, R_s?

  $s_0, s_1, s_2, s_3, susp, s_3$
- A, B, S, R_d?

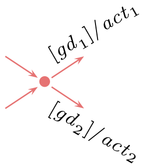
  $s_0, s_1, s_2, s_3, susp, s_3$
- A, B, C, D, E, R_s?

  $s_0, s_1, s_2, s_3, s_4, s_5, susp, s_3$
- A, B, C, D, R_d?

  $s_0, s_1, s_2, s_3, s_4, s_5, susp, s_5$

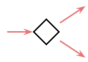
Junction and Choice

- Junction (**"static conditional branch"**):



  - **good**: abbreviation
  - unfolds to so many similar transitions with different guards, the unfolded transitions are then checked for enabledness
  - at best, start with trigger, branch into conditions, then apply actions

- Choice: (**"dynamic conditional branch"**)



  - **evil**: may get stuck
  - enters the transition **without knowing** whether there's an enabled path
  - at best, use "else" and convince yourself that it cannot get stuck
  - maybe even better: **avoid**

Note: not so sure about naming and symbols, e.g.,
I'd guessed it was just the other way round...

Entry and Exit Point, Submachine State, Terminate

- Hierarchical states can be **"folded"** for readability.
  (but: this can also hinder readability.)
- Can even be taken from a different state-machine for re-use. $\boxed{S : s}$
- **Entry/exit points** ○, ⊗
  - Provide connection points for finer integration into the current level, than just via initial state.
  - Semantically a bit tricky:
    - **First** the exit action of the exiting state,
    - **then** the actions of the transition,
    - **then** the entry actions of the entered state,
    - **then** action of the transition from the entry point to an internal state,
    - and **then** that internal state's entry action.
- **Terminate Pseudo-State** ×
  - When a terminate pseudo-state is reached, the object taking the transition is immediately killed.

References

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