Contents & Goals

Last Lecture:
- LSC concrete syntax.
- LSC intuitive semantics.

This Lecture:
- Educational Objectives: Capabilities for following tasks/questions.
  - What does this LSC mean?
  - Are this UML model's state machines consistent with the interactions?
  - Please provide a UML model which is consistent with this LSC.
  - What is: activation, hot/cold condition, pre-chart, etc.?

- Content:
  - Symbolic Büchi Automata (TBA) and its (accepted) language.
  - Words of a model.
  - LSC abstract syntax.
  - LSC formal semantics.
Excursus: Symbolic Büchi Automata (over Signature)
Symbolic Büchi Automata

Definition. A **Symbolic Büchi Automaton** (TBA) is a tuple

\[ B = (\mathit{Expr}_B(X), X, Q, q_{\text{ini}}, \rightarrow, Q_F) \]

where

- \( X \) is a set of logical variables,
- \( \mathit{Expr}_B(X) \) is a set of Boolean expressions over \( X \),
- \( Q \) is a finite set of states,
- \( q_{\text{ini}} \in Q \) is the initial state,
- \( \rightarrow \subseteq Q \times \mathit{Expr}_B(X) \times Q \) is the transition relation.

Transitions \((q, \psi, q')\) from \( q \) to \( q' \) are labelled with an expression \( \psi \in \mathit{Expr}_B(X) \).
- \( Q_F \subseteq Q \) is the set of fair (or accepting) states.

\[ L(A) = \{ 0 \} \]
\[ L(W) = \{ 0,1 \}^* \]
\[ L(A) = \{ 00 \}^* \]

\[ \sum_1 = \{ 0,0,0 \} \]
\[ \omega \in \sum_1^\omega \text{ infinite sequence of labels} \]
\[ \omega = 0101001 \in L(B) \]
\[ \omega = 010^\omega \in L(B) \]

\[ L_B(\Sigma) = \{ 0 \} \]
\[ L_B(\Sigma) = \{ 0 \}^* \]
\[ L_B(\Sigma) = \{ 0 \}^* \]
\[ \Sigma = \{ 1,2 \} \rightarrow 2 \]
Definition. Let $X$ be a set of logical variables and let $\text{Expr}_B(X)$ be a set of Boolean expressions over $X$.

A set $(\Sigma, \cdot \models \cdot)$ is called an alphabet for $\text{Expr}_B(X)$ if and only if

- for each $\sigma \in \Sigma$,
- for each expression $\text{expr} \in \text{Expr}_B$, and
- for each valuation $\beta : X \rightarrow \mathcal{D}(X)$ of logical variables to domain $\mathcal{D}(X)$,

either $\sigma \models_\beta \text{expr}$ or $\sigma \not\models_\beta \text{expr}$.

An infinite sequence

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^\omega$$

over $(\Sigma, \cdot \models \cdot)$ is called word for $\text{Expr}_B(X)$. 

\[ Q = \{ q_1, q_2, \ldots, q_7 \} \]
\[ Q_F = \{ q_3 \} \]
\[ X = \{ f, x, y \} \]
\[ \text{Expr}_B(X) = \{ a(x, y), b(x, y), \neg \text{expr}, c(y, x), d(y, z), f(y, x) \} \]

\[ Q = \{ q_1, q_2, \ldots, q_7 \} \]
\[ F = \{ q_3 \} \]
\[ X = \{ f, x, y \} \]
\[ \text{Expr}_B(X) = \{ a(x, y), b(x, y), \neg \text{expr}, c(y, x), d(y, z), f(y, x) \} \]
**Word Example**

Run of TBA over Word

**Definition.** Let $B = (Expr_B(X), X, Q, q_{ini}, \rightarrow, Q_F)$ be a TBA and

\[ w = \sigma_1, \sigma_2, \sigma_3, \ldots \in \sum^\omega \]

a word for $Expr_B(X)$.

An infinite sequence

\[ \rho = q_0, q_1, q_2, \ldots \in Q^\omega \]

is called run of $B$ over $w$ under valuation $\beta : X \rightarrow \mathcal{P}(X)$ if and only if

- $q_0 = q_{ini}$,
- for each $i \in \mathbb{N}_0$ there is a transition $(q_i, \psi_i, q_{i+1}) \in \rightarrow$ of $B$ such that $\sigma_i \models_\beta \psi_i$. 

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**See Slide 5a**
The Language of a TBA

Definition.
We say $B$ accepts word $w$ (under $\beta$) if and only if $B$ has a run
$$\varrho = (q_i)_{i \in \mathbb{N}_0}$$
over $w$ such that fair (or accepting) states are visited infinitely often by $\varrho$, i.e., such that
$$\forall i \in \mathbb{N}_0 \exists j > i : q_j \in Q_F.$$

We call the set $L_\beta(B) \subseteq \Sigma^\omega$ of words for $Expr_B(X)$ that are accepted by $B$ the language of $B$. 
Language of the Example TBA

\[ L_\beta(B) \] consists of the words

\[ w = (\sigma_i)_{i \in \mathbb{N}_0} \]

where for \( 0 \leq n < m < k < \ell \) we have

- for \( 0 \leq i < n \), \( \sigma_i \notin \beta_x(y) \)
- for \( n < i < m \), \( \sigma_i \notin \beta_y(x, y) \)
- for \( m < i < k \), \( \sigma_i \notin \beta_y(x) \)
- for \( k < i < \ell \), \( \sigma_i \notin \beta_y(x, y) \)
- \( \ldots \)

Course Map
Words over Signature

Definition. Let $\mathcal{S} = (\mathcal{F}, \mathcal{G}, V, atr, \mathcal{E})$ be a signature and $\mathcal{D}$ a structure of $\mathcal{S}$. A **word** over $\mathcal{S}$ and $\mathcal{D}$ is an infinite sequence

$(\sigma_i, cons_i, Snd_i)_{i \in \mathbb{N}_0}$

$\in \left( \Sigma_{\mathcal{D}} \times 2^{\mathcal{E}(\mathcal{G}) \times \text{Ev}(\mathcal{S}, \mathcal{D}) \times \mathcal{D}(\mathcal{G})} \times 2^{\mathcal{E}(\mathcal{G}) \times \text{Ev}(\mathcal{S}, \mathcal{D}) \times \mathcal{D}(\mathcal{G})} \right)$. 

Back to Main Track: Language of a Model
The Language of a Model

Recall: A UML model $\mathcal{M} = (C, D, \mathbb{M}, \mathbb{D})$ and a structure $\mathbb{D}$ denotes a set $[\mathcal{M}]$ of (initial and consecutive) computations of the form

$$(\sigma_0, \varepsilon_0) \xrightarrow{a_0} (\sigma_1, \varepsilon_1) \xrightarrow{a_1} (\sigma_2, \varepsilon_2) \xrightarrow{a_2} \ldots$$

where

$$a_i = (\text{cons}_i, \text{Snd}_i, u_i) \in \mathbb{D}(\mathbb{D}) \times \mathbb{D}(\mathbb{D}) \times \mathbb{D}(\mathbb{D}) \times \mathbb{D}(\mathbb{D}) \times \mathbb{D}(\mathbb{D}).$$

For the connection between models and interactions, we disregard the configuration of the ether and who made the step, and define as follows:

Definition. Let $\mathcal{M} = (C, D, \mathbb{M}, \mathbb{D})$ be a UML model and $\mathbb{D}$ a structure. Then

$$\mathcal{L}(\mathcal{M}) := \{ (\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma \times \tilde{A})^\omega \mid \exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow{u_0} (\sigma_1, \varepsilon_1) \cdots \in [\mathcal{M}] \}$$

is the language of $\mathcal{M}$.

Example: The Language of a Model

$$\mathcal{L}(\mathcal{M}) := \{ (\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma \times \tilde{A})^\omega \mid \exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow{u_0} (\sigma_1, \varepsilon_1) \cdots \in [\mathcal{M}] \}$$
Signal and Attribute Expressions

Let \( \mathcal{S} = (\mathcal{P}, \mathcal{E}, V, \text{atr}, \mathcal{E}) \) be a signature and \( X \) a set of logical variables.

The signal and attribute expressions \( \text{Expr}_{\mathcal{S}}(\mathcal{E}, X) \) are defined by the grammar:

\[
\psi ::= \text{true} \mid \text{expr} \mid E_{x,y}^1 \mid E_{x,y}^2 \mid \neg \psi \mid \psi_1 \lor \psi_2,
\]

where \( \text{expr} : \text{Bool} \in \text{Expr}_{\mathcal{S}}, E \in \mathcal{E}, x, y \in X \).

Satisfaction of Signal and Attribute Expressions

Let \( (\sigma, \text{cons}, \text{Snd}) \in \Sigma_{\mathcal{S}} \times \hat{\mathcal{A}} \) be a triple consisting of system state, consume set, and send set.

Let \( \beta : X \rightarrow \mathcal{S}(\mathcal{E}) \) be a valuation of the logical variables.

Then

\[
\begin{align*}
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} \text{true} \\
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} \neg \psi \text{ if and only if not } (\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi \\
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} \psi_1 \lor \psi_2 \text{ if and only if } \\
& \quad (\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi_1 \text{ or } (\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi_2 \\
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} \text{expr} \text{ if and only if } I[\text{expr}](\sigma, \beta) = 1 \\
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} E_{x,y}^1 \text{ if and only if } \exists \vec{d} \cdot (\beta(x), (E, \vec{d}), \beta(y)) \in \text{Snd} \\
(\sigma, \text{cons}, \text{Snd}) & \models_{\beta} E_{x,y}^2 \text{ if and only if } \exists \vec{d} \cdot (\beta(x), (E, \vec{d}), \beta(y)) \in \text{cons}
\end{align*}
\]
Let \((\sigma, \text{cons}, \text{Snd}) \in \Sigma_{\mathcal{I}} \times \hat{A}\) be a triple consisting of system state, consume set, and send set.

Let \(\beta : X \rightarrow \mathcal{P}(\mathcal{E})\) be a valuation of the logical variables.

Then

\(\models_{\beta} \text{true}\)

\(\models_{\beta} \neg \psi\) if and only if not \(\models_{\beta} \psi\)

\(\models_{\beta} \psi_1 \lor \psi_2\) if and only if \(\models_{\beta} \psi_1\) or \(\models_{\beta} \psi_2\)

\(\models_{\beta} \text{expr}\) if and only if \(I[\text{expr}](\sigma, \beta) = 1\)

\(\models_{\beta} E_x^1\) if and only if \(\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in \text{Snd}\)

\(\models_{\beta} E_x^2\) if and only if \(\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in \text{cons}\)

**Observation:** semantics of models keeps track of sender and receiver at sending and consumption time. We disregard the event identity.

**Alternative:** keep track of event identities.

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**TBA over Signature**

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**Definition.** A TBA

\[ \mathcal{B} = (\mathcal{B}_X(X), X, Q_{\text{init}}, \rightarrow, Q_F) \]

where \(\mathcal{B}_X(X)\) is the set of signal and attribute expressions \(\mathcal{B}_{\mathcal{S}}(\mathcal{X}, X)\) over signature \(\mathcal{S}\) is called TBA over \(\mathcal{S}\).

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Any word over \(\mathcal{S}\) and \(\mathcal{D}\) is then a word for \(\mathcal{B}\).

(By the satisfaction relation defined on the previous slide; \(\mathcal{D}(X) = \mathcal{D}(\mathcal{E})\).)

Thus a TBA over \(\mathcal{S}\) accepts words of models with signature \(\mathcal{S}\).

(By the previous definition of TBA.)
TBA over Signature Example

\[ \{ \sigma, \text{cons}, \text{Snd} \} \models_\beta \text{expr} \iff I[\text{expr}](\sigma, \beta) = 1; \]

\[ \{ \sigma, \text{cons}, \text{Snd} \} \models_\beta E_{x,y}^\gamma \iff (\beta(x), (E, \vec{d}), \beta(y)) \in \text{Snd} \]

**Course Map**
Example

Live Sequence Charts Abstract Syntax
**LSC Body: Abstract Syntax**

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg, Cond, LocInv})$$

- $I$ is a finite set of instance lines,
LSC Body: Abstract Syntax

Let \( \Theta = \{ \text{hot, cold} \} \). An LSC body is a tuple

\[
(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})
\]

- \( I \) is a finite set of instance lines,
- \((\mathcal{L}, \preceq)\) is a finite, non-empty, partially ordered set of locations;
  each \( l \in \mathcal{L} \) is associated with a temperature \( \theta(l) \in \Theta \) and an instance line \( i_l \in I \),
- \( \sim \subseteq \mathcal{L} \times \mathcal{L} \) is an equivalence relation on locations, the simultaneity relation,
Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple $(I, (L, \preceq), \sim, \mathcal{F}, \text{Msg}, \text{Cond}, \text{LocInv})$

- $I$ is a finite set of **instance lines**,
- $(L, \preceq)$ is a finite, non-empty,\n  partially ordered set of **locations**;
  each $l \in L$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq L \times L$ is an **equivalence relation**
  on locations, the **simultaneity** relation,
- $\mathcal{F} = (\mathcal{P}, \mathcal{E}, V, \text{atr}, \mathcal{E})$ is a signature,
- $\text{Msg} \subseteq L \times \mathcal{E} \times L$ is a set of **asynchronous messages** with $(l, b, l') \in \text{Msg}$ only if $l \preceq l'$,
  **Not**: instantaneous messages — could be linked to method/operation calls.
Let $\Theta = \{\text{hot, cold}\}$. An \textbf{LSC body} is a tuple $(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$

- $I$ is a finite set of \textit{instance lines},
- $(\mathcal{L}, \preceq)$ is a finite, non-empty, \textit{partially ordered} set of \textit{locations}; each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an \textit{equivalence relation} on locations, the \textit{simultaneity} relation,
- $\mathcal{S} = (\mathcal{P}, \mathcal{B}, V, atr, \delta)$ is a signature,
- $\text{Msg} \subseteq \mathcal{L} \times \mathcal{B} \times \mathcal{L}$ is a set of \textit{asynchronous messages} with $(l, b, l') \in \text{Msg}$ only if $l \preceq l'$, \textbf{Not}: instantaneous messages — could be linked to method/operation calls.
- $\text{Cond} \subseteq (2^{\mathcal{L}} \setminus \emptyset) \times \text{Expr}_{\mathcal{S}} \times \Theta$ is a set of \textit{conditions} where $\text{Expr}_{\mathcal{S}}$ are OCL expressions over $W = I \cup \{\text{self}\}$ with $(L, expr, \theta) \in \text{Cond}$ only if $l \sim l'$ for all $l, l' \in L$,
- $\text{LocInv} \subseteq \mathcal{L} \times \{\circ, \bullet\} \times \text{Expr}_{\mathcal{S}} \times \Theta \times \mathcal{L} \times \{\circ, \bullet\}$ is a set of \textit{local invariants},
Well-Formedness

Bondedness/no floating conditions: (could be relaxed a little if we wanted to)

- For each location \( l \in \mathcal{L} \), if \( l \) is the location of
  - a condition, i.e.
    \[ \exists (L, expr, \theta) \in \text{Cond} : l \in L, \text{ or} \]
  - a local invariant, i.e.
    \[ \exists (l_1, i_1, expr, \theta, l_2, i_2) \in \text{LocInv} : l \in \{l_1, l_2\}, \text{ or} \]
  then there is a location \( l' \) equivalent to \( l \), i.e. \( l \sim l' \), which is the location of
  - an instance head, i.e. \( l' \) is minimal wrt. \( \preceq \), or
  - a message, i.e.
    \[ \exists (l_1, b, l_2) \in \text{Msg} : l \in \{l_1, l_2\}. \]

Note: if messages in a chart are cyclic, then there doesn’t exist a partial order (so such charts don’t even have an abstract syntax).
**TBA-based Semantics of LSCs**

**Plan:**
- Given an LSC $L$ with body
  \[ (I, (\mathcal{D}, \preceq), \sim, \mathcal{R}, \text{Msg}, \text{Cond}, \text{LocInv}), \]
- construct a TBA $B_L$, and
- define $\mathcal{L}(L)$ in terms of $\mathcal{L}(B_L)$, in particular taking activation condition and activation mode into account.
- Then $\mathcal{M} \models L$ (universal) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(L)$. 
Recall: Intuitive Semantics

(i) Strictly After:

\[ a \rightarrow b \rightarrow a \]

(ii) Simultaneously: (simultaneous region)

\[ a \rightarrow expr_1 \rightarrow b \rightarrow c \]

(iii) Explicitly Unordered: (co-region)

\[ a \rightarrow \cdots \rightarrow b \]

Intuition: A computation path violates an LSC if the occurrence of some events doesn’t adhere to the partial order obtained as the transitive closure of (i) to (iii).

Examples: Semantics?

\[ \mathbb{E}_3 \]

\[ \mathbb{E}_2 \]

\[ \mathbb{E}_1 \]
Formal LSC Semantics: It’s in the Cuts!

Definition.
Let \( (I, (\mathcal{L}, \leq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv}) \) be an LSC body.
A non-empty set \( \emptyset \neq C \subseteq \mathcal{L} \) is called a cut of the LSC body iff

- it is downward closed, i.e.
  \[ \forall l, l' : l' \in C \land l \leq l' \implies l \in C, \]
- it is closed under simultaneity, i.e.
  \[ \forall l, l' : l' \in C \land l \sim l' \implies l \in C, \]
- it comprises at least one location per instance line, i.e.
  \[ \forall i \in I \exists l \in C : i_l = i. \]

A cut \( C \) is called hot, denoted by \( \theta(C) = \text{hot} \), if and only if at least one of its maximal elements is hot, i.e. if
\[ \exists l \in C : \theta(l) = \text{hot} \land \forall l' \in C : l \prec l' \]
Otherwise, \( C \) is called cold, denoted by \( \theta(C) = \text{cold} \).
Examples: Cut or Not Cut? Hot/Cold?

(i) non-empty set $\emptyset \neq C \subseteq \mathcal{L}$.
(ii) downward closed, i.e. $\forall l, l' \in C \land l \preceq l' \implies l \in C$
(iii) closed under simultaneity, i.e. $\forall l, l' \in C \land l \sim l' \implies l \in C$
(iv) at least one location per instance line, i.e. $\forall i \in I \exists l \in C : i_l = i$

$\mathcal{L} = \mathbb{R}$

$\mathcal{E}_1 = \emptyset$
$\mathcal{E}_2 = \{l_1, 0\}$
$\mathcal{E}_3 = \{l_1, 1\}$
$\mathcal{E}_4 = \{l_1, 0, l_2, 0, l_3, 0\}$
$\mathcal{E}_5 = \{l_1, 0, l_2, 0, l_3, 0\}$
$\mathcal{E}_6 = \{l_1, 0, l_2, 0, l_3, 1\}$
$\mathcal{E}_7 = \mathcal{L} \setminus \{l_1, 0, l_2, 0\}$

A Successor Relation on Cuts

The partial order of $(\mathcal{L}, \preceq)$ and the simultaneity relation "~" induce a direct successor relation on cuts of $\mathcal{L}$ as follows:

Definition. Let $C, C' \subseteq \mathcal{L}$ be cuts of an LSC body with locations $(\mathcal{L}, \preceq)$ and messages $\text{Msg}$. $C'$ is called direct successor of $C$ via fired-set $F$, denoted by $C \xhookrightarrow{F} C'$, if and only if

- $F \neq \emptyset$,
- $C' \setminus C = F$,
- for each message reception in $F$, the corresponding sending is already in $C$,
  $\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C$, and
- locations in $F$, that lie on the same instance line, are pairwise unordered, i.e.
  $\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\preceq l' \land l' \not\preceq l$
Properties of the Fired-set

$C \rightsquigarrow_F C'$ if and only if

- $F \neq \emptyset$,
- $C' \setminus C = F$,
- $\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C$, and
- $\forall l, l' \in F : l \neq l' \land i_l = i_{l'} \implies l \not\preceq l' \land l' \not\preceq l$

- **Note**: $F$ is closed under simultaneity.
- **Note**: locations in $F$ are direct $\preceq$-successors of locations in $C$, i.e.

\[ \forall l' \in F \exists l \in C : l \prec l' \land \exists l'' \in C : l' \prec l'' \prec l \]

Successor Cut Examples

(i) $F \neq \emptyset$,  
(ii) $C' \setminus C = F$,
(iii) $\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C$, and
(iv) $\forall l, l' \in F : l \neq l' \land i_l = i_{l'} \implies l \not\prec l' \land l' \not\prec l$
Idea: Accept Timed Words by Advancing the Cut

- Let $w = (\sigma_0, \text{cons}_0, \text{Snd}_0), (\sigma_1, \text{cons}_1, \text{Snd}_1), (\sigma_2, \text{cons}_2, \text{Snd}_2), \ldots$ be a word of a UML model and $\beta$ a valuation of $I \cup \{\text{self}\}$.

- Intuitively (and for now disregarding cold conditions), an LSC body $(I, (\mathcal{L}, \preceq), \sim, \mathcal{P}, \text{Msg}, \text{Cond}, \text{LocInv})$ is supposed to accept $w$ if and only if there exists a sequence $C_0 \xrightarrow{F_1} C_1 \xrightarrow{F_2} C_2 \cdots \xrightarrow{F_n} C_n$ and indices $0 = i_0 < i_1 < \cdots < i_n$ such that for all $0 \leq j < n$,

  - for all $i_j \leq k < i_{j+1}$, $(\sigma_k, \text{cons}_k, \text{Snd}_k), \beta$ satisfies the hold condition of $C_j$,
  - $(\sigma_{i_j}, \text{cons}_{i_j}, \text{Snd}_{i_j}), \beta$ satisfies the transition condition of $F_j$, $C_n$ is cold,
  - for all $i_n < k$, $(\sigma_k, \text{cons}_k, \text{Snd}_k), \beta$ satisfies the hold condition of $C_n$.

Language of LSC Body

The language of the body

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{P}, \text{Msg}, \text{Cond}, \text{LocInv})$$

of LSC $L$ is the language of the TBA

$$\mathcal{B}_L = (\text{Expr}_L(X), X, Q, q_{\text{ini}}, \rightarrow, Q_F)$$

with

- $\text{Expr}_L(X) = \text{Expr}_{\mathcal{P}}(\mathcal{P}, X)$
- $Q$ is the set of cuts of $(\mathcal{L}, \preceq)$, $q_{\text{ini}}$ is the instance heads cut,
- $F = \{C \in Q \mid \theta(C) = \text{cold}\}$ is the set of cold cuts of $(\mathcal{L}, \preceq)$,
- $\rightarrow$ as defined in the following, consisting of
  - loops $(q, \psi, q)$,
  - progress transitions $(q, \psi, q')$ corresponding to $q \xrightarrow{F} q'$, and
  - legal exits $(q, \psi, \mathcal{L})$. 
Language of LSC Body: Intuition

\[ B_L = (\text{Expr}_B(X), X, Q, q_{\text{init}}, \rightarrow, Q_F) \] with

- \( \text{Expr}_B(X) = \text{Expr}_{\mathcal{C}}(\mathcal{X}, X) \)
- \( Q \) is the set of cuts of \( (\mathcal{X}, \preceq) \), \( q_{\text{init}} \) is the instance heads cut,
- \( F = \{ C \in Q \mid \theta(C) = \text{cold} \} \) is the set of cold cuts,
- \( \rightarrow \) consists of
  - loops \( (q, \psi, q) \),
  - progress transitions \( (q, \psi, q') \) corresponding to \( q \rightarrow_F q' \), and
  - legal exits \( (q, \psi, \mathcal{L}) \).

**Step I: Only Messages**
Some Helper Functions

- **Message-expressions of a location:**

\[
\mathcal{E}(l) := \{ E^l_{i,j,j'} \mid (l, E, l') \in \text{Msg} \} \cup \{ E^l_{i,j,j'} \mid (l', E, l) \in \text{Msg} \},
\]

\[
\mathcal{E}\{l_1, \ldots, l_n\} := \mathcal{E}(l_1) \cup \cdots \cup \mathcal{E}(l_n).
\]

\[
\bigvee \emptyset := \text{true}; \bigvee \{E^1_{i_1,j_1,i'_1}, \ldots, F^1_{k_1,j_1,i'_1}, \ldots\} := \bigvee_{1 \leq j < k} E^1_{i_j,j_j,i'_j} \lor \bigvee_{k \leq j} F^1_{i_j,j_j,i'_j}
\]

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Loops

- How long may we **legally** stay at a cut \( q \)?
Loops

- How long may we legally stay at a cut \( q \)?
- **Intuition**: those \((\sigma_i, \text{cons}_i, \text{Snd}_i)\) are allowed to fire the self-loop \((q, \psi, q)\) where
  - \(\text{cons}_i \cup \text{Snd}_i\) comprises only irrelevant messages:
    - **weak mode**: no message from a direct successor cut is in,
    - **strict mode**: no message occurring in the LSC is in,
  - \(\sigma_i\) satisfies the local invariants active at \( q \)
  And nothing else.

- **Formally**: Let \( F := F_1 \cup \cdots \cup F_n \)
  be the union of the firedsets of \( q \).
  - \( \psi := \neg(\bigvee \delta(F)) \land \bigwedge \psi(q) \).
    
    If \( F = \emptyset \), then \( \psi = \text{true} \).
Progress

- When do we move from $q$ to $q'$?

**Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \xrightarrow{F} q'$ and

- $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (strict mode),

- $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$. 
**Progress**

- When do we move from \( q \) to \( q' \)?
- **Intuition:** those \( (\sigma_i, \text{cons}_i, \text{Snd}_i) \) fire the progress transition \( (q, \psi, q') \) for which there exists a firedset \( F \) such that \( q \rightarrow_F q' \) and
  - \( \text{cons}_i \cup \text{Snd}_i \) comprises exactly the messages that distinguish \( F \) from other firedsets of \( q \) (weak mode), and in addition no message occurring in the LSC is in \( \text{cons}_i \cup \text{Snd}_i \) (strict mode),
  - \( \psi \) satisfies the local invariants and conditions relevant at \( q' \).
- **Formally:** Let \( F, F_1, \ldots, F_n \) be the firedsets of \( q \) and let \( q \rightarrow_F q' \) (unique).
  - \( \psi := \bigwedge \mathcal{E}(F) \land \neg\left( \bigvee \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \right) \land \psi(q, q') \).

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**Step II: Conditions and Local Invariants**
Some More Helper Functions

- **Constraints relevant at cut** $q$:
  
  $\psi_q(q) = \{ \psi \mid \exists l \in q, l' \notin q \mid (l, \psi, \theta, l') \in \text{LocInv} \lor (l', \psi, \theta, l) \in \text{LocInv} \}$,
  
  $\psi(q) = \psi_{\text{hot}}(q) \cup \psi_{\text{cold}}(q)$
  
  $\bigwedge \emptyset := \text{false}; \bigwedge \{ \psi_1, \ldots, \psi_n \} := \bigwedge_{1 \leq i \leq n} \psi_i$

Loops with Conditions

- How long may we **legally** stay at a cut $q$?
- **Intuition:** those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where
  
  - $\text{cons}_i \cup \text{Snd}_i$ comprises only irrelevant messages:
    - **weak mode:** no message from a direct successor cut is in,
    - **strict mode:** no message occurring in the LSC is in,
  
  - $\sigma_i$ satisfies the local invariants active at $q$
  
  And nothing else.
- **Formally:** Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of $q$.
  
  $\psi := \neg(\bigvee F) \land \bigwedge \psi(q)$.

  \[= \text{true if } F = \emptyset\]
Loops with Conditions

- How long may we legally stay at a cut \( q \)?
- **Intuition:** those \((\sigma_i, \text{cons}_i, \text{Snd}_i)\) are allowed to fire the self-loop \((q, \psi, q)\) where
  - \( \text{cons}_i \cup \text{Snd}_i \) comprises only irrelevant messages:
    - **weak mode:** no message from a direct successor cut is in,
    - **strict mode:** no message occurring in the LSC is in,
  - \( \sigma_i \) satisfies the local invariants active at \( q \)

And nothing else.

- **Formally:** Let \( F := F_1 \cup \cdots \cup F_n \) be the union of the firedsets of \( q \).
  - \( \psi := \neg(\bigvee E(F)) \land \psi(q) \).

\( = \text{true if } F = \emptyset \)
Even More Helper Functions

- **Constraints** relevant when moving from $q$ to cut $q'$:

$$
\psi_0(q, q') = \{ \psi \mid \exists L \subseteq \mathcal{L} \mid (L, \psi, \theta) \in \text{Cond} \land L \cap (q' \setminus q) \neq \emptyset \}
\cup \psi(q')
\setminus \{ \psi \mid \exists l \in q' \setminus q, l' \in \mathcal{L} \mid (l, \cdot, \text{expr}, \theta, l') \in \text{LocInv} \lor (l', \cdot, \text{expr}, \theta, \cdot, l) \in \text{LocInv} \}
\cup \{ \psi \mid \exists l \in q' \setminus q, l' \in \mathcal{L} \mid (l, \cdot, \text{expr}, \theta, l') \in \text{LocInv} \lor (l', \cdot, \text{expr}, \theta, \cdot, l) \in \text{LocInv} \}
\psi(q, q') = \psi_{\text{hot}}(q, q') \cup \psi_{\text{cold}}(q, q')
\psi_{\text{hot}}(q, q') \cup \psi_{\text{cold}}(q, q')
$$

Progress with Conditions

- When do we move from $q$ to $q'$?
- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \rightarrow_F q'$ and
  - $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (strict mode),
  - $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.
- **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \rightarrow_F q'$ (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \left( \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \right) \setminus \mathcal{E}(F) \right) \land \bigwedge \psi(q, q')$. 

- $v = 0$
Progress with Conditions

- When do we move from $q$ to $q'$?
- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \xrightarrow{F} q'$ and
  - $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (**strict mode**).
  - $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.
- **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \xrightarrow{F} q'$ (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \{ \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \} \setminus \mathcal{E}(F) \right) \land \psi(q, q')$. 

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Progress with Conditions

- When do we move from $q$ to $q'$?
- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \xrightarrow{F} q'$ and
  - $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (**strict mode**).
  - $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.
- **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \xrightarrow{F} q'$ (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \{ \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \} \setminus \mathcal{E}(F) \right) \land \psi(q, q')$. 

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Step III: Cold Conditions and Cold Local Invariants

Legal Exits

- When do we take a legal exit from $q$?
Legal Exits

• When do we take a legal exit from $q$?
• **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the legal exit transition $(q, \psi, L)$
  • for which there exists a firedset $F$ and some $q'$ such that $q \rightsquigarrow_F q'$ and
  - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (strict mode) and
  - at least one cold condition or local invariant relevant when moving to $q'$ is violated, or
  • for which there is no matching firedset and at least one cold local invariant relevant at $q$ is violated.

**Formally**: Let $F_1, \ldots, F_n$ be the firedsets of $q$ with $q \rightsquigarrow_{F_i} q'$. 

$$\psi := \bigvee_{i=1}^n \delta(F_i) \land \neg (\bigvee \delta'(F_i) \cup \cdots \cup \delta'(F_n)) \land \psi_{\text{cold}}(q, q')$$

$$\lor \neg (\bigvee \delta'(F_i)) \land \psi_{\text{cold}}(q)$$
Finally: The LSC Semantics

A full LSC $L$ consist of

- a **body** $(I, (\mathcal{L}, \preceq), \sim, \mathcal{F}, \text{Msg}, \text{Cond}, \text{LocInv})$,
- an **activation condition** (here: event) $ac = E_{i_1,i_2}^?, E \in \mathcal{F}$, $i_1, i_2 \in I$,
- an **activation mode**, either **initial** or **invariant**,
- a **chart mode**, either **existential** (cold) or **universal** (hot).
Finally: The LSC Semantics

A full LSC $L$ consists of

- a body $(I, (\mathcal{L}, \preceq), \sim, \mathcal{F}, \text{Msg}, \text{Cond}, \text{LocInv})$,
- an activation condition (here: event) $ac = E_{i_1, i_2} \in \mathcal{F}, i_1, i_2 \in I$,
- an activation mode, either initial or invariant,
- a chart mode, either existential (cold) or universal (hot).

A set $W$ of words over $\mathcal{F}$ and $\mathcal{D}$ satisfies $L$, denoted $W \models L$, iff $L$

- universal ($\equiv$ hot), initial, and
  \[ \forall w \in W \forall \beta : I \rightarrow \text{dom}(\sigma(w^n)) \bullet w \text{ activates } L \implies w \in L_\beta(\mathcal{B}_L). \]
- existential ($\equiv$ cold), initial, and
  \[ \exists w \in W \exists \beta : I \rightarrow \text{dom}(\sigma(w^0)) \bullet w \text{ activates } L \land w \in L_\beta(\mathcal{B}_L). \]

- universal ($\equiv$ hot), invariant, and
  \[ \forall w \in W \forall k \in \mathbb{N}_0 \forall \beta : I \rightarrow \text{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \implies w/k \in L_\beta(\mathcal{B}_L). \]
- existential ($\equiv$ cold), invariant, and
  \[ \exists w \in W \exists k \in \mathbb{N}_0 \exists \beta : I \rightarrow \text{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \land w/k \in L_\beta(\mathcal{B}_L). \]

References
References


