

# *Software Design, Modelling and Analysis in UML*

## *Lecture 18: Live Sequence Charts II*

2013-01-22

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## Contents & Goals

### Last Lecture:

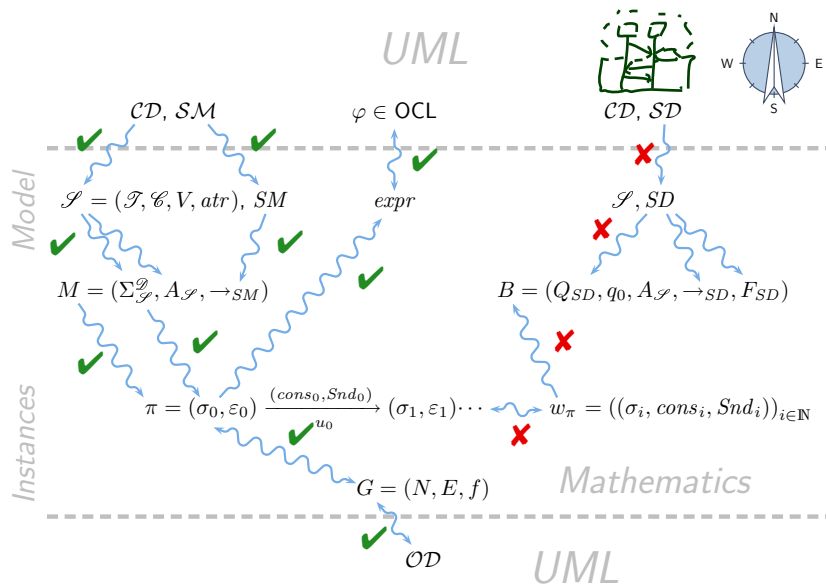
- LSC concrete syntax.
- LSC intuitive semantics.

### This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this LSC mean?
  - Are this UML model's state machines consistent with the interactions?
  - Please provide a UML model which is consistent with this LSC.
  - What is: activation, hot/cold condition, pre-chart, etc.?
- **Content:**
  - Symbolic Büchi Automata (TBA) and its (accepted) language.
  - Words of a model.
  - LSC abstract syntax.
  - LSC formal semantics.

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# Course Map



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## Excursus: Symbolic Büchi Automata (over Signature)

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# Symbolic Büchi Automata

**Definition.** A **Symbolic Büchi Automaton** (TBA) is a tuple

$$\mathcal{B} = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{\text{ini}}, \rightarrow, Q_F)$$

where

- $X$  is a set of logical variables,
- $\text{Expr}_{\mathcal{B}}(X)$  is a set of Boolean expressions over  $X$ ,
- $Q$  is a finite set of **states**,
- $q_{\text{ini}} \in Q$  is the initial state,
- $\rightarrow \subseteq Q \times \text{Expr}_{\mathcal{B}}(X) \times Q$  is the **transition relation**.  
Transitions  $(q, \psi, q')$  from  $q$  to  $q'$  are labelled with an expression  $\psi \in \text{Expr}_{\mathcal{B}}(X)$ .
- $Q_F \subseteq Q$  is the set of **fair** (or accepting) states.

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$\Sigma = \{0, 1\}$   
 $L(A) = 0^*$   
 $L(A) = (01)^*$   
 $L(A) = (01)^*0$

$w = 011010$

*Büchi*  
infinite words

$\Sigma = \{0, 1\}$   
 $w \in \Sigma^{\omega}$  ← infinite sequences of letters  
 $w = 01010101\dots \in L(\mathcal{B})$   
 $w = 01^{\omega} \in L(\mathcal{B})$   
 $w = 10^{\omega} \in L(\mathcal{B})$

*symbolic*

$\text{Expr}(X) = \{\text{even}(x), \text{odd}(x)\}$   
 $X = \{x\}$   
 $\Sigma = (\{1, 2\} \rightarrow \mathbb{Z})$   
 $\omega = (p: 1 \mapsto 0, 2 \mapsto 0), \{ \sigma_A$   
 $(\beta_1: 1 \mapsto 1, 2 \mapsto -1),$   
 $(\beta_2: 1 \mapsto 3, 2 \mapsto 0),$   
 $(\beta_3: 1 \mapsto 4, 2 \mapsto 1),$   
 $(\beta_4: 1 \mapsto 6, 2 \mapsto 0),$   
 $(\beta_5: 1 \mapsto 7, 2 \mapsto 0)$

$L_{\beta}(\Sigma) \not\subseteq \omega$   
 $x \mapsto 1$   
 $L_{\beta'}(\Sigma) \ni \omega$   
 $x \mapsto 2$

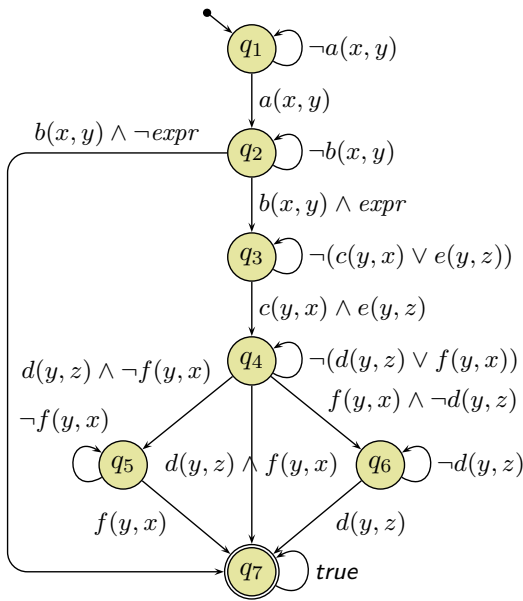
$\text{runs}_{\beta'}(0^{\omega}): q_1^+ q_2^{\omega} | q_3^{\omega}$

$(01)^+ (001)^{\omega} \in L(\mathcal{B})$

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## TBA Example

$(Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F), (q, \psi, q') \in \rightarrow,$



$$Q = \{q_1, q_2, \dots, q_7\}$$

$$q_{ini} = q_1$$

$$Q_F = \{q_7\}$$

$$X = \{x, y\}$$

$$Expr_{\mathcal{B}}(X): \psi := a(z_1, z_2) \mid b(z_1, z_2) \mid \dots$$

$$\mid f(z_1, z_2) \mid expr \mid$$

$$\neg \psi \mid \psi_1 \wedge \psi_2, z_1, z_2 \in X$$

$$\rightarrow = \{(q_2, \neg b(x, y), q_2), \dots\}$$

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## Word

**Definition.** Let  $X$  be a set of logical variables and let  $Expr_{\mathcal{B}}(X)$  be a set of Boolean expressions over  $X$ .

A set  $(\Sigma, \cdot \models \cdot)$  is called an **alphabet** for  $Expr_{\mathcal{B}}(X)$  if and only if

- for each  $\sigma \in \Sigma$ ,
- for each expression  $expr \in Expr_{\mathcal{B}}$ , and
- for each valuation  $\beta : X \rightarrow \mathcal{D}(X)$  of logical variables to domain  $\mathcal{D}(X)$ ,

either  $\sigma \models_{\beta} expr$  or  $\sigma \not\models_{\beta} expr$ .

An **infinite sequence**

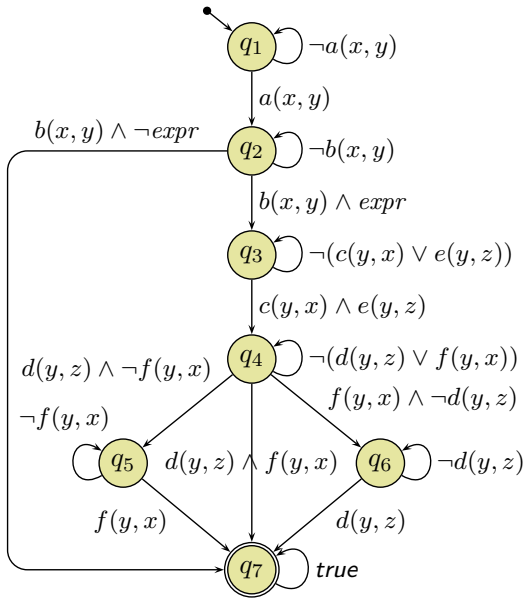
$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^{\omega}$$

over  $(\Sigma, \cdot \models \cdot)$  is called **word** for  $Expr_{\mathcal{B}}(X)$ .

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## Word Example



see Slide 5a

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## Run of TBA over Word

**Definition.** Let  $\mathcal{B} = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$  be a TBA and

a word for  $\text{Expr}_{\mathcal{B}}(X)$ .

An infinite sequence

$$w = \sigma_1, \sigma_2, \sigma_3, \dots \in \Sigma^\omega$$

$$q = q_0, q_1, q_2, \dots \in Q^\omega$$

is called **run** of  $\mathcal{B}$  over  $w$  under valuation  $\beta : X \rightarrow \mathcal{D}(X)$  if and only if

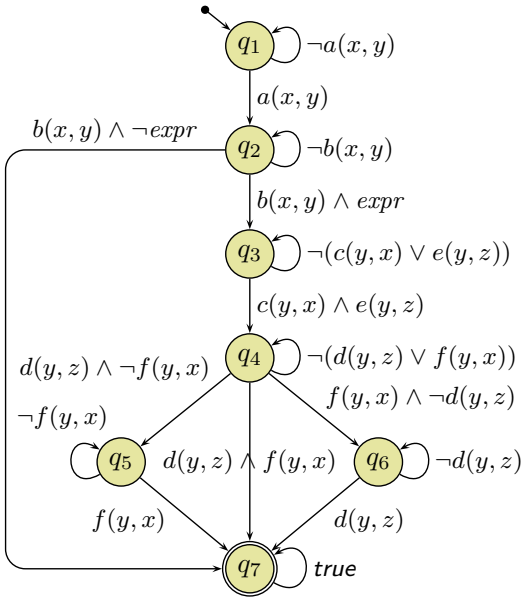
- $q_0 = q_{ini}$ ,
- for each  $i \in \mathbb{N}_0$  there is a transition  $(q_i, \psi_i, q_{i+1}) \in \rightarrow$  of  $\mathcal{B}$  such that  $\sigma_i \models_{\beta} \psi_i$ .

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## Run Example

$\varrho = q_0, q_1, q_2, \dots \in Q^\omega$  s.t.  $\sigma_i \models_\beta \psi_i, i \in \mathbb{N}_0$ .



see Slide 5a

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## The Language of a TBA

### Definition.

We say  $\mathcal{B}$  **accepts** word  $w$  (under  $\beta$ ) if and only if  $\mathcal{B}$  **has a run**

$$\varrho = (q_i)_{i \in \mathbb{N}_0}$$

over  $w$  such that fair (or accepting) states are **visited infinitely often** by  $\varrho$ , i.e., such that

$$\forall i \in \mathbb{N}_0 \exists j > i : q_j \in Q_F.$$

We call the set  $\mathcal{L}_\beta(\mathcal{B}) \subseteq \Sigma^\omega$  of words for  $\text{Expr}_\mathcal{B}(X)$  that are accepted by  $\mathcal{B}$  the **language of  $\mathcal{B}$** .

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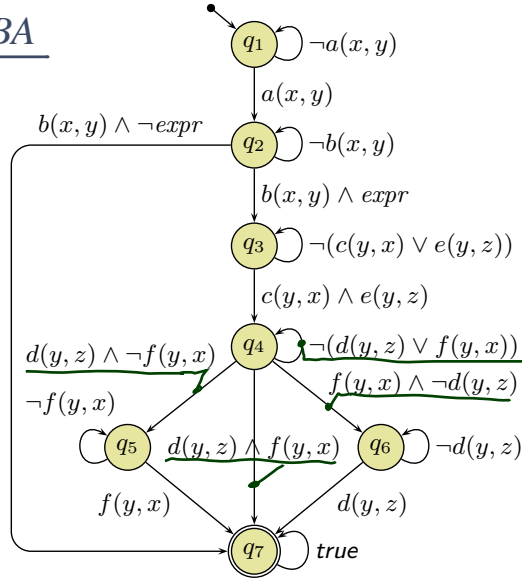
# Language of the Example TBA

$\mathcal{L}_\beta(\mathcal{B})$  consists of the words

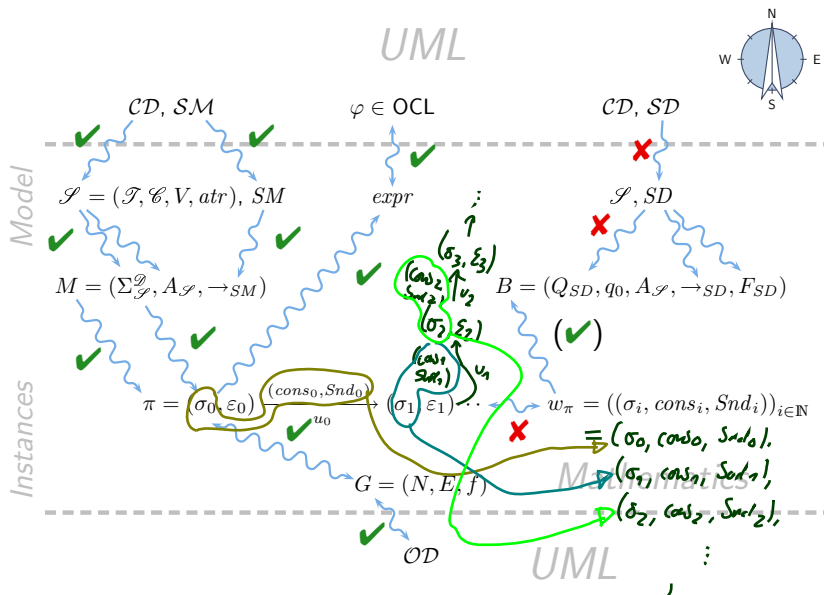
$$w = (\sigma_i)_{i \in \mathbb{N}_0}$$

where for  $0 \leq n < m < k < \ell$  we have

- for  $0 \leq i < n$ ,  $\sigma_i \not\models_\beta a(x,y)$
- $\sigma_n \models_\beta a(x,y)$
- for  $n < i < m$ ,  $\sigma_i \not\models_\beta b(x,y)$
- $\sigma_m \models_\beta b(x,y) \wedge \neg expr$
- for  $m < i < k$ ,  $\sigma_i \models_\beta (c(y,x) \vee e(y,z))$
- $\sigma_k \models_\beta c(y,x) \wedge e(y,z)$
- for  $k < i < \ell$ ,  $\sigma_i \not\models_\beta d(y,z) \vee f(y,x)$
- ...



# Course Map



## Back to Main Track: Language of a Model

### Words over Signature

**Definition.** Let  $\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr, \mathcal{E})$  be a signature and  $\mathcal{D}$  a structure of  $\mathcal{S}$ . A **word** over  $\mathcal{S}$  and  $\mathcal{D}$  is an infinite sequence

$$(\sigma_i, cons_i, Snd_i)_{i \in \mathbb{N}_0}$$
$$\in \left( \Sigma_{\mathcal{S}}^{\mathcal{D}} \times \underbrace{2^{\mathcal{D}(\mathcal{C}) \times Evs(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{C})} \times 2^{\mathcal{D}(\mathcal{C}) \times Evs(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{C})}}_{=: \mathcal{A}} \right)^{\omega}$$



## The Language of a Model

**Recall:** A UML model  $\mathcal{M} = (\mathcal{C}\mathcal{D}, \mathcal{SM}, \mathcal{O}\mathcal{D})$  and a structure  $\mathcal{D}$  denotes a set  $[[\mathcal{M}]]$  of (initial and consecutive) **computations** of the form

$$(\sigma_0, \varepsilon_0) \xrightarrow{a_0} (\sigma_1, \varepsilon_1) \xrightarrow{a_1} (\sigma_2, \varepsilon_2) \xrightarrow{a_2} \dots \text{ where}$$

$$a_i = (\text{cons}_i, \text{Snd}_i, u_i) \in \underbrace{2^{\mathcal{D}(\mathcal{C}) \times \text{Evs}(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{C})} \times 2^{\mathcal{D}(\mathcal{C}) \times \text{Evs}(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{C})} \times \mathcal{D}(\mathcal{C})}_{=: \tilde{A}}.$$

For the connection between models and interactions, we **disregard** the configuration of **the ether** and **who** made the step, and define as follows:

**Definition.** Let  $\mathcal{M} = (\mathcal{C}\mathcal{D}, \mathcal{SM}, \mathcal{O}\mathcal{D})$  be a UML model and  $\mathcal{D}$  a structure. Then

$$\mathcal{L}(\mathcal{M}) := \{(\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma_{\mathcal{D}}^{\mathcal{D}} \times \tilde{A})^\omega \mid$$

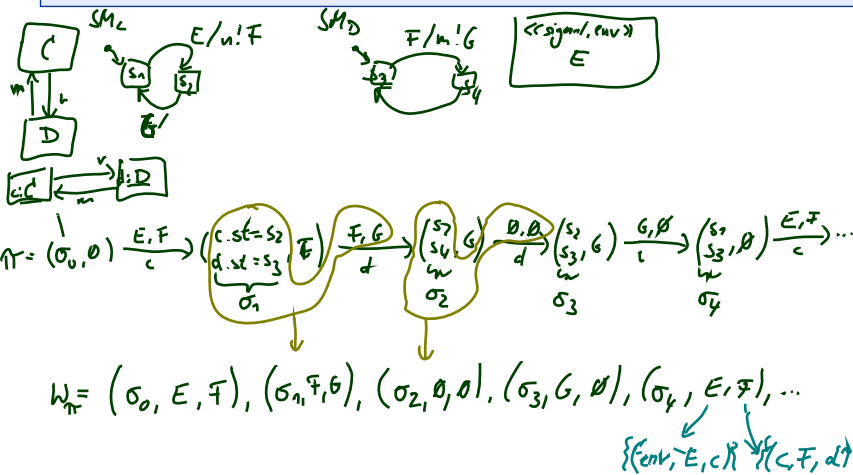
$$\exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(\text{cons}_0, \text{Snd}_0)} (\sigma_1, \varepsilon_1) \dots \in [[\mathcal{M}]]\}$$

is the **language** of  $\mathcal{M}$ .

## Example: The Language of a Model

$$\mathcal{L}(\mathcal{M}) := \{(\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma_{\mathcal{D}}^{\mathcal{D}} \times \tilde{A})^\omega \mid$$

$$\exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(\text{cons}_0, \text{Snd}_0)} (\sigma_1, \varepsilon_1) \dots \in [[\mathcal{M}]]\}$$



## Signal and Attribute Expressions

- Let  $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr, \mathcal{E})$  be a signature and  $X$  a set of logical variables,
- The signal and attribute expressions  $Expr_{\mathcal{S}}(\mathcal{C}, X)$  are defined by the grammar:

$$\psi ::= \text{true} \mid expr \mid E_{x,y}^! \mid E_{x,y}^? \mid \neg\psi \mid \psi_1 \vee \psi_2,$$

where  $expr : Bool \in Expr_{\mathcal{S}}, E \in \mathcal{C}, x, y \in X$ .

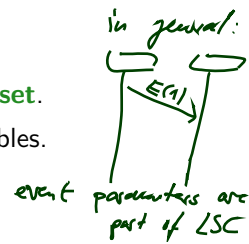
## Satisfaction of Signal and Attribute Expressions

- Let  $(\sigma, cons, Snd) \in \Sigma_{\mathcal{S}} \times \tilde{A}$  be a triple consisting of **system state**, **consume set**, and **send set**.
- Let  $\beta : X \rightarrow \mathcal{D}(\mathcal{C})$  be a valuation of the logical variables.

Then

- $(\sigma, cons, Snd) \models_{\beta} \text{true}$
- $(\sigma, cons, Snd) \models_{\beta} \neg\psi$  if and only if not  $(\sigma, cons, Snd) \models_{\beta} \psi$
- $(\sigma, cons, Snd) \models_{\beta} \psi_1 \vee \psi_2$  if and only if  $(\sigma, cons, Snd) \models_{\beta} \psi_1$  or  $(\sigma, cons, Snd) \models_{\beta} \psi_2$

- $(\sigma, cons, Snd) \models_{\beta} expr$  if and only if  $I[expr](\sigma, \beta) = 1$ .
- $(\sigma, cons, Snd) \models_{\beta} E_{x,y}^!$  if and only if  $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in Snd$
- $(\sigma, cons, Snd) \models_{\beta} E_{x,y}^?$  if and only if  $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in cons$



for simplicity, disregard parameters

## Satisfaction of Signal and Attribute Expressions

- Let  $(\sigma, cons, Snd) \in \Sigma_{\mathcal{E}} \times \tilde{A}$  be a triple consisting of **system state**, **consume set**, and **send set**.
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Then

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- $(\sigma, cons, Snd) \models_{\beta} expr$  if and only if  $I[expr](\sigma, \beta) = 1$
- $(\sigma, cons, Snd) \models_{\beta} E_{x,y}^!$  if and only if  $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in Snd$
- $(\sigma, cons, Snd) \models_{\beta} E_{x,y}^?$  if and only if  $\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in cons$

**Observation:** semantics of models **keeps track** of sender and receiver at sending and consumption time. We disregard the event identity.

**Alternative:** keep track of event identities.

## TBA over Signature

**Definition.** A TBA

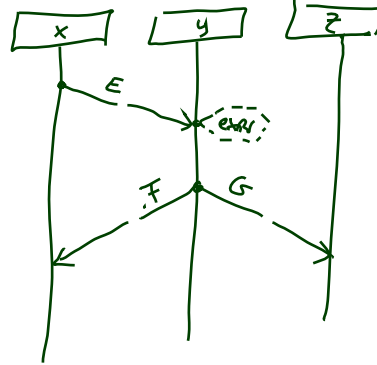
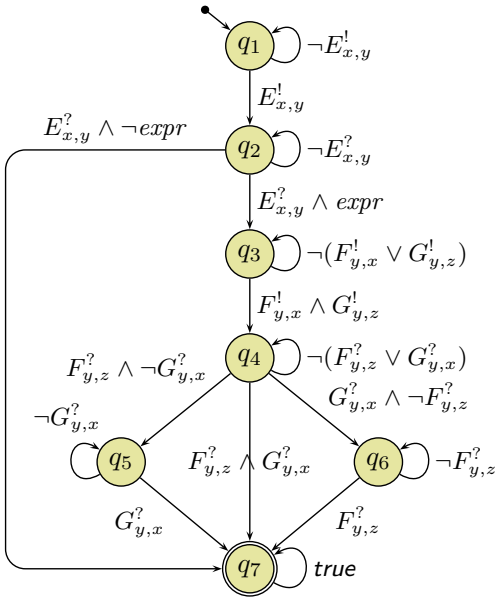
$$\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

where  $Expr_{\mathcal{B}}(X)$  is the set of **signal and attribute expressions**  $Expr_{\mathcal{S}}(\mathcal{E}, X)$  over signature  $\mathcal{S}$  is called **TBA over  $\mathcal{S}$** .

- Any word over  $\mathcal{S}$  and  $\mathcal{D}$  is then a word for  $\mathcal{B}$ .  
(By the satisfaction relation defined on the previous slide;  $\mathcal{D}(X) = \mathcal{D}(\mathcal{C})$ .)
- Thus a TBA over  $\mathcal{S}$  accepts words of models with signature  $\mathcal{S}$ .  
(By the previous definition of TBA.)

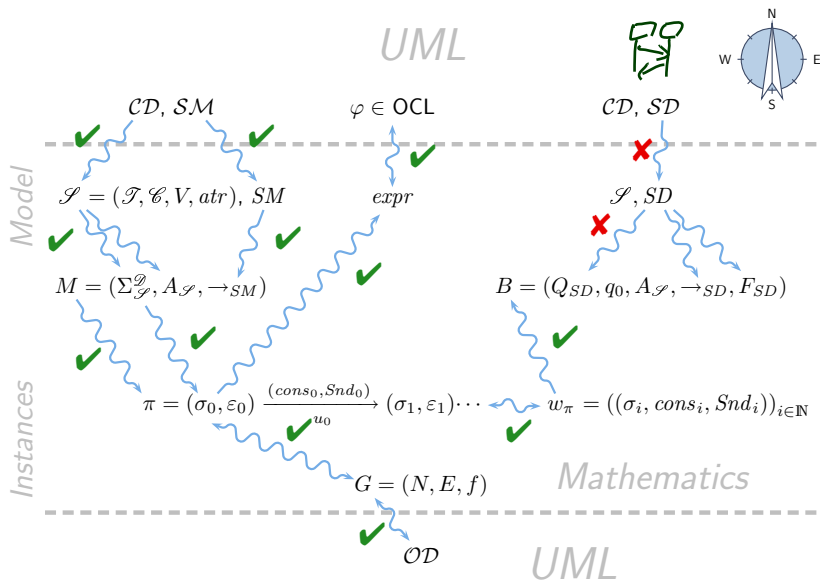
# TBA over Signature Examp

$(\sigma, cons, Snd) \models_{\beta} expr$  iff  $I[expr](\sigma, \beta) = 1$ ;  
 $(\sigma, cons, Snd) \models_{\beta} E_{x,y}^1$  iff  $(\beta(x), (E, \vec{d}), \beta(y)) \in Snd$



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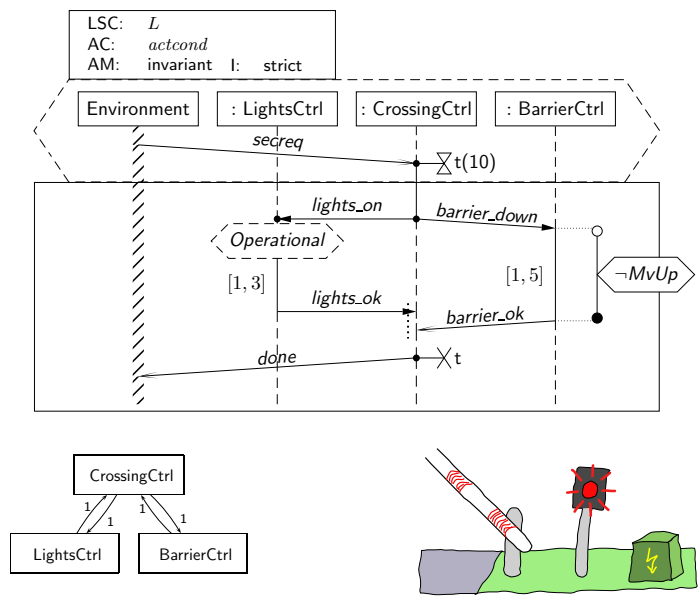
# Course Map



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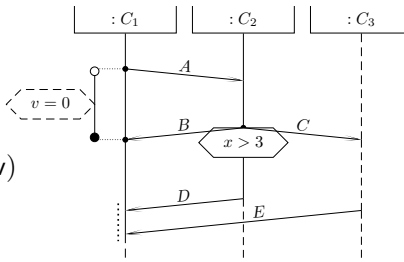
# Live Sequence Charts Abstract Syntax

## Example



## LSC Body: Abstract Syntax

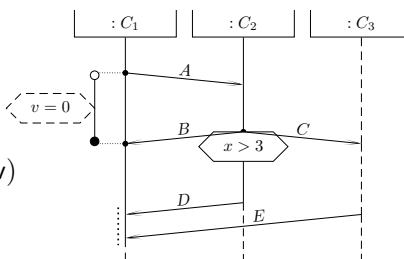
Let  $\Theta = \{\text{hot, cold}\}$ . An **LSC body** is a tuple  
 $(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$



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- $I$  is a finite set of **instance lines**,

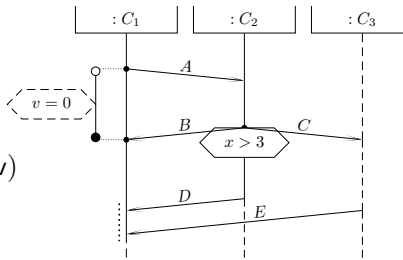


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- $I$  is a finite set of **instance lines**,
- $(\mathcal{L}, \preceq)$  is a finite, non-empty, **partially ordered** set of **locations**; each  $l \in \mathcal{L}$  is associated with a temperature  $\theta(l) \in \Theta$  and an instance line  $i_l \in I$ ,

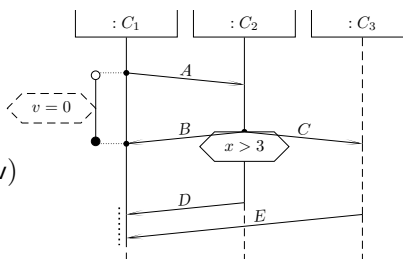


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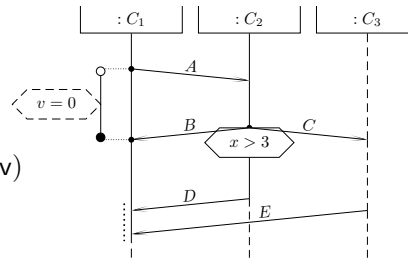


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- $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, \text{atr}, \mathcal{E})$  is a signature,

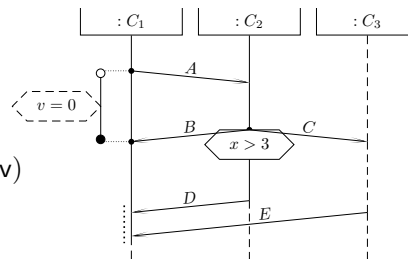


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- $\text{Msg} \subseteq \mathcal{L} \times \mathcal{E} \times \mathcal{L}$  is a set of **asynchronous messages** with  $(l, b, l') \in \text{Msg}$  only if  $l \preceq l'$ ,  
**Not: instantaneous messages** — could be linked to method/operation calls.



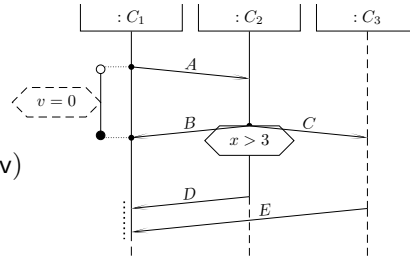


## LSC Body: Abstract Syntax

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**Not: instantaneous messages** — could be linked to method/operation calls.
- $\text{Cond} \subseteq (2^{\mathcal{L}} \setminus \emptyset) \times \text{Expr}_{\mathcal{S}} \times \Theta$  is a set of **conditions** where  $\text{Expr}_{\mathcal{S}}$  are OCL expressions over  $W = I \cup \{\text{self}\}$  with  $(L, \text{expr}, \theta) \in \text{Cond}$  only if  $l \sim l'$  for all  $l, l' \in L$ ,

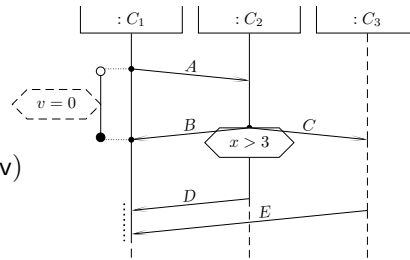


## LSC Body: Abstract Syntax

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- $(\mathcal{L}, \preceq)$  is a finite, non-empty, **partially ordered** set of **locations**; each  $l \in \mathcal{L}$  is associated with a temperature  $\theta(l) \in \Theta$  and an instance line  $i_l \in I$ ,
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$  is an **equivalence relation** on locations, the **simultaneity** relation,
- $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, \text{atr}, \mathcal{E})$  is a signature,
- $\text{Msg} \subseteq \mathcal{L} \times \mathcal{E} \times \mathcal{L}$  is a set of **asynchronous messages** with  $(l, b, l') \in \text{Msg}$  only if  $l \preceq l'$ ,  
**Not: instantaneous messages** — could be linked to method/operation calls.
- $\text{Cond} \subseteq (2^{\mathcal{L}} \setminus \emptyset) \times \text{Expr}_{\mathcal{S}} \times \Theta$  is a set of **conditions** where  $\text{Expr}_{\mathcal{S}}$  are OCL expressions over  $W = I \cup \{\text{self}\}$  with  $(L, \text{expr}, \theta) \in \text{Cond}$  only if  $l \sim l'$  for all  $l, l' \in L$ ,
- $\text{LocInv} \subseteq \mathcal{L} \times \{\circ, \bullet\} \times \text{Expr}_{\mathcal{S}} \times \Theta \times \mathcal{L} \times \{\circ, \bullet\}$  is a set of **local invariants**,



## Well-Formedness

**Bondedness/no floating conditions:** (could be relaxed a little if we wanted to)

- For each location  $l \in \mathcal{L}$ , **if**  $l$  is the location of

- a **condition**, i.e.

$$\exists (L, \text{expr}, \theta) \in \text{Cond} : l \in L, \text{ or}$$

- a **local invariant**, i.e.

$$\exists (l_1, i_1, \text{expr}, \theta, l_2, i_2) \in \text{LocInv} : l \in \{l_1, l_2\}, \text{ or}$$

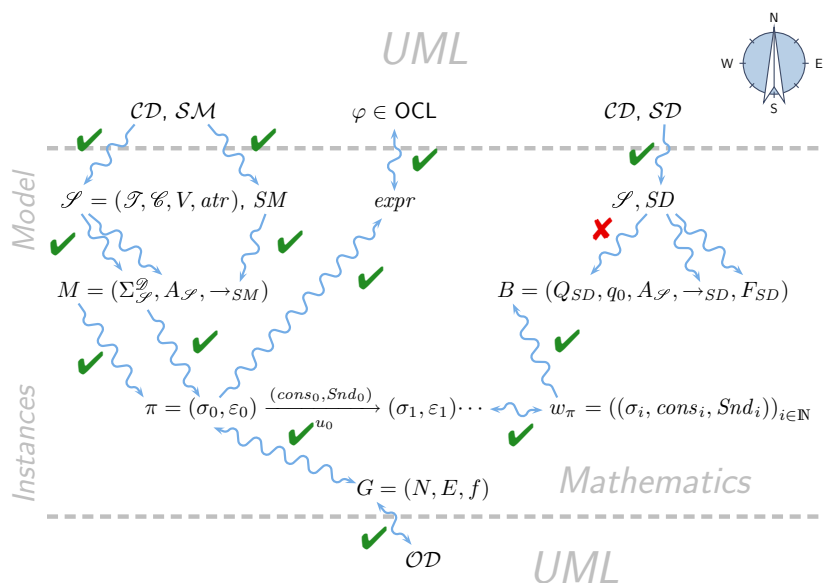
**then** there is a location  $l'$  **equivalent** to  $l$ , i.e.  $l \sim l'$ , which is the location of

- an **instance head**, i.e.  $l'$  is minimal wrt.  $\preceq$ , or
- a **message**, i.e.

$$\exists (l_1, b, l_2) \in \text{Msg} : l \in \{l_1, l_2\}.$$

**Note:** if messages in a chart are **cyclic**, then there doesn't exist a partial order (so such charts **don't even have** an abstract syntax).

## Course Map



## Live Sequence Charts Semantics

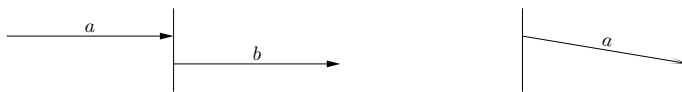
### TBA-based Semantics of LSCs

**Plan:**

- Given an LSC  $L$  with body
$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv}),$$
- construct a TBA  $\mathcal{B}_L$ , and
- define  $\mathcal{L}(L)$  **in terms of**  $\mathcal{L}(\mathcal{B}_L)$ ,  
in particular taking activation condition and activation mode into account.
- Then  $\mathcal{M} \models L$  (universal) if and only if  $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(L)$ .

## Recall: Intuitive Semantics

(i) **Strictly After:**



(ii) **Simultaneously:** (simultaneous region)



(iii) **Explicitly Unordered:** (co-region)

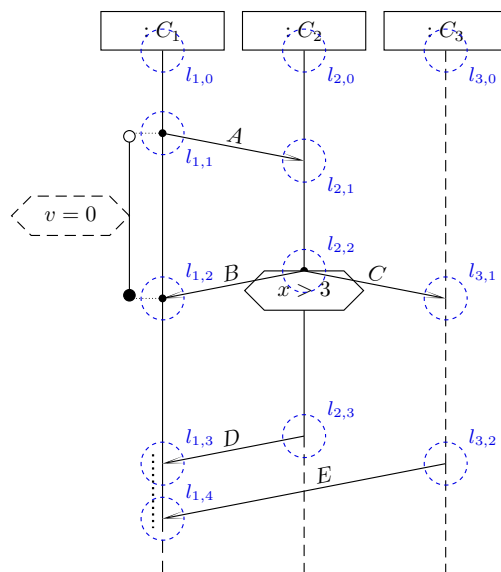


**Intuition:** A computation path **violates** an LSC if the occurrence of some events doesn't adhere to the partial order obtained as the **transitive closure** of (i) to (iii).

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## Examples: Semantics?



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## Formal LSC Semantics: It's in the Cuts!

### Definition.

Let  $(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$  be an LSC body.

A non-empty set  $\emptyset \neq C \subseteq \mathcal{L}$  is called a **cut** of the LSC body iff

- it is **downward closed**, i.e.

$$\forall l, l' : l' \in C \wedge l \preceq l' \implies l \in C,$$

- it is **closed** under **simultaneity**, i.e.

$$\forall l, l' : l' \in C \wedge l \sim l' \implies l \in C, \text{ and}$$

- it comprises at least **one location per instance line**, i.e.

$$\forall i \in I \exists l \in C : i_l = i.$$

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- it comprises at least **one location per instance line**, i.e.

$$\forall i \in I \exists l \in C : i_l = i.$$

A cut  $C$  is called **hot**, denoted by  $\theta(C) = \text{hot}$ , if and only if at least one of its maximal elements is hot, i.e. if

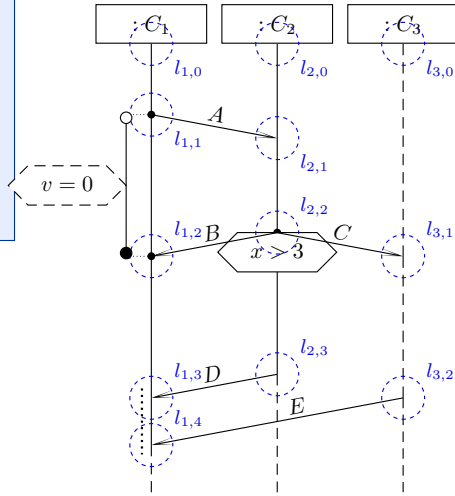
$$\exists l \in C : \theta(l) = \text{hot} \wedge \nexists l' \in C : l \prec l'$$

Otherwise,  $C$  is called **cold**, denoted by  $\theta(C) = \text{cold}$ .

## Examples: Cut or Not Cut? Hot/Cold?

- (i) **non-empty** set  $\emptyset \neq C \subseteq \mathcal{L}$ ,
- (ii) **downward closed**, i.e.  
 $\forall l, l' : l' \in C \wedge l \preceq l' \implies l \in C$
- (iii) **closed under simultaneity**, i.e.  
 $\forall l, l' : l' \in C \wedge l \sim l' \implies l \in C$
- (iv) at least **one location per instance line**, i.e.  
 $\forall i \in I \exists l \in C : i_l = i$ ,

- $C_0 = \emptyset$
- $C_1 = \{l_{1,0}, l_{2,0}, l_{3,0}\}$
- $C_2 = \{l_{1,1}, l_{2,1}, l_{3,0}\}$
- $C_3 = \{l_{1,0}, l_{1,1}\}$
- $C_4 = \{l_{1,0}, l_{1,1}, l_{2,0}, l_{3,0}\}$
- $C_5 = \{l_{1,0}, l_{1,1}, l_{2,0}, l_{2,1}, l_{3,0}\}$
- $C_6 = \mathcal{L} \setminus \{l_{1,3}, l_{2,3}\}$
- $C_7 = \mathcal{L}$



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## A Successor Relation on Cuts

The partial order of  $(\mathcal{L}, \preceq)$  and the simultaneity relation " $\sim$ " induce a **direct successor relation** on cuts of  $\mathcal{L}$  as follows:

**Definition.** Let  $C, C' \subseteq \mathcal{L}$  be cuts of an LSC body with locations  $(\mathcal{L}, \preceq)$  and messages  $\text{Msg}$ .  
 $C'$  is called **direct successor** of  $C$  **via fired-set**  $F$ , denoted by  $C \rightsquigarrow_F C'$ , if and only if

- $F \neq \emptyset$ ,
- $C' \setminus C = F$ ,
- for each message reception in  $F$ , the corresponding sending is already in  $C$ ,

$$\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C, \text{ and}$$

- locations in  $F$ , that lie on the same instance line, are pairwise unordered, i.e.

$$\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\preceq l' \wedge l' \not\preceq l$$

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## Properties of the Fired-set

$C \rightsquigarrow_F C'$  if and only if

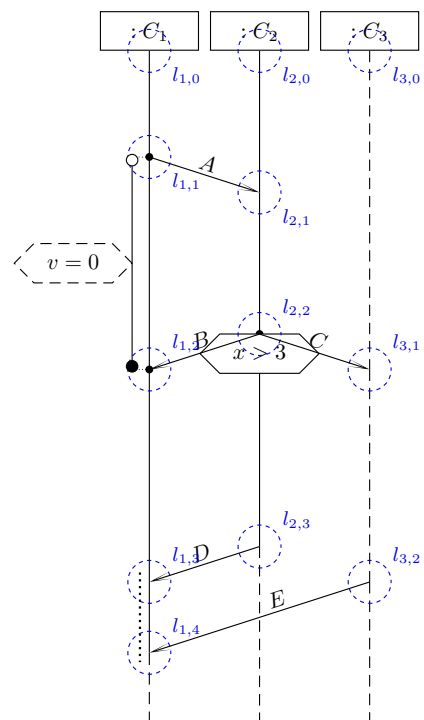
- $F \neq \emptyset$ ,
- $C' \setminus C = F$ ,
- $\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C$ , and
- $\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\leq l' \wedge l' \not\leq l$

- **Note:**  $F$  is closed under simultaneity.
- **Note:** locations in  $F$  are direct  $\preceq$ -successors of locations in  $C$ , i.e.

$$\forall l' \in F \exists l \in C : l \prec l' \wedge \nexists l'' \in C : l' \prec l'' \prec l$$

## Successor Cut Examples

- (i)  $F \neq \emptyset$ , (ii)  $C' \setminus C = F$ ,
- (iii)  $\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C$ , and
- (iv)  $\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \not\leq l' \wedge l' \not\leq l$



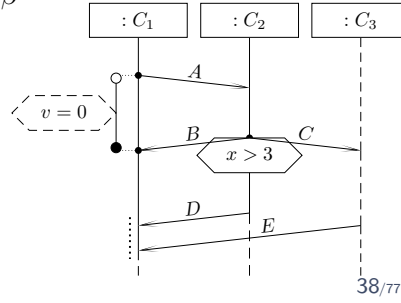
## Idea: Accept Timed Words by Advancing the Cut

- Let  $w = (\sigma_0, cons_0, Snd_0), (\sigma_1, cons_1, Snd_1), (\sigma_2, cons_2, Snd_2), \dots$  be a word of a UML model and  $\beta$  a valuation of  $I \cup \{self\}$ .
- Intuitively** (and for now **disregarding** cold conditions), an LSC body  $(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$  is **supported** to **accept**  $w$  if and only if there exists a sequence

$$C_0 \rightsquigarrow_{F_1} C_1 \rightsquigarrow_{F_2} C_2 \cdots \rightsquigarrow_{F_n} C_n$$

and indices  $0 = i_0 < i_1 < \dots < i_n$  such that for all  $0 \leq j < n$ ,

- for all  $i_j \leq k < i_{j+1}$ ,  $(\sigma_k, cons_k, Snd_k), \beta$  satisfies the **hold condition** of  $C_j$ ,
- $(\sigma_{i_j}, cons_{i_j}, Snd_{i_j}), \beta$  satisfies the **transition condition** of  $F_j$ ,
- $C_n$  is cold,
- for all  $i_n < k$ ,  $(\sigma_k, cons_{i_j}, Snd_{i_j}), \beta$  satisfies the **hold condition** of  $C_n$ .



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## Language of LSC Body

The **language** of the body

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$$

of LSC  $L$  is the language of the TBA

$$\mathcal{B}_L = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

with

- $\text{Expr}_{\mathcal{B}}(X) = \text{Expr}_{\mathcal{S}}(\mathcal{S}, X)$
- $Q$  is the set of cuts of  $(\mathcal{L}, \preceq)$ ,  $q_{ini}$  is the **instance heads cut**,
- $F = \{C \in Q \mid \theta(C) = \text{cold}\}$  is the set of cold cuts of  $(\mathcal{L}, \preceq)$ ,
- $\rightarrow$  as defined in the following, consisting of
  - loops**  $(q, \psi, q)$ ,
  - progress transitions**  $(q, \psi, q')$  corresponding to  $q \rightsquigarrow_F q'$ , and
  - legal exits**  $(q, \psi, \mathcal{L})$ .

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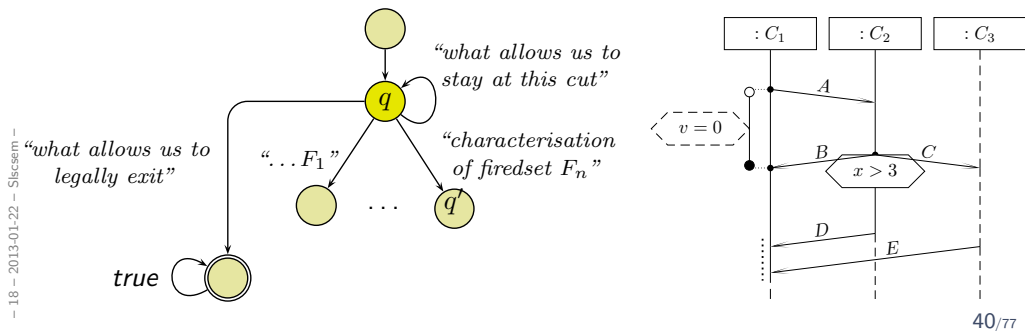
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## Language of LSC Body: Intuition

$\mathcal{B}_L = (\text{Expr}_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$  with

- $\text{Expr}_{\mathcal{B}}(X) = \text{Expr}_{\mathcal{S}}(\mathcal{S}, X)$
- $Q$  is the set of cuts of  $(\mathcal{L}, \preceq)$ ,  $q_{ini}$  is the **instance heads** cut,
- $F = \{C \in Q \mid \theta(C) = \text{cold}\}$  is the set of cold cuts,
- $\rightarrow$  consists of
  - **loops**  $(q, \psi, q)$ ,
  - **progress transitions**  $(q, \psi, q')$  corresponding to  $q \rightsquigarrow_F q'$ , and
  - **legal exits**  $(q, \psi, \mathcal{L})$ .



## Step I: Only Messages

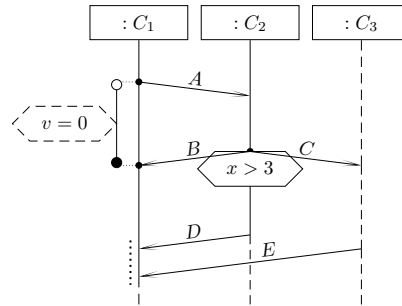
## Some Helper Functions

- **Message-expressions of a location:**

$$\mathcal{E}(l) := \{E_{i_l, i_{l'}}^! \mid (l, E, l') \in \text{Msg}\} \cup \{E_{i_{l'}, i_l}^? \mid (l', E, l) \in \text{Msg}\},$$

$$\mathcal{E}(\{l_1, \dots, l_n\}) := \mathcal{E}(l_1) \cup \dots \cup \mathcal{E}(l_n).$$

$$\bigvee \emptyset := \text{true}; \bigvee \{E_{i_1, i_{12}}^!, \dots, F_{k, i_{k1}, i_{k2}}^?, \dots\} := \bigvee_{1 \leq j < k} E_{i_{j1}, i_{j2}}^! \vee \bigvee_{k \leq j} F_{i_{j1}, i_{j2}}^?$$

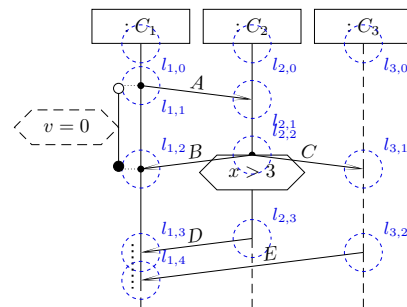


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## Loops

- How long may we **legally** stay at a cut  $q$ ?



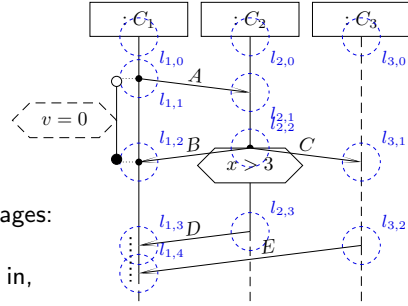
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## Loops

- How long may we **legally** stay at a cut  $q$ ?
- **Intuition:** those  $(\sigma_i, cons_i, Snd_i)$  are allowed to fire the self-loop  $(q, \psi, q)$  where
  - $cons_i \cup Snd_i$  comprises only irrelevant messages:
    - **weak mode:** no message from a direct successor cut is in,
    - **strict mode:** no message occurring in the LSC is in,
  - $\sigma_i$  satisfies the local invariants active at  $q$

And nothing else.



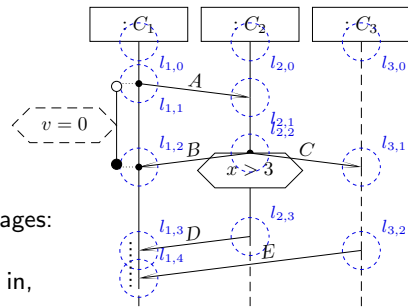
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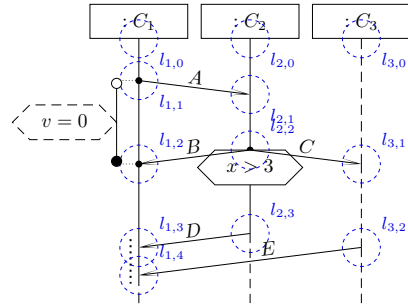
- **Formally:** Let  $F := F_1 \cup \dots \cup F_n$  be the union of the firedsets of  $q$ .

$$\psi := \underbrace{\neg(\bigvee \mathcal{E}(F))}_{= \text{true if } F = \emptyset} \wedge \psi(q).$$



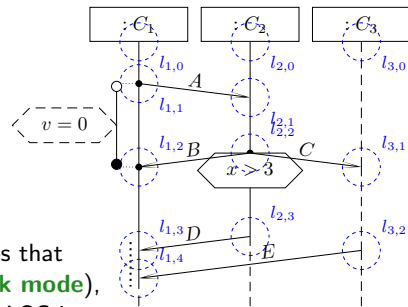
## Progress

- When do we move from  $q$  to  $q'$ ?



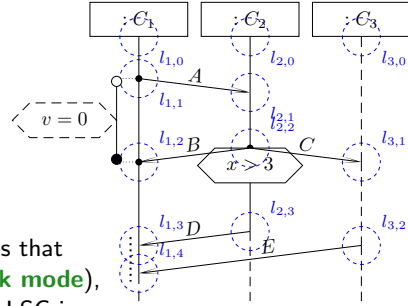
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  - $cons_i \cup Snd_i$  comprises exactly the messages that distinguish  $F$  from other firedsets of  $q$  (**weak mode**), and in addition no message occurring in the LSC is in  $cons_i \cup Snd_i$  (**strict mode**),
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  - $\sigma_i$  satisfies the local invariants and conditions relevant at  $q'$ .
- **Formally:** Let  $F, F_1, \dots, F_n$  be the firedsets of  $q$  and let  $q \rightsquigarrow_F q'$  (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \wedge \neg(\bigvee(\mathcal{E}(F_1) \cup \dots \cup \mathcal{E}(F_n)) \setminus \mathcal{E}(F)) \wedge \bigwedge \psi(q, q')$ .



## Step II: Conditions and Local Invariants

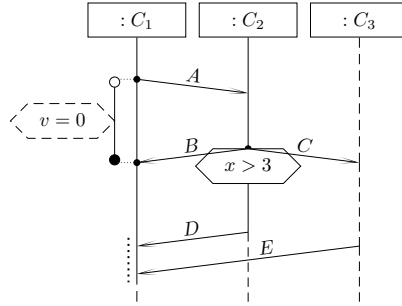
## Some More Helper Functions

- **Constraints** relevant **at** cut  $q$ :

$$\psi_\theta(q) = \{\psi \mid \exists l \in q, l' \notin q \mid (l, \psi, \theta, l') \in \text{LocInv} \vee (l', \psi, \theta, l) \in \text{LocInv}\},$$

$$\psi(q) = \psi_{\text{hot}}(q) \cup \psi_{\text{cold}}(q)$$

$$\bigwedge \emptyset := \text{false}; \quad \bigwedge \{\psi_1, \dots, \psi_n\} := \bigwedge_{1 \leq i \leq n} \psi_i$$



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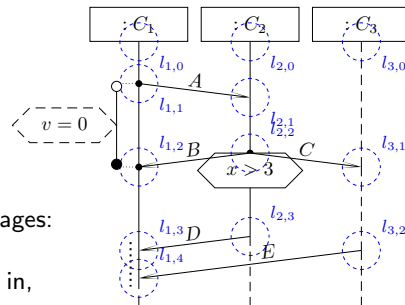
## Loops with Conditions

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And nothing else.

- **Formally:** Let  $F := F_1 \cup \dots \cup F_n$  be the union of the firedsets of  $q$ .

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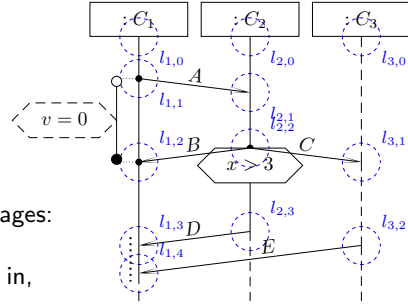
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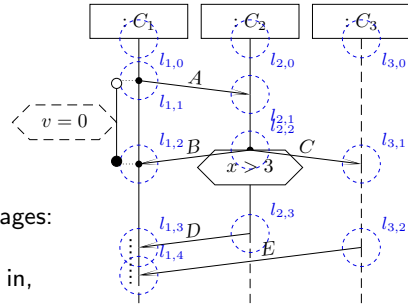
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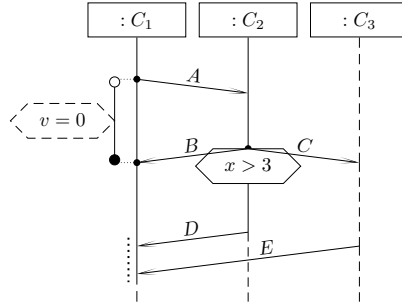


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## Even More Helper Functions

- **Constraints** relevant when moving from  $q$  to cut  $q'$ :

$$\begin{aligned} \psi_\theta(q, q') &= \{\psi \mid \exists L \subseteq \mathcal{L} \mid (L, \psi, \theta) \in \text{Cond} \wedge L \cap (q' \setminus q) \neq \emptyset\} \\ &\cup \psi_\theta(q') \\ &\setminus \{\psi \mid \exists l \in q' \setminus q, l' \in \mathcal{L} \mid (l, \circ, \text{expr}, \theta, l') \in \text{Loclnv} \vee (l', \text{expr}, \theta, \circ, l) \in \text{Loclnv}\} \\ &\cup \{\psi \mid \exists l \in q' \setminus q, l' \in \mathcal{L} \mid (l, \bullet, \text{expr}, \theta, l') \in \text{Loclnv} \vee (l', \text{expr}, \theta, \bullet, l) \in \text{Loclnv}\} \\ \psi(q, q') &= \psi_{\text{hot}}(q, q') \cup \psi_{\text{cold}}(q, q') \end{aligned}$$

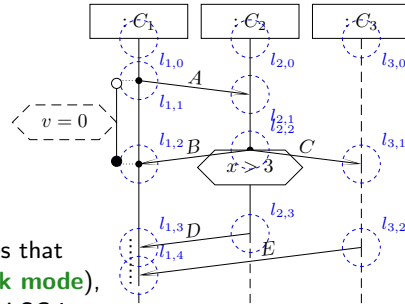


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## Progress with Conditions

- When do we move from  $q$  to  $q'$ ?
- **Intuition:** those  $(\sigma_i, \text{cons}_i, \text{Snd}_i)$  fire the progress transition  $(q, \psi, q')$  for which there exists a firedset  $F$  such that  $q \rightsquigarrow_F q'$  and
  - $\text{cons}_i \cup \text{Snd}_i$  comprises exactly the messages that distinguish  $F$  from other firedsets of  $q$  (**weak mode**), and in addition no message occurring in the LSC is in  $\text{cons}_i \cup \text{Snd}_i$  (**strict mode**),
  - $\sigma_i$  satisfies the local invariants and conditions relevant at  $q'$ .
- **Formally:** Let  $F, F_1, \dots, F_n$  be the firedsets of  $q$  and let  $q \rightsquigarrow_F q'$  (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \wedge \neg(\bigvee(\mathcal{E}(F_1) \cup \dots \cup \mathcal{E}(F_n)) \setminus \mathcal{E}(F)) \wedge \bigwedge \psi(q, q')$ .



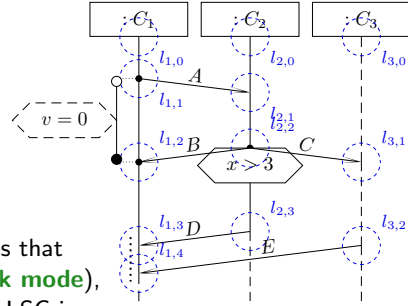
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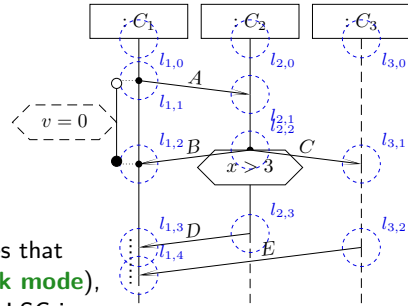
## Progress with Conditions

- When do we move from  $q$  to  $q'$ ?
- **Intuition:** those  $(\sigma_i, cons_i, Snd_i)$  fire the progress transition  $(q, \psi, q')$  for which there exists a firedset  $F$  such that  $q \rightsquigarrow_F q'$  and
  - $cons_i \cup Snd_i$  comprises exactly the messages that distinguish  $F$  from other firedsets of  $q$  (**weak mode**), and in addition no message occurring in the LSC is in  $cons_i \cup Snd_i$  (**strict mode**),
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## Progress with Conditions

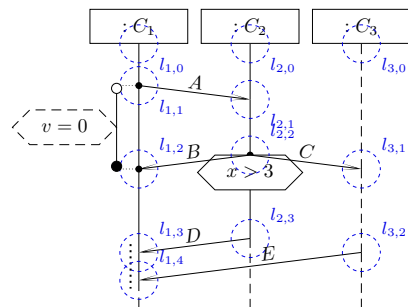
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### Step III: Cold Conditions and Cold Local Invariants

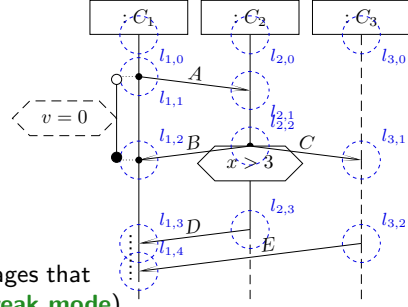
#### Legal Exits

- When do we take a legal exit from  $q$ ?



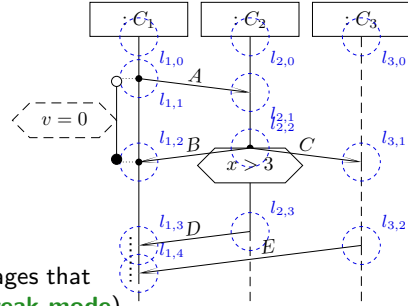
## Legal Exits

- When do we take a legal exit from  $q$ ?
- **Intuition:** those  $(\sigma_i, cons_i, Snd_i)$  fire the legal exit transition  $(q, \psi, \mathcal{L})$ 
  - for which there exists a firedset  $F$  and some  $q'$  such that  $q \rightsquigarrow_F q'$  and
    - $cons_i \cup Snd_i$  comprises exactly the messages that distinguish  $F$  from other firedsets of  $q$  (**weak mode**), and in addition no message occurring in the LSC is in  $cons_i \cup Snd_i$  (**strict mode**) and
    - at least one cold condition or local invariant relevant when moving to  $q'$  is violated, or
  - for which there is no matching firedset and at least one cold local invariant relevant at  $q$  is violated.



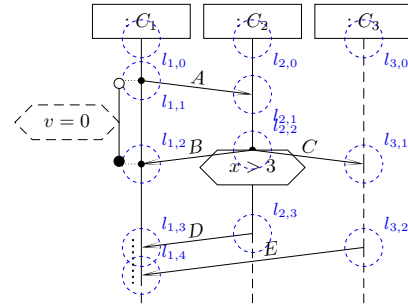
## Legal Exits

- When do we take a legal exit from  $q$ ?
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  - for which there is no matching firedset and at least one cold local invariant relevant at  $q$  is violated.



- **Formally:** Let  $F_1, \dots, F_n$  be the firedsets of  $q$  with  $q \rightsquigarrow_{F_i} q'_i$ .
  - $\psi := \bigvee_{i=1}^n \bigwedge \mathcal{E}(F_i) \wedge \neg(\bigvee(\mathcal{E}(F_1) \cup \dots \cup \mathcal{E}(F_n)) \setminus \mathcal{E}(F_i)) \wedge \bigvee \psi_{\text{cold}}(q, q'_i) \vee \neg(\bigvee \mathcal{E}(F_i)) \wedge \bigvee \psi_{\text{cold}}(q)$

## Example



## Finally: The LSC Semantics

A **full LSC**  $L$  consist of

- a **body**  $(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$ ,
- an **activation condition** (here: event)  $ac = E_{i_1, i_2}^?$ ,  $E \in \mathcal{E}$ ,  $i_1, i_2 \in I$ ,
- an **activation mode**, either **initial** or **invariant**,
- a **chart mode**, either **existential** (cold) or **universal** (hot).

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A set  $W$  of words over  $\mathcal{S}$  and  $\mathcal{D}$  **satisfies**  $L$ , denoted  $W \models L$ , iff  $L$

- **universal** (= hot), **initial**, and  
 $\forall w \in W \forall \beta : I \rightarrow \text{dom}(\sigma(w^0)) \bullet w \text{ activates } L \implies w \in \mathcal{L}_\beta(\mathcal{B}_L).$
- **existential** (= cold), **initial**, and  
 $\exists w \in W \exists \beta : I \rightarrow \text{dom}(\sigma(w^0)) \bullet w \text{ activates } L \wedge w \in \mathcal{L}_\beta(\mathcal{B}_L).$
- **universal** (= hot), **invariant**, and  
 $\forall w \in W \forall k \in \mathbb{N}_0 \forall \beta : I \rightarrow \text{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \implies w/k \in \mathcal{L}_\beta(\mathcal{B}_L).$
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