Contents & Goals

Last Lecture:

- LSC concrete syntax.
- LSC intuitive semantics.

This Lecture:

- Educational Objectives: Capabilities for following tasks/questions.
  - What does this LSC mean?
  - Are this UML model’s state machines consistent with the interactions?
  - Please provide a UML model which is consistent with this LSC.
  - What is: activation, hot/cold condition, pre-chart, etc.?

- Content:
  - Symbolic Büchi Automata (TBA) and its (accepted) language.
  - Words of a model.
  - LSC abstract syntax.
  - LSC formal semantics.
\[ \mathcal{I} = (\mathcal{I}, \mathcal{C}, \mathcal{V}, atr), SM \]

\[ M = (\Sigma_\mathcal{I}, A_\mathcal{I}, \rightarrow_\mathcal{SM}) \]

\[ \pi = (\sigma_0, \varepsilon_0) \xrightarrow{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots \]

\[ w_\pi = ((\sigma_i, cons_i, Snd_i))_{i \in \mathbb{N}} \]

\[ CD, SM \]

\[ \varphi \in \text{OCL} \]

\[ CD, SD \]

\[ B = (Q_{SD}, q_0, A_\mathcal{I}, \rightarrow_{SD}, F_{SD}) \]

\[ G = (N, E, f) \]

\[ \text{OD} \]
Excursus: Symbolic Büchi Automata (over Signature)
Symbolic Büchi Automata

Definition. A **Symbolic Büchi Automaton** (TBA) is a tuple

\[ \mathcal{B} = (\text{Expr}_\mathcal{B}(X), X, Q, q_{ini}, \to, Q_F) \]

where

- \( X \) is a set of logical variables,
- \( \text{Expr}_\mathcal{B}(X) \) is a set of Boolean expressions over \( X \),
- \( Q \) is a finite set of **states**,
- \( q_{ini} \in Q \) is the initial state,
- \( \to \subseteq Q \times \text{Expr}_\mathcal{B}(X) \times Q \) is the **transition relation**. Transitions \((q, \psi, q')\) from \( q \) to \( q' \) are labelled with an expression \( \psi \in \text{Expr}_\mathcal{B}(X) \).
- \( Q_F \subseteq Q \) is the set of **fair** (or accepting) states.
\[ \Sigma = \{0, 1\} \]

\[ L(A) = 0^* \]

\[ L(A) = (01)^* \]

\[ L(A) = (01)^*0 \]

\[ \nu = 011010 \]

\[ B = \]

\[ \Sigma = \{0, 1\} \]

\[ w \in \Sigma_0^\omega \] infinite sequences of letters

\[ w = 01010101... \in L(B) \]

\[ w = 01^\omega \notin L(B) \]

\[ w = 01^\omega \notin L(B) \]

\[ \mu_{w_1}(B) \leq Q^\omega \]

\[ \mu_{w_2}(B) = 0^\omega \]

\[ \mu_{w_3}(B) = 0\cdot 2^\omega \]

\[ \mu_{w_4}(B) = 0 \cdot 1^\omega \]

\[ \mu_{w_5}(B) = (11)^+ (00)^\omega \]

\[ \in L(B) \]
(Expr_B(X), X, Q, q_{ini}, \rightarrow, Q_F), (q, \psi, q') \in \rightarrow,

Q = \{ q_1, q_2, \ldots, q_7 \}

q_{ini} = q_2

Q_F = \{ q_7 \}

X = \{ x, y \}

Expr_B(X): \psi = a(x, y) \land b(y, z) \land c(y, z) \land d(y, z) \land e(y, z)

\rightarrow = \{ (z_2, \neg b(x, y), \psi) \mid z_2, z_2 \in X \}
Definition. Let $X$ be a set of logical variables and let $Expr_B(X)$ be a set of Boolean expressions over $X$.

A set $(\Sigma, \cdot \models \cdot)$ is called an alphabet for $Expr_B(X)$ if and only if

- for each $\sigma \in \Sigma$,
- for each expression $expr \in Expr_B$, and
- for each valuation $\beta : X \to \mathcal{D}(X)$ of logical variables to domain $\mathcal{D}(X)$,

either $\sigma \models_\beta expr$ or $\sigma \not\models_\beta expr$.

An infinite sequence

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^\omega$$

over $(\Sigma, \cdot \models \cdot)$ is called word for $Expr_B(X)$. 
\[
\text{Word Example}
\]

\[
\begin{align*}
q_1 & : \neg a(x, y) \\
q_2 & : \neg b(x, y) \\
q_3 & : \neg (c(y, x) \lor e(y, z)) \\
q_4 & : \neg (d(y, z) \lor f(y, x)) \\
q_5 & : d(y, z) \land f(y, x) \\
q_6 & : d(y, z) \\
q_7 & : \text{true}
\end{align*}
\]

\[b(x, y) \land \neg \text{expr} \]

\[b(x, y) \land \text{expr} \]

\[\neg a(x, y) \land \neg b(x, y) \land (c(y, x) \land \neg e(y, z)) \land (d(y, z) \land \neg f(y, x)) \land \neg f(y, x) \land d(y, z) \land f(y, x) \land \neg d(y, z) \land f(y, x) \land d(y, z) \land \text{true}\]
**Definition.** Let $\mathcal{B} = (Expr_\mathcal{B}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ be a TBA and

$$w = \sigma_1, \sigma_2, \sigma_3, \ldots \in \Sigma$$

a word for $Expr_\mathcal{B}(X)$.

An infinite sequence

$$\varrho = q_0, q_1, q_2, \ldots \in Q^\omega$$

is called run of $\mathcal{B}$ over $w$ under valuation $\beta : X \rightarrow \mathcal{D}(X)$ if and only if

- $q_0 = q_{ini}$,
- for each $i \in \mathbb{N}_0$ there is a transition $(q_i, \psi_i, q_{i+1}) \in \rightarrow$ of $\mathcal{B}$ such that $\sigma_i \models_\beta \psi_i$. 
$\varrho = q_0, q_1, q_2, \ldots \in Q^\omega$ s.t. $\sigma_i \models^\beta \psi_i$, $i \in \mathbb{N}_0$. 

See Slide 5a
Definition.
We say \( \mathcal{B} \) accepts word \( w \) (under \( \beta \)) if and only if \( \mathcal{B} \) has a run
\[
\varrho = (q_i)_{i \in \mathbb{N}_0}
\]
over \( w \) such that fair (or accepting) states are visited infinitely often by \( \varrho \), i.e., such that
\[
\forall i \in \mathbb{N}_0 \exists j > i : q_j \in Q_F.
\]

We call the set \( \mathcal{L}_\beta(\mathcal{B}) \subseteq \Sigma^\omega \) of words for \( Expr_\mathcal{B}(X) \) that are accepted by \( \mathcal{B} \) the language of \( \mathcal{B} \).
\[ \mathcal{L}_\beta(\mathcal{B}) \] consists of the words

\[ w = (\sigma_i)_{i \in \mathbb{N}_0} \]

where for \(0 \leq n < m < k < \ell\) we have

- for \(0 \leq i < n\), \(\sigma_i \not\vdash \beta \text{a}(x,y)\)
- \(\sigma_n \vdash \beta \text{a}(x,y)\)
- for \(n < i < m\), \(\sigma_i \not\vdash \beta \text{b}(x,y)\)
- \(\sigma_m \vdash \beta \text{b}(x,y) \land \text{expr}\)
- for \(m < i < k\), \(\sigma_i \not\vdash \beta (\text{c}(y,x) \lor \text{e}(y,z))\)
- \(\sigma_k \vdash \beta (\text{c}(y,x) \land \text{e}(y,z))\)
- for \(k < i < \ell\), \(\sigma_i \not\vdash \beta \text{d}(y,z) \lor \text{f}(y,x)\)
- \(\sigma_{\ell} \vdash \beta \text{f}(y,x) \land \text{true}\)

...
Back to Main Track: Language of a Model
**Definition.** Let $\mathcal{I} = (\mathcal{I}, C, V, atr, \mathcal{E})$ be a signature and $\mathcal{D}$ a structure of $\mathcal{I}$. A **word** over $\mathcal{I}$ and $\mathcal{D}$ is an infinite sequence $(\sigma_i, cons_i, Snd_i)_{i \in \mathbb{N}_0}$

$$\in \left( \sum_{\mathcal{D}} \times 2^{\mathcal{D}(C) \times Evs(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(C)} \times 2^{\mathcal{D}(C) \times Evs(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(C)} \right)^\omega.$$
**The Language of a Model**

**Recall:** A UML model \( \mathcal{M} = (\mathcal{C} \mathcal{D}, \mathcal{S} \mathcal{M}, \mathcal{O} \mathcal{D}) \) and a structure \( \mathcal{D} \) denotes a set \([\mathcal{M}]\) of (initial and consecutive) **computations** of the form

\[
(\sigma_0, \varepsilon_0) \xrightarrow{a_0} (\sigma_1, \varepsilon_1) \xrightarrow{a_1} (\sigma_2, \varepsilon_2) \xrightarrow{a_2} \ldots \quad \text{where}
\]

\[
a_i = (\text{cons}_i, \text{Snd}_i, u_i) \in 2^{\mathcal{D}(\mathcal{C}) \times \text{Ev}(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{D})} \times 2^{\mathcal{D}(\mathcal{D}) \times \text{Ev}(\mathcal{E}, \mathcal{D}) \times \mathcal{D}(\mathcal{D})} \times \mathcal{D}(\mathcal{C}).
\]

For the connection between models and interactions, we **disregard** the configuration of the ether and who made the step, and define as follows:

**Definition.** Let \( \mathcal{M} = (\mathcal{C} \mathcal{D}, \mathcal{S} \mathcal{M}, \mathcal{O} \mathcal{D}) \) be a UML model and \( \mathcal{D} \) a structure. Then

\[
\mathcal{L}(\mathcal{M}) := \{(\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma^\mathcal{D} \times \tilde{A})^\omega \mid
\]

\[
\exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow{u_0} \cdots \in [\mathcal{M}]
\]

is the **language** of \( \mathcal{M} \).
Example: The Language of a Model

\[ \mathcal{L}(\mathcal{M}) := \{(\sigma_i, \text{cons}_i, \text{Snd}_i)_{i \in \mathbb{N}_0} \in (\Sigma^2 \times \tilde{A})^\omega \mid \exists (\varepsilon_i, u_i)_{i \in \mathbb{N}_0} : (\sigma_0, \varepsilon_0) \xrightarrow{(\text{cons}_0, \text{Snd}_0)} u_0 \rightarrow (\sigma_1, \varepsilon_1) \cdots \in \llbracket \mathcal{M} \rrbracket \} \]
Signal and Attribute Expressions

- Let $\mathcal{I} = (\mathcal{T}, \mathcal{C}, V, atr, \mathcal{E})$ be a signature and $X$ a set of logical variables,

- The signal and attribute expressions $Expr_\mathcal{I}(\mathcal{E}, X)$ are defined by the grammar:

  $$\psi ::= \text{true} \mid expr \mid E_{x,y}^1 \mid E_{x,y}^2 \mid \neg \psi \mid \psi_1 \lor \psi_2,$$

  where $expr : \text{Bool} \in Expr_\mathcal{I}$, $E \in \mathcal{E}$, $x, y \in X$. 
Satisfaction of Signal and Attribute Expressions

- Let $(σ, cons, Snd) ∈ Σ_φ × ˜A$ be a triple consisting of system state, consume set, and send set.
- Let $β : X → ℰ(ℰ)$ be a valuation of the logical variables.

Then

- $(σ, cons, Snd) |=_β \text{true}$
- $(σ, cons, Snd) |=_β \neg ψ$ if and only if not $(σ, cons, Snd) |=_β ψ$
- $(σ, cons, Snd) |=_β ψ_1 \lor ψ_2$ if and only if
  $$(σ, cons, Snd) |=_β ψ_1 \text{ or } (σ, cons, Snd) |=_β ψ_2$$
- $(σ, cons, Snd) |=_β expr$ if and only if $I[expr](σ, β) = 1$
- $(σ, cons, Snd) |=_β E^!_{x,y}$ if and only if $∃ \vec{d} • (β(x), (E, \vec{d}), β(y)) ∈ Snd$
- $(σ, cons, Snd) |=_β E^?_{x,y}$ if and only if $∃ \vec{d} • (β(x), (E, \vec{d}), β(y)) ∈ cons$
Satisfaction of Signal and Attribute Expressions

- Let \((\sigma, \text{cons}, \text{Snd}) \in \Sigma \times \tilde{A}\) be a triple consisting of system state, consume set, and send set.
- Let \(\beta : X \to \mathcal{D}(\mathcal{C})\) be a valuation of the logical variables.

Then
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} \text{true}\)
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} \neg \psi\) if and only if not \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi\)
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi_1 \lor \psi_2\) if and only if
  \[(\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi_1\] or \[(\sigma, \text{cons}, \text{Snd}) \models_{\beta} \psi_2\]
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} \text{expr}\) if and only if \(I[expr](\sigma, \beta) = 1\)
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} E_x^l_{x,y}\) if and only if \(\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in \text{Snd}\)
- \((\sigma, \text{cons}, \text{Snd}) \models_{\beta} E_x^?_{x,y}\) if and only if \(\exists \vec{d} \bullet (\beta(x), (E, \vec{d}), \beta(y)) \in \text{cons}\)

Observation: semantics of models keeps track of sender and receiver at sending and consumption time. We disregard the event identity.
Alternative: keep track of event identities.
\textbf{TBA over Signature}

\textbf{Definition.} A TBA

\[
\mathcal{B} = (\text{Expr}_B(X), X, Q, q_{\text{ini}}, \rightarrow, Q_F)
\]

where $\text{Expr}_B(X)$ is the set of \textbf{signal and attribute expressions} $\text{Expr}_\mathcal{I}(\mathcal{E}, X)$ over signature $\mathcal{I}$ is called \textbf{TBA over $\mathcal{I}$}.

- Any word over $\mathcal{I}$ and $\mathcal{D}$ is then a word for $\mathcal{B}$.
  (By the satisfaction relation defined on the previous slide; $\mathcal{D}(X) = \mathcal{D}(\mathcal{E})$.)

- Thus a TBA over $\mathcal{I}$ accepts words of models with signature $\mathcal{I}$.
  (By the previous definition of TBA.)
TBA over Signature Examp

\[(\sigma, \text{cons}, \text{Snd}) \models_{\beta} \text{expr} \iff I[[\text{expr}]](\sigma, \beta) = 1;\]

\[(\sigma, \text{cons}, \text{Snd}) \models_{\beta} E_{x,y}^! \iff (\beta(x), (E, \bar{d}), \beta(y)) \in \text{Snd}\]
\[ \mathcal{I} = (\mathcal{I}, C, V, \text{attr}), \text{SM} \]

\[ M = (\Sigma_{\mathcal{I}}, A_{\mathcal{I}}, \rightarrow_{\text{SM}}) \]

\[ \mathcal{I}, \text{SD} \]

\[ B = (Q_{SD}, q_0, A_{\mathcal{I}}, \rightarrow_{SD}, F_{SD}) \]

\[ \mathcal{I} \subseteq \text{UML} \]

\[ \varphi \in \text{OCL} \]

\[ \mathcal{S} = (\mathcal{S}, \mathcal{E}, \mathcal{V}, \text{attr}) \]

\[ \mathcal{P} = (\sigma_0, \varepsilon_0) \xrightarrow{ \text{cons}_0, \text{Snd}_0 } (\sigma_1, \varepsilon_1) \cdots \]

\[ w_\pi = ((\sigma_i, \text{cons}_i, \text{Snd}_i))_{i \in \mathbb{N}} \]

\[ G = (N, E, f, \text{OD}) \]

\[ \mathcal{O} \subseteq \text{UML} \]
Live Sequence Charts Abstract Syntax
Example

LSC: \( L \)
AC: \( actcond \)
AM: invariant \( I: \) strict

Environment : LightsCtrl : CrossingCtrl : BarrierCtrl

\[ \text{secreq} \]
\[ \text{lights\_on} \]
\[ \text{lights\_ok} \]
\[ \text{barrier\_down} \]
\[ \text{barrier\_ok} \]
\[ \text{done} \]
\[ \text{t(10)} \]
\[ \text{t} \]

\[ \text{t} \]

\[ \neg \text{MvUp} \]
LSC Body: Abstract Syntax

Let \( \Theta = \{ \text{hot, cold} \} \). An **LSC body** is a tuple

\[
(I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})
\]
Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of **instance lines**,
LSC Body: Abstract Syntax

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of **instance lines**,
- $(\mathcal{L}, \preceq)$ is a finite, non-empty, partially ordered set of **locations**;
  - each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$, 

\[
\begin{align*}
A & \quad v = 0 \\
B & \quad x > 3 \\
C & \quad D \quad E
\end{align*}
\]
LSC Body: Abstract Syntax

Let $\Theta = \{\text{hot, cold}\}$. An LSC body is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of instance lines,
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  each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an equivalence relation on locations, the simultaneity relation,
**LSC Body: Abstract Syntax**

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of **instance lines**, 
- $(\mathcal{L}, \preceq)$ is a finite, non-empty, partially ordered set of **locations**; each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$, 
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an **equivalence relation** on locations, the **simultaneity** relation, 
- $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, \text{atr}, \mathcal{E})$ is a signature,
**LSC Body: Abstract Syntax**

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of **instance lines**, 
- $(\mathcal{L}, \preceq)$ is a finite, non-empty, **partially ordered** set of **locations**;
  - each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an **equivalence relation** on locations, the **simultaneity** relation,
- $\mathcal{I} = (\mathcal{I}, \mathcal{C}, V, \text{atr}, \mathcal{E})$ is a signature,
- $\text{Msg} \subseteq \mathcal{L} \times \mathcal{E} \times \mathcal{L}$ is a set of **asynchronous messages** with $(l, b, l') \in \text{Msg}$ only if $l \preceq l'$,

**Not:** **instantaneous messages** — could be linked to method/operation calls.
LSC Body: Abstract Syntax

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (L, \preceq), \sim, L, \text{Msg}, \text{Cond}, \text{LocInv})$$

- $I$ is a finite set of **instance lines**, 
- $(L, \preceq)$ is a finite, non-empty, partially ordered set of **locations**;
  each $l \in L$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq L \times L$ is an **equivalence relation** on locations, the simultaneity relation,
- $L = (I, C, V, \text{atr}, E)$ is a signature,
- $\text{Msg} \subseteq L \times E \times L$ is a set of asynchronous messages with $(l, b, l') \in \text{Msg}$ only if $l \preceq l'$,
  **Not:** instantaneous messages — could be linked to method/operation calls.
- $\text{Cond} \subseteq (2^L \setminus \emptyset) \times \text{Expr}_{\mathcal{G}} \times \Theta$ is a set of **conditions**
  where $\text{Expr}_{\mathcal{G}}$ are OCL expressions over $W = I \cup \{\text{self}\}$
  with $(L, \text{expr}, \theta) \in \text{Cond}$ only if $l \sim l'$ for all $l, l' \in L$, 

\[
v = 0
\]

\[
x > 3
\]
LSC Body: Abstract Syntax

Let $\Theta = \{\text{hot, cold}\}$. An **LSC body** is a tuple

$$(I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg, Cond, LocInv})$$

- $I$ is a finite set of *instance lines*,
- $(\mathcal{L}, \preceq)$ is a finite, non-empty, *partially ordered* set of *locations*;
  each $l \in \mathcal{L}$ is associated with a temperature $\theta(l) \in \Theta$ and an instance line $i_l \in I$,
- $\sim \subseteq \mathcal{L} \times \mathcal{L}$ is an *equivalence relation* on locations, the *simultaneity* relation,
- $\mathcal{S} = (\mathcal{I}, \mathcal{E}, V, \text{atr}, \mathcal{E})$ is a signature,
- $\text{Msg} \subseteq \mathcal{L} \times \mathcal{E} \times \mathcal{L}$ is a set of *asynchronous messages* with $(l,b,l') \in \text{Msg}$ only if $l \preceq l'$,
  **Not**: instantaneous messages — could be linked to method/operation calls.
- $\text{Cond} \subseteq (2^{\mathcal{L}} \setminus \emptyset) \times \text{Expr}_{\mathcal{S}} \times \Theta$ is a set of *conditions*
  where $\text{Expr}_{\mathcal{S}}$ are OCL expressions over $W = I \cup \{\text{self}\}$
  with $(L, expr, \theta) \in \text{Cond}$ only if $l \sim l'$ for all $l, l' \in L$,
- $\text{LocInv} \subseteq \mathcal{L} \times \{\circ, \bullet\} \times \text{Expr}_{\mathcal{S}} \times \Theta \times \mathcal{L} \times \{\circ, \bullet\}$
  is a set of *local invariants*. 
Well-Formedness

**Bondedness/no floating conditions:** (could be relaxed a little if we wanted to)

- For each location \( l \in \mathcal{L} \), **if** \( l \) is the location of
  - a **condition**, i.e.
    \[
    \exists (L, expr, \theta) \in \text{Cond} : l \in L, \text{ or}
    \]
  - a **local invariant**, i.e.
    \[
    \exists (l_1, i_1, expr, \theta, l_2, i_2) \in \text{LocInv} : l \in \{l_1, l_2\}, \text{ or}
    \]

  **then** there is a location \( l' \) **equivalent** to \( l \), i.e. \( l \sim l' \), which is the location of
  - an **instance head**, i.e. \( l' \) is minimal wrt. \( \preceq \), or
  - a **message**, i.e.
    \[
    \exists (l_1, b, l_2) \in \text{Msg} : l \in \{l_1, l_2\}.
    \]

**Note:** if messages in a chart are **cyclic**, then there doesn’t exist a partial order (so such charts **don’t even have** an abstract syntax).
Course Map

\[ G = (N, E, f) \]

\[ \mathcal{I} = (\mathcal{I}, \mathcal{C}, V, atr), SM \]

\[ M = (\Sigma_{\mathcal{I}}, A_{\mathcal{I}}, \rightarrow_{SM}) \]

\[ \pi = (\sigma_0, \varepsilon_0) \xrightarrow{\text{cons}_0, \text{Snd}_0} (\sigma_1, \varepsilon_1) \cdots \xrightarrow{u_0} (\sigma_i, \varepsilon_i) \]

\[ w_{\pi} = ((\sigma_i, \text{cons}_i, \text{Snd}_i))_{i \in \mathbb{N}} \]

\[ \mathcal{J} \subseteq \varphi \in \text{OCL} \]

\[ CD, SM \]

\[ CD, SD \]

\[ B = (Q_{SD}, q_0, A_{\mathcal{I}}, \rightarrow_{SD}, F_{SD}) \]

\[ \text{Model} \]

\[ \text{Instances} \]

\[ \text{UML} \]
Live Sequence Charts Semantics
Plan:

- Given an LSC $L$ with body
  
  \[(I, (\mathcal{L}, \preceq), \sim, \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv}),\]

- construct a TBA $B_L$, and

- define $\mathcal{L}(L)$ in terms of $\mathcal{L}(B_L)$,
  in particular taking activation condition and activation mode into account.

- Then $\mathcal{M} \models L$ (universal) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(L)$. 

Recall: Intuitive Semantics

(i) **Strictly After:**

(ii) **Simultaneously:** (simultaneous region)

(iii) **Explicitly Unordered:** (co-region)

**Intuition:** A computation path violates an LSC if the occurrence of some events doesn’t adhere to the partial order obtained as the transitive closure of (i) to (iii).
Examples: Semantics?
Definition.
Let \((I, (\mathcal{L}, \preceq), \sim, \mathcal{L}, \text{Msg}, \text{Cond}, \text{LocInv})\) be an LSC body. A non-empty set \(\emptyset \neq C \subseteq \mathcal{L}\) is called a cut of the LSC body iff

- it is **downward closed**, i.e.
  \[
  \forall l, l' : l' \in C \land l \preceq l' \implies l \in C,
  \]

- it is **closed** under simultaneity, i.e.
  \[
  \forall l, l' : l' \in C \land l \sim l' \implies l \in C, \text{ and}
  \]

- it comprises at least one location per instance line, i.e.
  \[
  \forall i \in I \exists l \in C : i_l = i.
  \]
Definition.
Let \((I, (\mathcal{L}, \preceq), \sim, \mathcal{S}, \text{Msg}, \text{Cond}, \text{LocInv})\) be an LSC body. A non-empty set \(\emptyset \neq C \subseteq \mathcal{L}\) is called a cut of the LSC body iff

- it is **downward closed**, i.e.
  \[
  \forall l, l' : l' \in C \land l \preceq l' \implies l \in C,
  \]

- it is **closed** under **simultaneity**, i.e.
  \[
  \forall l, l' : l' \in C \land l \sim l' \implies l \in C, \text{ and}
  \]

- it comprises at least one **location per instance line**, i.e.
  \[
  \forall i \in I \ \exists l \in C : i_l = i.
  \]

A cut \(C\) is called **hot**, denoted by \(\theta(C) = \text{hot}\), if and only if at least one of its maximal elements is hot, i.e. if

\[
\exists l \in C : \theta(l) = \text{hot} \land \nexists l' \in C : l \prec l'
\]

Otherwise, \(C\) is called **cold**, denoted by \(\theta(C) = \text{cold}\).
Examples: Cut or Not Cut? Hot/Cold?

(i) **non-empty** set \( \emptyset \neq C \subseteq \mathcal{L} \),

(ii) **downward closed**, i.e.
\[
\forall l, l' : l' \in C \land l \preceq l' \implies l \in C
\]

(iii) **closed under simultaneity**, i.e.
\[
\forall l, l' : l' \in C \land l \sim l' \implies l \in C
\]

(iv) **at least one location per instance line**, i.e.
\[
\forall i \in I \exists l \in C : i_l = i,
\]

- \( C_0 = \emptyset \)
- \( C_1 = \{l_1,0, l_2,0, l_3,0\} \)
- \( C_2 = \{l_1,1, l_2,1, l_3,0\} \)
- \( C_3 = \{l_1,0, l_1,1\} \)
- \( C_4 = \{l_1,0, l_1,1, l_2,0, l_3,0\} \)
- \( C_5 = \{l_1,0, l_1,1, l_2,0, l_2,1, l_3,0\} \)
- \( C_6 = \mathcal{L} \setminus \{l_1,3, l_2,3\} \)
- \( C_7 = \mathcal{L} \)
A Successor Relation on Cuts

The partial order of \((\mathcal{L}, \preceq)\) and the simultaneity relation \(\sim\) induce a direct successor relation on cuts of \(\mathcal{L}\) as follows:

**Definition.** Let \(C, C' \subseteq \mathcal{L}\) be cuts of an LSC body with locations \((\mathcal{L}, \preceq)\) and messages \(\text{Msg}\).

\(C'\) is called **direct successor** of \(C\) via fired-set \(F\), denoted by \(C \rightsquigarrow_F C'\), if and only if

- \(F \neq \emptyset\),
- \(C' \setminus C = F\),
- for each message reception in \(F\), the corresponding sending is already in \(C\),

\[\forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C,\] and

- locations in \(F\), that lie on the same instance line, are pairwise unordered, i.e.

\[\forall l, l' \in F : l \neq l' \wedge i_l = i_{l'} \implies l \nparallel l' \wedge l' \nparallel l\]
Properties of the Fired-set

\[ C \rightsquigarrow_F C' \text{ if and only if} \]

- \[ F \neq \emptyset, \]
- \[ C' \setminus C = F, \]
- \[ \forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C, \text{ and} \]
- \[ \forall l, l' \in F : l \neq l' \land i_l = i_{l'} \implies l \not\preceq l' \land l' \not\preceq l \]

- **Note**: \( F \) is closed under simultaneity.
- **Note**: locations in \( F \) are direct \( \preceq \)-successors of locations in \( C \), i.e.

\[ \forall l' \in F \exists l \in C : l \prec l' \land \not\exists l'' \in C : l' \prec l'' \prec l \]
Successor Cut Examples

(i) \( F \neq \emptyset \),  
(ii) \( C' \setminus C = F \),
(iii) \( \forall (l, E, l') \in \text{Msg} : l' \in F \implies l \in C \), and
(iv) \( \forall l, l' \in F : l \neq l' \land i_l = i_{l'} \implies l \not\preceq l' \land l' \not\preceq l \)
Idea: Accept Timed Words by Advancing the Cut

• Let \( w = (\sigma_0, \text{cons}_0, \text{Snd}_0), (\sigma_1, \text{cons}_1, \text{Snd}_1), (\sigma_2, \text{cons}_2, \text{Snd}_2), \ldots \) be a word of a UML model and \( \beta \) a valuation of \( I \cup \{\text{self}\} \).

• Intuitively (and for now disregarding cold conditions), an LSC body \((I, (\mathcal{L}, \leq), \sim, \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv})\) is supposed to accept \( w \) if and only if there exists a sequence

\[
C_0 \rightsquigarrow_{F_1} C_1 \rightsquigarrow_{F_2} C_2 \cdots \rightsquigarrow_{F_n} C_n
\]

and indices \( 0 = i_0 < i_1 < \cdots < i_n \) such that for all \( 0 \leq j < n \),

- for all \( i_j \leq k < i_{j+1} \), \((\sigma_k, \text{cons}_k, \text{Snd}_k), \beta\) satisfies the hold condition of \( C_j \),
- \((\sigma_{i_j}, \text{cons}_{i_j}, \text{Snd}_{i_j}), \beta\) satisfies the transition condition of \( F_j \),
- \( C_n \) is cold,
- for all \( i_n < k \), \((\sigma_k, \text{cons}_{i_j}, \text{Snd}_{i_j}), \beta\) satisfies the hold condition of \( C_n \).
Language of LSC Body

The language of the body

\[(I, (\mathcal{L}, \preceq), \sim, \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv})\]

of LSC \(L\) is the language of the TBA

\[\mathcal{B}_L = (\text{Expr}_B(X), X, Q, q_{ini}, \rightarrow, Q_F)\]

with

- \(\text{Expr}_B(X) = \text{Expr}_\mathcal{I}(\mathcal{I}, X)\)
- \(Q\) is the set of cuts of \((\mathcal{L}, \preceq)\), \(q_{ini}\) is the instance heads cut,
- \(F = \{C \in Q \mid \theta(C) = \text{cold}\}\) is the set of cold cuts of \((\mathcal{L}, \preceq)\),
- \(\rightarrow\) as defined in the following, consisting of
  - loops \((q, \psi, q)\),
  - progress transitions \((q, \psi, q')\) corresponding to \(q \leadsto_F q'\), and
  - legal exits \((q, \psi, \mathcal{L})\).
Language of LSC Body: Intuition

\[ \mathcal{B}_L = (\text{Expr}_B(X), X, Q, q_{\text{ini}}, \rightarrow, Q_F) \]

with

- \( \text{Expr}_B(X) = \text{Expr}_\mathcal{L}(\mathcal{L}, X) \)
- \( Q \) is the set of cuts of \((\mathcal{L}, \preceq)\), \( q_{\text{ini}} \) is the instance heads cut,
- \( F = \{ C \in Q \mid \theta(C) = \text{cold} \} \) is the set of cold cuts,
- \( \rightarrow \) consists of
  - loops \((q, \psi, q)\),
  - progress transitions \((q, \psi, q')\) corresponding to \( q \leadsto_F q' \), and
  - legal exits \((q, \psi, \mathcal{L})\).

"what allows us to stay at this cut"

"what allows us to legally exit"

"characterisation of firedset \( F_n \)"

"\( \ldots F_1 \)"

true

\[ v = 0 \]
Step I: Only Messages
Some Helper Functions

- **Message-expressions of a location:**

\[ \mathcal{E}(l) := \{ E_{i_1, i_{l'}}^! \mid (l, E, l') \in \text{Msg} \} \cup \{ E_{i_{l'}, i_l}^? \mid (l', E, l) \in \text{Msg} \}, \]

\[ \mathcal{E}(\{l_1, \ldots, l_n\}) := \mathcal{E}(l_1) \cup \cdots \cup \mathcal{E}(l_n). \]

\[ \bigvee \emptyset := \text{true}; \bigvee \{E_{i_{11}, i_{12}}, \ldots, F_{i_{k1}, i_{k2}}, \ldots\} := \bigvee_{1 \leq j < k} E_{i_{j1}, i_{j2}} \bigvee_{k \leq j} F_{i_{j1}, i_{j2}} \]

\[ v = 0 \]

\[ x > 3 \]

\[ D \]

\[ E \]
Loops

- How long may we **legally** stay at a cut $q$?
Loops

- How long may we **legally** stay at a cut $q$?
- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where
  - $\text{cons}_i \cup \text{Snd}_i$ comprises only irrelevant messages:
    - **weak mode**:
      no message from a direct successor cut is in,
    - **strict mode**:
      no message occurring in the LSC is in,
  - $\sigma_i$ satisfies the local invariants active at $q$

And nothing else.
Loops

- How long may we **legally** stay at a cut $q$?
- **Intuition**: those $(\sigma_i, con_{\sigma_i}, Snd_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where
  - $con_{\sigma_i} \cup Snd_i$ comprises only irrelevant messages:
    - **weak mode**: no message from a direct successor cut is in,
    - **strict mode**: no message occurring in the LSC is in,
  - $\sigma_i$ satisfies the local invariants active at $q$

And nothing else.

- **Formally**: Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of $q$.
  - $\psi := \neg (\bigvee E(F)) \wedge \psi(q)$.
    - $= true$ if $F = \emptyset$
• When do we move from $q$ to $q'$?
When do we move from $q$ to $q'$?

**Intuition:** those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \xrightarrow{F} q'$ and

- $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (weak mode), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (strict mode),

- $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$. 
Progress

- When do we move from $q$ to $q'$?

  - **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \leadsto_F q'$ and
    - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**),
    - $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.

- **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \leadsto_F q'$ (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee (\mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n)) \setminus \mathcal{E}(F) \right) \land \psi(q, q')$. 
Step II: Conditions and Local Invariants
Some More Helper Functions

- **Constraints** relevant at cut $q$:

$$\psi_\theta(q) = \{\psi \mid \exists l \in q, l' \notin q \mid (l, \psi, \theta, l') \in \text{LocInv} \lor (l', \psi, \theta, l) \in \text{LocInv}\}$$

$$\psi(q) = \psi_{\text{hot}}(q) \cup \psi_{\text{cold}}(q)$$

$$\bigwedge \emptyset := \text{false}; \quad \bigwedge \{\psi_1, \ldots, \psi_n\} := \bigwedge_{1 \leq i \leq n} \psi_i$$
Loops with Conditions

- How long may we **legally** stay at a cut $q$?
- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where
  - $\text{cons}_i \cup \text{Snd}_i$ comprises only irrelevant messages:
    - **weak mode**:
      no message from a direct successor cut is in,
    - **strict mode**:
      no message occurring in the LSC is in,
  - $\sigma_i$ satisfies the local invariants active at $q$

And nothing else.

- **Formally**: Let $F := F_1 \cup \cdots \cup F_n$
  be the union of the firedsets of $q$.
  - $\psi := \neg(\bigvee E(F)) \land \psi(q)$.
  - $\iff true$ if $F = \emptyset$
Loops with Conditions

- How long may we **legally** stay at a cut $q$?

- **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where
  - $\text{cons}_i \cup \text{Snd}_i$ comprises only irrelevant messages:
    - **weak mode**: no message from a direct successor cut is in,
    - **strict mode**: no message occurring in the LSC is in,
  - $\sigma_i$ satisfies the local invariants active at $q$

And nothing else.

- **Formally**: Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of $q$.
  - $\psi := \neg(\bigvee \varepsilon^c(F)) \land \psi(q)$.
    - $=true$ if $F=\emptyset$
Loops with Conditions

- How long may we **legally** stay at a cut $q$?

**Intuition**: those $(\sigma_i, cons_i, Snd_i)$ are allowed to fire the self-loop $(q, \psi, q)$ where

- $cons_i \cup Snd_i$ comprises only irrelevant messages:
  - **weak mode**: no message from a direct successor cut is in,
  - **strict mode**: no message occurring in the LSC is in,
- $\sigma_i$ satisfies the local invariants active at $q$

And nothing else.

**Formally**: Let $F := F_1 \cup \cdots \cup F_n$ be the union of the firedsets of $q$.

- $\psi := \neg(\bigvee \mathcal{E}(F)) \land \land \psi(q)$.

$$= \text{true if } F = \emptyset$$
Even More Helper Functions

- **Constraints** relevant when moving from $q$ to cut $q'$:

$$
\psi_\theta(q, q') = \{ \psi \mid \exists L \subseteq L \mid (L, \psi, \theta) \in \text{Cond} \land L \cap (q' \setminus q) \neq \emptyset \}
\cup \psi_\theta(q')
\setminus \{ \psi \mid \exists l \in q' \setminus q, l' \in L \mid (l, \circ, expr, \theta, l') \in \text{LocInv} \lor (l', expr, \theta, \circ, l) \in \text{LocInv} \}
\cup \{ \psi \mid \exists l \in q' \setminus q, l' \in L \mid (l, \bullet, expr, \theta, l') \in \text{LocInv} \lor (l', expr, \theta, \bullet, l) \in \text{LocInv} \}
$$

$$\psi(q, q') = \psi_{\text{hot}}(q, q') \cup \psi_{\text{cold}}(q, q')$$
When do we move from $q$ to $q'$?

**Intuition:** those $(\sigma_i, cons_i, Snd_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \leadsto_F q'$ and

- $cons_i \cup Snd_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**),
- $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.

**Formally:** Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \leadsto_F q'$ (unique).

- $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \left( \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \right) \setminus \mathcal{E}(F) \right) \land \psi(q, q')$. 

$\mathcal{E}(F)$ is the set of enabled transitions in the firedset $F$. \(\psi\) represents a condition that must be satisfied for the transition to occur.
Progress with Conditions

- When do we move from $q$ to $q'$?

- **Intuition**: those $(\sigma_i, cons_i, Snd_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \leadsto_F q'$ and
  - $cons_i \cup Snd_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $cons_i \cup Snd_i$ (**strict mode**),
  - $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.

- **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \leadsto_F q'$ (unique).
  - $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \left( \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \right) \setminus \mathcal{E}(F) \right) \land \bigwedge \psi(q, q')$. 

- $v = 0$

- $x > 3$

- $l_{1,0}$
- $l_{1,1}$
- $l_{1,2}$
- $l_{1,3}$
- $l_{2,0}$
- $l_{2,1}$
- $l_{2,2}$
- $l_{2,3}$
- $l_{3,0}$
- $l_{3,1}$
- $l_{3,2}$
• When do we move from $q$ to $q'$?

• **Intuition**: those $(\sigma_i, \text{cons}_i, \text{Snd}_i)$ fire the progress transition $(q, \psi, q')$ for which there exists a firedset $F$ such that $q \rightsquigarrow_F q'$ and
  
  • $\text{cons}_i \cup \text{Snd}_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $\text{cons}_i \cup \text{Snd}_i$ (**strict mode**),

  • $\sigma_i$ satisfies the local invariants and conditions relevant at $q'$.

• **Formally**: Let $F, F_1, \ldots, F_n$ be the firedsets of $q$ and let $q \rightsquigarrow_F q'$ (unique).

  • $\psi := \bigwedge \mathcal{E}(F) \land \neg \left( \bigvee \left( \mathcal{E}(F_1) \cup \cdots \cup \mathcal{E}(F_n) \right) \setminus \mathcal{E}(F') \right) \land \bigwedge \psi(q, q')$. 

Step III: Cold Conditions and Cold Local Invariants
Legal Exits

- When do we take a legal exit from $q$?
Legal Exits

- When do we take a legal exit from $q$?

**Intuition**: those $(\sigma, \text{cons}_i, Snd_i)$ fire the legal exit transition $(q, \psi, \mathcal{L})$

- for which there exists a firedset $F$ and some $q'$ such that $q \sim F q'$ and

  - $\text{cons}_i \cup Snd_i$ comprises exactly the messages that distinguish $F$ from other firedsets of $q$ (**weak mode**), and in addition no message occurring in the LSC is in $\text{cons}_i \cup Snd_i$ (**strict mode**) and

  - at least one cold condition or local invariant relevant when moving to $q'$ is violated, or

  - for which there is no matching firedset and at least one cold local invariant relevant at $q$ is violated.
Legal Exits

- When do we take a legal exit from \( q \)?

  **Intuition:** those \((\sigma_i, \text{cons}_i, \text{Snd}_i)\) fire the legal exit transition \((q, \psi, L)\)

  - for which there exists a firedset \( F \) and some \( q' \) such that \( q \xrightarrow{F} q' \) and
    - \( \text{cons}_i \cup \text{Snd}_i \) comprises exactly the messages that distinguish \( F \) from other firedsets of \( q \) (weak mode), and in addition no message occurring in the LSC is in \( \text{cons}_i \cup \text{Snd}_i \) (strict mode) and
    - at least one cold condition or local invariant relevant when moving to \( q' \) is violated, or
  - for which there is no matching firedset and
    - at least one cold local invariant relevant at \( q \) is violated.

- **Formally:** Let \( F_1, \ldots, F_n \) be the firedsets of \( q \) with \( q \xrightarrow{F_i} q'_i \).

  - \( \psi := \bigvee_{i=1}^{n} \bigwedge E(F_i) \land \neg(\bigvee E(F_i) \cup \cdots \cup E(F_n)) \setminus E(F_i) \bigwedge \bigvee \psi_{\text{cold}}(q, q'_i) \land \bigvee \psi_{\text{cold}}(q) \)
Example
Finally: The LSC Semantics

A **full LSC** $L$ consist of

- a **body** $(I, (\mathcal{L}, \preceq), \sim, \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv})$,
- an **activation condition** (here: event) $ac = E_{i_1,i_2}^?, E \in \mathcal{E}, i_1, i_2 \in I$,
- an **activation mode**, either **initial** or **invariant**,
- a **chart mode**, either **existential** (cold) or **universal** (hot).
Finally: The LSC Semantics

A full LSC $L$ consists of

- a **body** $(I, (\mathcal{L}, \preceq), \sim, \mathcal{I}, \text{Msg}, \text{Cond}, \text{LocInv})$,
- an **activation condition** (here: event) $ac = E^?_{i_1,i_2}, \ E \in \mathcal{E}, \ i_1,i_2 \in I$,
- an **activation mode**, either **initial** or **invariant**,
- a **chart mode**, either **existential** (cold) or **universal** (hot).

A set $W$ of words over $\mathcal{I}$ and $\varnothing$ **satisfies** $L$, denoted $W \models L$, iff $L$

- **universal** (= hot), **initial**, and
  \[
  \forall w \in W \ \forall \beta : I \rightarrow \text{dom}(\sigma(w^0)) \bullet w \text{ activates } L \implies w \in \mathcal{L}_\beta(\mathcal{B}_L).
  \]

- **existential** (= cold), **initial**, and
  \[
  \exists w \in W \ \exists \beta : I \rightarrow \text{dom}(\sigma(w^0)) \bullet w \text{ activates } L \land w \in \mathcal{L}_\beta(\mathcal{B}_L).
  \]

- **universal** (= hot), **invariant**, and
  \[
  \forall w \in W \ \forall k \in \mathbb{N}_0 \ \forall \beta : I \rightarrow \text{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \implies w/k \in \mathcal{L}_\beta(\mathcal{B}_L).
  \]

- **existential** (= cold), **invariant**, and
  \[
  \exists w \in W \ \exists k \in \mathbb{N}_0 \ \exists \beta : I \rightarrow \text{dom}(\sigma(w^k)) \bullet w/k \text{ activates } L \land w/k \in \mathcal{L}_\beta(\mathcal{B}_L).
  \]
References
References


