Software Design, Modelling and Analysis in UML

Lecture 02: Semantical Model

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Contents & Goals

Last Lecture:

- Motivation: model-based development of things (houses, software) to cope with complexity, detect errors early
- Model-based (or -driven) Software Engineering
- UML Mode of the Lecture: Blueprint.

This Lecture:

- **Educational Objectives:** Capabilities for these tasks/questions:
  - Why is UML of the form it is?
  - Shall one feel bad if not using all diagrams during software development?
  - What is a signature, an object, a system state, etc.?
    What's the purpose of signature, object, etc. in the course?
  - How do Basic Object System Signatures relate to UML class diagrams?
- **Content:**
  - Brief history of UML
  - Course map revisited
  - Basic Object System Signature, Structure, and System State

Why (of all things) UML?

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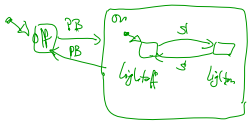
- Note: being a **modelling** languages doesn't mean being graphical (or: being a visual formalism [Harel]).
- For instance, [Kastens and Büning, 2008] also name:
  - Sets, Relations, Functions
  - Terms and Algebras
  - Propositional and Predicate Logic
  - Graphs
  - XML Schema, Entity Relation Diagrams, UML Class Diagrams
  - Finite Automata, Petri Nets, UML State Machines
- **Pro**: visual formalisms are found appealing and easier to **grasp**. Yet they are not necessarily easier to **write**!
- **Beware**: you may meet people who dislike visual formalisms just for being graphical — maybe because it is easier to "trick" people with a meaningless picture than with a meaningless formula.

  More serious: it's maybe easier to misunderstand a picture than a formula.

A Brief History of UML

- Boxes/lines and finite automata are used to visualise software **for ages**.
- **1970's**, **Software Crisis**™
  — Idea: learn from engineering disciplines to handle growing complexity.

  Languages: **Flowcharts, Nassi-Shneiderman, Entity-Relation Diagrams**
- Mid **1980**'s: **Statecharts** [Harel, 1987], **StateMate**™ [Harel et al., 1990]



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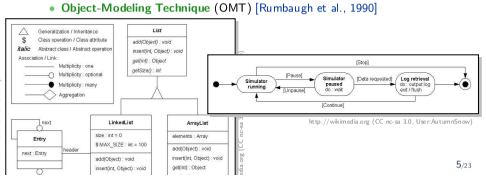
- Early **1990's**, advent of **Object-Oriented**-Analysis/Design/Programming
  — Inflation of notations and methods, most prominent:
  - **Object-Modeling Technique** (OMT) [Rumbaugh et al., 1990]



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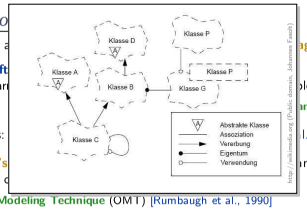

http://uebriends.net (Public domain, Johannes Finch)

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  - Object-Oriented Software Engineering (OOSE) [Jacobson et al., 1992]

  Each "persuasion" selling books, tools, seminars...
- Late 1990's: joint effort UML 0.x, 1.x
  Standards published by **Object Management Group** (OMG), "*international, open membership, not-for-profit* **computer industry** *consortium*".
- Since 2005: UML 2.x

UML Overview [OMG, 2007b, 684]

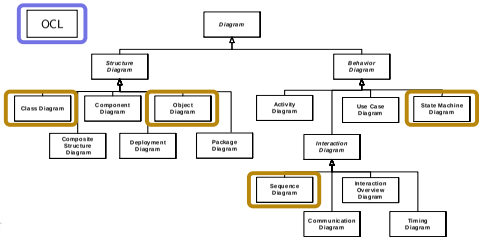


Figure A.5 - The taxonomy of structure and behavior diagram

[Dobing and Parsons, 2006]

Common Expectations on UML

- Easily writeable, readable even by customers
- Powerful enough to bridge the gap between idea and implementation
- Means to tame complexity by separation of concerns ("views")
- Unambiguous
- Standardised, exchangeable between modelling tools
- UML standard says how to develop software
- Using UML leads to better software
- ...

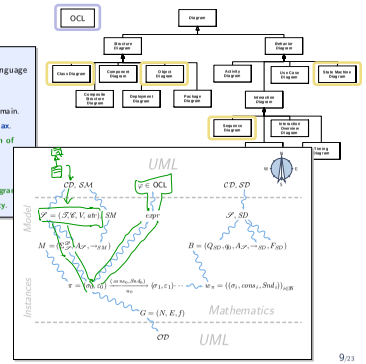
We will see...

Seriously: After the course, you should have an own opinion on each of these claims. In how far/in what sense does it hold? Why? Why not? How can it be achieved? Which ones are really only hopes and expectations? ...?

Course Map Revisited

The Plan

Recall:
- Overall aim: a formal language for software blueprints.
- Approach:
  (i) Common semantical domain.
  (ii) UML fragments as syntax.
  (iii) Abstract representation of diagrams.
  (iv) Informal semantics: UML standard
  (v) assign meaning to diagrams
  (vi) Define, e.g., consistency.



UML: Semantic Areas

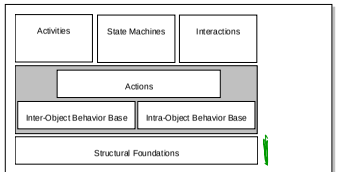


Figure 6.1 - A schematic of the UML semantic areas and their dependencies

[OMG, 2007b, 11]

Common Semantical Domain

Basic Object System Signature

Definition. A (Basic) Object System Signature is a quadruple

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$$

where

- $\mathscr{T}$ is a set of (basic) types,
- $\mathscr{C}$ is a finite set of classes,
- V is a finite set of typed attributes, i.e., each $v \in V$ has type
  - $\tau \in \mathscr{T}$ or
  - $C_{0,1}$ or C_*, where $C \in \mathscr{C}$,

  (written $v : \tau$ or $v : C_{0,1}$ or $v : C_*$),
- $atr : \mathscr{C} \to 2^V$ maps each class to its set of attributes.

for each class $D \in \mathscr{C}$ there are two different types: $D_{0,1}$ or: D_* or: $\triangle D$, $\boxed{D}$, $D_{\circledast}$

in other words: typeof: $V \to \mathscr{T} \cup \{D_{0,1}, D_* \mid D \in \mathscr{C}\}$

total function powerset of V

Note: Inspired by OCL 2.0 standard [OMG, 2006], Annex A.

Basic Object System Signature Example

$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$, (both finite),
- typed attributes V, τ from $\mathscr{T}$ or $C_{0,1}$ or C_*, $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

Example:

basic types classes attributes attribute x their type Int atr atr(C) = {p,n} atr(D)={x}

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

maps to

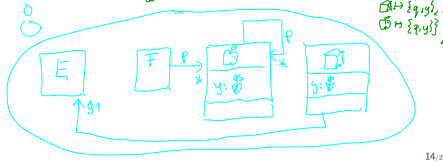
Basic Object System Signature Another Example

$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$, (both finite),
- typed attributes V, τ from $\mathscr{T}$ or $C_{0,1}$ or C_*, $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

Example:

$\mathscr{S}_1 = (\{\ldots\}, \{E, F, \ldots\}, \{y : \ldots, p : \ldots, q : E_{0,*}\}, \{E \mapsto \emptyset, F \mapsto \{p\}, \ldots \mapsto \{q, y\}, \ldots \mapsto \{z, y\}\})$



Basic Object System Structure

Definition. A Basic Object System Structure of

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$$

is a domain function $\mathscr{D}$ which assigns to each type a domain, i.e.

- $\tau \in \mathscr{T}$ is mapped to $\mathscr{D}(\tau)$,
- $C \in \mathscr{C}$ is mapped to an infinite set $\mathscr{D}(C)$ of (object) identities.

  Note: Object identities only have the "=" operation; object identities of different classes are disjoint, i.e. $\forall C, D \in \mathscr{C} : C \neq D \to \mathscr{D}(C) \cap \mathscr{D}(D) = \emptyset$.
- C_* **and** $C_{0,1}$ for $C \in \mathscr{C}$ are mapped to $2^{\mathscr{D}(C)}$.

We use $\mathscr{D}(\mathscr{C})$ to denote $\bigcup_{C \in \mathscr{C}} \mathscr{D}(C)$; analogously $\mathscr{D}(\mathscr{C}_*)$.

Note: We identify objects and object identities, because both uniquely determine each other (cf. OCL 2.0 standard).

Basic Object System Structure Example

Wanted: a structure for signature

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

Recall: by definition, seek a $\mathscr{D}$ which maps

- $\tau \in \mathscr{T}$ to **some** $\mathscr{D}(\tau)$,
- $c \in \mathscr{C}$ to **some** identities $\mathscr{D}(C)$ (infinite, disjoint for different classes),
- C_* and $C_{0,1}$ for $C \in \mathscr{C}$ to $\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) = 2^{\mathscr{D}(C)}$.

$$\begin{aligned}
\mathscr{D}(Int) &= \mathbb{Z} \\
\mathscr{D}(C) &= \mathbb{N}^+ \times \{C\} = \{1_C, 2_C, 3_C, \ldots\} \\
\mathscr{D}(D) &= \mathbb{N}^+ \times \{D\} = \{1_D, 2_D, 3_D, \ldots\} \\
\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) &= 2^{\mathbb{N}^+ \times \{C\}} \\
\mathscr{D}(D_{0,1}) = \mathscr{D}(D_*) &= 2^{\mathscr{D}(D)}
\end{aligned}$$

alternative choice:

$\mathscr{D}_2(Int) = \{-1, 1, 2, \ldots, 1, 2, 3\}$

$\mathscr{D}_2(C) = \{1, 3, 5, \ldots\}$

$\mathscr{D}_2(D) = \{2, 4, 6, \ldots\}$

$2^{\mathscr{D}_2(C)}$

$2^{\mathscr{D}_2(D)}$

System State

all object identities

partial function

partial function from V to types/domains

Definition. Let $\mathscr{D}$ be a structure of $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$.
A system state of $\mathscr{S}$ wrt. $\mathscr{D}$ is a **type-consistent** mapping

$$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*))).$$

That is, for each $u \in \mathscr{D}(C)$, $C \in \mathscr{C}$, if $u \in \mathrm{dom}(\sigma)$

- $\mathrm{dom}(\sigma(u)) = atr(C)$
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$
- $\sigma(u)(v) \in \mathscr{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathscr{C}$

We call $u \in \mathscr{D}(\mathscr{C})$ alive in σ if and only if $u \in \mathrm{dom}(\sigma)$.

We use $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ to denote the set of all system states of $\mathscr{S}$ wrt. $\mathscr{D}$.

System State Example

Signature, **Structure**:

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, \ldots\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, \ldots\}$$

Wanted: $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that

- $\mathrm{dom}(\sigma(u)) = atr(C)$,
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$,
- $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ with $D \in \mathscr{C}$.

- $\sigma_1 = \emptyset$ ← empty function
- $\sigma_2 = \{1_C \mapsto \{p \mapsto \{1_D\}, n \mapsto \{5_C, 6_C\}\}, 2_D \mapsto \{x \mapsto 3\}\}$

one way to read out:

= object 1_C has a p-link to 1_C (i.e. to itself)

= object 1_C refers to objects $5_C, 6_C$ via link n

Wrt. $\mathscr{D}_2$:

- $\sigma_3 = \{5 \mapsto \{p \mapsto \{1_D\}, n \mapsto \emptyset\}\}$

System State Example

Signature, **Structure**:

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- **Concrete**, **explicit**:

  $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$.

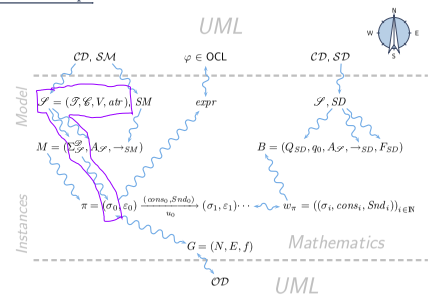
- **Alternative**: **symbolic** system state

  $\sigma = \{c_1 \mapsto \{p \mapsto \emptyset, n \mapsto \{c_2\}\}, c_2 \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, d \mapsto \{x \mapsto 23\}\}$

  assuming $c_1 \neq c_2 \in \mathscr{D}(C)$, $d \in \mathscr{D}(D)$, $c_1 \neq c_2$.

You Are Here.

Course Map



References

References

[Booch, 1993] Booch, G. (1993). *Object-oriented Analysis and Design with Applications*. Prentice-Hall.

[Dobing and Parsons, 2006] Dobing, B. and Parsons, J. (2006). How UML is used. *Communications of the ACM*, 49(5):109–114.

[Harel, 1987] Harel, D. (1987). Statecharts: A visual formalism for complex systems. *Science of Computer Programming*, 8(3):231–274.

[Harel et al., 1990] Harel, D., Lachover, H., et al. (1990). Statemate: A working environment for the development of complex reactive systems. *IEEE Transactions on Software Engineering*, 16(4):403–414.

[Jacobson et al., 1992] Jacobson, I., Christerson, M., and Jonsson, P. (1992). *Object-Oriented Software Engineering - A Use Case Driven Approach*. Addison-Wesley.

[Kastens and Büning, 2008] Kastens, U. and Büning, H. K. (2008). *Modellierung, Grundlagen und Formale Methoden*. Carl Hanser Verlag München, 2nd edition.

[OMG, 2006] OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.

[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.

[Rumbaugh et al., 1990] Rumbaugh, J., Blaha, M., Premerlani, W., Eddy, F., and Lorensen, W. (1990). *Object-Oriented Modeling and Design*. Prentice Hall.