Software Design, Modelling and Analysis in UML

Lecture 04: OCL Cont'd, Object Diagrams

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:

- OCL Syntax

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is an object diagram? What are object diagrams good for?
  - When is an object diagram called partial? What are partial ones good for?
  - When is an object diagram an object diagram (wrt. what)?
  - Is this an object diagram wrt. to that other thing?
  - How are system states and object diagrams related?
  - What does it mean that an OCL expression is satisfiable?
  - When is a set of OCL constraints said to be consistent?
  - Can you think of an object diagram which violates this OCL constraint?
- **Content:**
  - OCL Semantics
  - Object Diagrams
  - Example: Object Diagrams for Documentation
  - OCL: consistency, satisfiability

OCL Semantics [OMG, 2006]

The Task

OCL Syntax 1/4: Expressions

$expr ::=$	
w	$: \tau(w)$
$\mid expr_1 =_\tau expr_2$	$: \tau \times \tau \to Bool$
$\mid \mathsf{oclIsUndefined}_\tau(expr_1)$	$: \tau \to Bool$
$\mid \{expr_1, \ldots, expr_n\}$	$: \tau \times \cdots \times \tau \to Set(\tau)$
$\mid \mathsf{isEmpty}(expr_1)$	$: Set(\tau) \to Bool$
$\mid \mathsf{size}(expr_1)$	$: Set(\tau) \to Int$
$\mid \mathsf{allInstances}_C$	$: Set(\tau_C)$
$\mid v(expr_1)$	$: \tau_C \to \tau(v)$
$\mid r_1(expr_1)$	$: \tau_C \to \tau_D$
$\mid r_2(expr_1)$	$: \tau_C \to Set(\tau_D)$

Where, given $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$,

- $W \supseteq \{self\}$ is a set of typed logical variables, w has type $\tau(w)$
- τ is any type from $\mathscr{T} \cup T_B \cup T_\mathscr{C} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_\mathscr{C}\}$
  - T_B is a set of basic types, in the following we use $T_B = \{Bool, Int, String\}$
  - $T_\mathscr{C} = \{\tau_C \mid C \in \mathscr{C}\}$ is the set of object types,
  - $Set(\tau_0)$ denotes the set-of-τ_0 type for $\tau_0 \in T_B \cup T_\mathscr{C}$ (sufficient because of "flattening" (cf. standard))
- $v : \tau(v) \in atr(C)$, $\tau(v) \in \mathscr{T}$,
- $r_1 : D_{0,1} \in atr(C)$,
- $r_2 : D_* \in atr(C)$,
- $C, D \in \mathscr{C}$.

- Given an OCL expression $expr$, a system state $\sigma \in \Sigma^\mathscr{D}_\mathscr{S}$, and a valuation of logical variables β, define

  $I[\![\cdot]\!](\cdot, \cdot) : OCLExpressions(\mathscr{S}) \times \Sigma^\mathscr{D}_\mathscr{S} \times (W \to I(\mathscr{T} \cup T_B \cup T_\mathscr{C})) \to I(Bool)$

  such that

  $$I[\![expr]\!](\sigma, \beta) \in \{\mathsf{true}, \mathsf{false}, \bot_{Bool}\}.$$ ≈ I(Bool)

Basically business as usual...

(i) Equip each OCL (!) **basic type** with a reasonable **domain**, i.e. define function

$$I_{(i)} \text{ with } \mathrm{dom}(I) = T_B$$

(ii) Equip each **object type** τ_C with a reasonable **domain**, i.e. define function

$$I_{(ii)} \text{ with } \mathrm{dom}(I) = \tau_C$$

(most reasonable: $\mathscr{D}(C)$ determined by structure $\mathscr{D}$ of $\mathscr{S}$).

(iii) Equip each **set type** $Set(\tau_0)$ with reasonable **domain**, i.e. define function

$$I_{(iii)} \text{ with } \mathrm{dom}(I) = \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathscr{C}}\}$$

(iv) Equip each **arithmetical operation** with a reasonable **interpretation** (that is, with a **function** operating on the corresponding **domains**).

$$I_{(iv)} \text{ with } \mathrm{dom}(I) = \{+, -, \leq, \dots\}, \text{ e.g., } I(+) \in I(Int) \times I(Int) \to I(Int)$$

(v) **Set operations** similar: $I_{(v)}$ with $\mathrm{dom}(I) = \{\text{isEmpty}, \dots\}$

(vi) Equip each **expression** with a reasonable **interpretation**, i.e. define function

$$I_{(vi)} : Expr \times \Sigma^{\mathscr{D}}_{\mathscr{S}} \times (W \to I(\mathscr{T} \cup T_B \cup T_{\mathscr{C}})) \to I(Bool)$$

...except for OCL being a **three-valued logic**, and the "iterate" expression.

(i) Domains of Basic Types (of OCL)

Recall:

- $T_B = \{Bool, Int, String\}$

assume both sets disjoint

"undefined"

We set:

- $I_{(i)}(Bool) := \{true, false\} \cup \{\bot_{Bool}\}$
- $I(Int) := \mathbb{Z} \cup \{\bot_{Int}\}$
- $I(String) := \dots \cup \{\bot_{String}\}$

finite sequences of characters

We may omit index τ of $\bot_\tau$ if it is clear from context.

(ii) Domains of Object and (iii) Set Types

- Now we need a structure $\mathscr{D}$ of our signature $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$.
- **Recall:** $\mathscr{D}$ assigns an (infinite) domain $\mathscr{D}(C)$ to each class $C \in \mathscr{C}$.

- Let τ_C be an (OCL) **object type** for a class $C \in \mathscr{C}$.
- We set

$$I_{(ii)}(\tau_C) := \mathscr{D}(C) \dot\cup \{\bot_{\tau_C}\}$$

- Let τ be a type from $T_B \cup T_{\mathscr{C}}$. (2^A is powerset of A)
- We set

$$I_{(iii)}(Set(\tau)) := 2^{I(\tau)} \dot\cup \{\bot_{Set(\tau)}\}$$

Note: in the OCL standard, only **finite** subsets of $I(\tau)$.
But infinity doesn't scare **us**, so we simply allow it.

(iv) Interpretation of Arithmetic Operations

- **Literals** map to fixed values: ($I_{(iv)}(Bool)$, $I_{(iv)}(Int) = \mathbb{Z} \cup \{\bot_{Int}\}$)

$$I_{(iv)}(\mathbf{true}) := true, \quad I(\mathbf{false}) := false, \quad I(\mathbf{0}) := 0, \quad I(\mathbf{1}) := 1, \ldots$$

$$I(\mathbf{OclUndefined}_\tau) := \bot_\tau \in I(\tau) \quad (I_{(i)} \cup I_{(ii)} \cup I_{(iii)})$$

- **Boolean operations** (defined point-wise for $x_1, x_2 \in I(\tau)$):

$$I(=_\tau)(x_1, x_2) := \begin{cases} true & \text{, if } x_1 \neq \bot_\tau \neq x_2 \text{ and } x_1 = x_2 \\ false & \text{, if } x_1 \neq \bot_\tau \neq x_2 \text{ and } x_1 \neq x_2 \\ \bot_{Bool} & \text{, otherwise} \end{cases}$$

- **Integer operations** (defined point-wise for $x_1, x_2 \in I(Int)$):

$$I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & \text{, if } x_1 \neq \bot \neq x_2 \\ \bot & \text{, otherwise} \end{cases}$$

Note: There is a **common principle**.
Namely, the **interpretation** of an operation $\omega : \tau_1 \times \ldots \tau_n \to \tau$
is a function $I_{(iv)}(\omega) : I(\tau_1) \times \cdots \times I(\tau_n) \to I(\tau)$ on corresponding semantical domain(s).

(iv) Interpretation of OclIsUndefined

- The **is-undefined** predicate (defined point-wise for $x \in I(\tau)$):

$$I(\mathsf{oclIsUndefined}_\tau)(x) := \begin{cases} true & \text{, if } x = \bot_\tau \\ false & \text{, otherwise} \end{cases}$$

(v) Interpretation of Set Operations

Basically the same principle as with arithmetic operations...

Let $\tau \in T_B \cup T_{\mathscr{C}}$.

- **Set comprehension** $(x_1, \ldots, x_n \in I(\tau))$:

$$\left(I(\{\}^\tau_n)\right)(x_1, \ldots, x_n) := \{x_1, \ldots, x_n\}$$

  for all $n \in \mathbb{N}_0$

- **Empty-ness check** $(x \in I(Set(\tau)))$:

$$I(\mathsf{isEmpty}^\tau)(x) := \begin{cases} true & \text{, if } x = \emptyset \\ \bot_{Bool} & \text{, if } x = \bot_{Set(\tau)} \\ false & \text{, otherwise} \end{cases}$$

- **Counting** $(x \in I(Set(\tau)))$: (cardinality)

$$\left(I(\mathsf{size}^\tau)\right)(x) := |x| \text{ if } x \neq \bot_{Set(\tau)} \text{ and } \bot_{Int} \text{ otherwise}$$

(vi) Putting It All Together

OCL Syntax 1/4: Expressions

$expr ::=$

w	$: \tau(w)$
$\mid expr_1 =_\tau expr_2$	$: \tau \times \tau \to Bool$
$\mid \text{oclIsUndefined}_\tau(expr_1)$	$: \tau \to Bool$
$\mid \{expr_1, \ldots, expr_n\}$	$: \tau \times \cdots \times \tau \to Set(\tau)$
$\mid \text{isEmpty}(expr_1)$	$: Set(\tau) \to Bool$
$\mid \text{size}(expr_1)$	$: Set(\tau) \to Int$
$\mid \text{allInstances}_C$	$: Set(\tau_C)$
$\mid v(expr_1)$	$: \tau_C \to \tau(v)$
$\mid r_1(expr_1)$	$: \tau_C \to \tau_D$
$\mid r_2(expr_1)$	$: \tau_C \to Set(\tau_D)$

Where, given $\mathscr{S} = (\mathscr{T}, \mathscr{C},$

- $W \supseteq \{self\}$ is a set of logical variables, w has
- τ is any type from $\mathscr{T} \cup$ $\cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup$
  - T_B is a set of basic the following we use $T_B = \{Bool, Int, St$
  - $T_\mathscr{C} = \{\tau_C \mid C \in \mathscr{C}\}$ set of object types,
  - $Set(\tau_0)$ denotes the set-of-τ_0 type for $\tau_0 \in T_B \cup T_\mathscr{C}$ (sufficient because o "flattening" (cf. sta
- $v : \tau(v) \in atr(C), \tau(v)$
- $r_1 : D_{0,1} \in atr(C),$
- $r_2 : D_* \in atr(C),$
- $C, D \in \mathscr{C}$.

OCL Syntax 2/4: Constants, Arithmetical Operat

For example:

$expr ::= \ldots$

$\mid$ true, false	$: Bool$
$\mid expr_1$ {and, or, implies} $expr_2$	$: Bool \times Bool -$
$\mid$ not $expr_1$	$: Bool \to Bool$
$\mid 0, -1, 1, -2, 2, \ldots$	$: Int$
$\mid$ OclUndefined	$: \tau$
$\mid expr_1 \{+, -, \ldots\} expr_2$	$: Int \times Int \to I$
$\mid expr_1 \{<, \leq, \ldots\} expr_2$	$: Int \times Int \to I$

Generalised notation:

$expr ::= \omega(expr_1, \ldots, expr_n) \quad : \tau_1 \times \cdots \times \tau_n -$

with $\omega \in \{+, -, \ldots\}$

OCL Syntax 3/4: Iterate

$expr ::= \cdots \mid expr_1\text{->iterate}(w_1 : \tau_1 ; w_2 : \tau_2 = expr_2 \mid expr_3)$

or, with a little renaming,

$expr ::= \cdots \mid expr_1\text{->iterate}(iter : \tau_1; result : \tau_2 = expr_2 \mid expr_3)$

OCL Syntax 4/4: Context

$context ::= \text{context } w_1 : \tau_1, \ldots, w_n : \tau_n \text{ inv} : expr$

where $w \in W$ and $\tau_i \in T_\mathscr{C}$, $1 \leq i \leq n$, $n \geq 0$.

Valuations of Logical Variables

- **Recall**: we have typed logical variables $(w \in) W$, $\tau(w)$ is the type of w.
- By β, we denote a valuation of the logical variables, i.e. for each $w \in W$,

$$\beta(w) \in I(\tau(w)).$$

$$\beta : W \longrightarrow \bigcup_{w \in W} I(\tau(w))$$

$W = \{ x : Int, self_C : \tau_C \}$

$\beta : W \longrightarrow I(Int) \cup I(\tau_C)$

Example:

- $\beta(x) = 27 \in I(Int)$
- $\beta(self_C) = 1_C \in I(\tau_C) = \mathcal{D}(C) \cup \{\bot\}$

- $\beta_2(x) = \bot_{Int}$
- $\beta_2(self) = \bot_{\tau_C}$

(vi) Putting It All Together...

$$expr ::= w \mid \omega(expr_1, \ldots, expr_n) \mid \mathsf{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1\text{->}\mathsf{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)$$

- $I[\![w]\!](\sigma, \beta) := \beta(w)$
- $I[\![\omega(expr_1, \ldots, expr_n)]\!](\sigma, \beta) := I(\omega)(I[\![expr_1]\!](\sigma,\beta), \ldots, I[\![expr_n]\!](\sigma,\beta))$
- $I[\![\mathsf{allInstances}_C]\!](\sigma, \beta) := dom(\sigma) \cap \mathcal{D}(C)$

Note: in the OCL standard, $\mathrm{dom}(\sigma)$ is assumed to be **finite**.
Again: doesn't scare us.

$\mathcal{Y} = (\emptyset, \{C, D\}, \emptyset, \emptyset)$

- $\sigma_1 = \{ 1_C \mapsto \emptyset,\ 3_C \mapsto \emptyset,\ 27_C \mapsto \emptyset,\ 5_D \mapsto \emptyset \}$
- $W = \{ x : Int,\ c : \tau_C \}$
- $\beta_1 = \{ x \mapsto 13,\ c \mapsto 3_C \}$ (**)
- $I[\![allInstances_D]\!](\sigma_1, \beta_1) = dom(\sigma_1) \cap \mathcal{D}(D) = \{5_D\}$ (*)
- $I[\![size(\underbrace{allInstances_D}_{expr_1})]\!](\sigma_1, \beta_1) = (I(size))(I[\![allInstances_D]\!](\sigma_1, \beta_1))$
  $= (I(size))(\{5_D\}) = |\{5_D\}| = 1$ (***)
  (by (*); by def. of $I(size)$)
- $I[\![x > size(allInstances_D)]\!](\sigma_1, \beta_1) = (I(>))(I[\![x]\!](\sigma_1,\beta_1), I[\![size(allInstances_D)]\!](\sigma_1,\beta_1))$
  $>(x, size(allInstances_D))$
  $= I(>)(\beta_1(x), 1) = I(>)(13, 1)$ (**)
  $= true$ (by def. of $I(>)$; by def. of $I[\![\cdot]\!]$ (***))
- $I[\![1/(size(allInstances_D) - 1)]\!](\sigma_1, \beta_1) = \bot_{Int}$
  $I[\![1]\!](\sigma_1,\beta_1) = I(1) = 1 \in I(Int)$ (ω)

assuming $I(/)(x_1, x_2) = \begin{cases} x_1/x_2 & \text{if } x_1 \neq \bot \text{ and } x_2 \neq \bot \text{ and } x_2 \neq 0 \\ \bot_{Int} & \text{otherwise} \end{cases}$

(vi) Putting It All Together...

$$expr ::= w \mid \omega(expr_1, \ldots, expr_n) \mid \text{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1\text{->iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)$$

Assume $expr_1 : \tau_C$ for some $C \in \mathscr{C}$. Set $u_1 := I[\![expr_1]\!](\sigma, \beta) \in \mathscr{D}(\tau_C)$.

- $I[\![v(expr_1)]\!](\sigma,\beta) := \begin{cases} \sigma(u_1)(v) & \text{, if } u_1 \in dom(\sigma) \\ \bot_\tau & \text{, otherwise} \end{cases}$ assuming $v:\tau$
- $I[\![r_1(expr_1)]\!](\sigma,\beta) := \begin{cases} u & \text{, if } u_1 \in dom(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \bot & \text{, otherwise} \end{cases}$ $r_1 : C_{0,1}$
- $I[\![r_2(expr_1)]\!](\sigma,\beta) := \begin{cases} \sigma(u_1)(r_2) & \text{, if } u_1 \in dom(\sigma) \\ \bot_{Set(\tau_C)} & \text{, otherwise} \end{cases}$

  (Recall: σ evaluates r_2 of type C_* to a set)

$\mathcal{Y} = (\{Int, Colour\}, \{C, D\}, \{ms : C_{0,*}, sl : C_{*,1}, r : Int, c : Colour\}, \{C \mapsto \{ms, sl, r\}, D \mapsto \{c\}\})$

$\mathcal{D}(Int) = \mathbb{Z}$, $\mathcal{D}(colour) = \{red, green, blue\}$

$\sigma_2 = \{1_C \mapsto \{ms \mapsto \emptyset, sl \mapsto \{2_C, 3_C\}, r \mapsto 9\},$
$2_C \mapsto \{ms \mapsto \{1_C\}, sl \mapsto \emptyset, r \mapsto 5\},$
$3_C \mapsto \{ms \mapsto \{4_C\}, sl \mapsto \emptyset, r \mapsto 3\},$
$5_D \mapsto \{c \mapsto blue\}\}$

$\beta_2 = \{p : \tau_C \to 1_C, q : \tau_D \to 5_D, x : Int \to 4, d : colour \to green, m : \tau_C \to 2_C\}$

- $I[\![c(q)]\!](\sigma_2, \beta_2) = \sigma_2(5_D)(c) = blue$, $I[\![q]\!](\sigma_2, \beta_2) = 5_D$
- $I[\![c(q) = d]\!](\sigma_2, \beta_2) = false$
- $I[\![sl(p)]\!](\sigma_2, \beta_2) = \{2_C, 3_C\}$
- $I[\![r(ms(m))]\!](\sigma_2, \beta_2) = 9$, $I[\![ms(m)]\!](\sigma_2, \beta_2) = 1_C$
- $I[\![r(ms(p))]\!](\sigma_2, \beta_2) = \bot_{Int}$, $I[\![ms(p)]\!](\sigma_2, \beta_2) = \bot_{\tau_C}$

(vi) Putting It All Together...

$$expr ::= w \mid \omega(expr_1, \ldots, expr_n) \mid \mathsf{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1\text{->iterate}(v_1 : \tau_1 \,;\, v_2 : \tau_2 = expr_2 \mid expr_3)$$

- $I[\![expr_1\text{->iterate}(v_1 : \tau_1 \,;\, v_2 : \tau_2 = expr_2 \mid expr_3)]\!](\sigma, \beta)$

$$:= \begin{cases} I[\![expr_2]\!](\sigma, \beta) & \text{, if } I[\![expr_1]\!](\sigma, \beta) = \emptyset \\ iterate(hlp, v_1, v_2, expr_3, \sigma, \beta') & \text{, otherwise} \end{cases}$$

where $\beta' = \beta[hlp \mapsto I[\![expr_1]\!](\sigma, \beta), v_2 \mapsto I[\![expr_2]\!](\sigma, \beta)]$ and

- $iterate(hlp, v_1, v_2, expr_3, \sigma, \beta')$

$$:= \begin{cases} I[\![expr_3]\!](\sigma, \beta'[v_1 \mapsto x]) & \text{, if } \beta'(hlp) = \{x\} \\ I[\![expr_3]\!](\sigma, \beta'') & \text{, if } \beta'(hlp) = X \,\dot\cup\, \{x\} \text{ and } X \neq \emptyset \end{cases}$$

where $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto iterate(hlp, v_1, v_2, expr_3, \sigma, \beta'[hlp \mapsto X])]$

(vi) Putting It All Together...

$$expr ::= w \mid \omega(expr_1, \ldots, expr_n) \mid \mathsf{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1\text{->iterate}(v_1 : \tau_1 \,;\, v_2 : \tau_2 = expr_2 \mid expr_3)$$

- $I[\![expr_1\text{->iterate}(v_1 : \tau_1 \,;\, v_2 : \tau_2 = expr_2 \mid expr_3)]\!](\sigma, \beta)$

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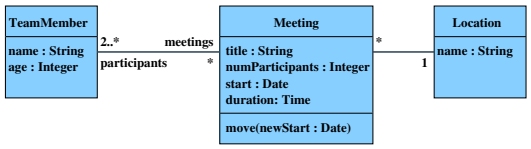
Quiz: Is (our) I a function?

Example



- `context TeamMember inv: age => 18`
- `context Meeting inv: duration > 0`

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