

# *Software Design, Modelling and Analysis in UML*

## *Lecture 04: OCL Cont'd, Object Diagrams*

2013-11-04

Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

- 04 - 2013-11-04 - main -

## Contents & Goals

### Last Lecture:

- OCL Syntax

### This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is an object diagram? What are object diagrams good for?
  - When is an object diagram called partial? What are partial ones good for?
  - When is an object diagram an object diagram (wrt. what)?
  - Is this an object diagram wrt. to that other thing?
  - How are system states and object diagrams related?
  - What does it mean that an OCL expression is satisfiable?
  - When is a set of OCL constraints said to be consistent?
  - Can you think of an object diagram which violates this OCL constraint?
- **Content:**
  - OCL Semantics
  - Object Diagrams
  - Example: Object Diagrams for Documentation
  - OCL : consistency, satisfiability

- 04 - 2013-11-04 - Prelim -

## OCL Semantics [OMG, 2006]

### The Task

OCL Syntax 1/4: Expressions

$expr ::=$		
$w$	$: \tau(w)$	
$  expr_1 = expr_2$	$: \tau \times \tau \rightarrow Bool$	
$  oclIsUndefined_r(expr_1)$	$: \tau \rightarrow Bool$	
$  \{expr_1, \dots, expr_n\}$	$: \tau \times \dots \times \tau \rightarrow Set(\tau)$	
$  isEmpty(expr_1)$	$: Set(\tau) \rightarrow Bool$	
$  size(expr_1)$	$: Set(\tau) \rightarrow Int$	
$  allInstances_C$	$: Set(\tau_C)$	
$  v(expr_1)$	$: \tau_C \rightarrow \tau(v)$	
$  r_1(expr_1)$	$: \tau_C \rightarrow \tau_D$	
$  r_2(expr_1)$	$: \tau_C \rightarrow Set(\tau_D)$	

Where, given  $\mathcal{S} = (\mathcal{F}, \mathcal{C}, V, atr)$ ,

- $W \supseteq \{self\}$  is a set of typed logical variables,  $w$  has type  $\tau(w)$
- $\tau$  is any type from  $\mathcal{F} \cup T_B \cup T_{\mathcal{C}}$   
 $\cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$
- $T_B$  is a set of basic types, in the following we use  
 $T_B = \{Bool, Int, String\}$
- $T_{\mathcal{C}} = \{\tau_C \mid C \in \mathcal{C}\}$  is the set of object types,
- $Set(\tau_0)$  denotes the set-of- $\tau_0$  type for  $\tau_0 \in T_B \cup T_{\mathcal{C}}$   
(sufficient because of "flattening" (cf. standard))
- $v : \tau(v) \in atr(C), \tau(v) \in \mathcal{S}$ ,
- $r_1 : D_{0,1} \in atr(C)$ ,
- $r_2 : D_* \in atr(C)$ ,
- $C, D \in \mathcal{C}$ .

- 04 - 2013-11-04 - SoDsem -
7/30

- Given an OCL expression  $expr$ , a system state  $\sigma \in \Sigma_{\mathcal{S}}$ , and a valuation of logical variables  $\beta$ , define

$$I[\![\cdot]\!](\cdot, \cdot) : OCLExpressions(\mathcal{S}) \times \Sigma_{\mathcal{S}} \times (W \rightarrow I(\mathcal{F} \cup T_B \cup T_{\mathcal{C}})) \rightarrow I(Bool)$$

such that

$$I[\![expr]\!](\sigma, \beta) \in \{true, false, \perp_{Bool}\} = I(Bool)$$

## Basically business as usual...

- (i) Equip each OCL (!) **basic type** with a reasonable **domain**, i.e. define function

$$I_{(i)} \text{ with } \text{dom}(I) = T_B$$

- (ii) Equip each **object type**  $\tau_C$  with a reasonable **domain**, i.e. define function

$$I_{(ii)} \text{ with } \text{dom}(I) = \tau_C$$

(most reasonable:  $\mathcal{D}(C)$  determined by structure  $\mathcal{D}$  of  $\mathcal{S}$ ).

- (iii) Equip each **set type**  $\text{Set}(\tau_0)$  with reasonable **domain**, i.e. define function

$$I_{(iii)} \text{ with } \text{dom}(I) = \{\text{Set}(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{E}}\}$$

- (iv) Equip each **arithmetical operation** with a reasonable **interpretation** (that is, with a **function** operating on the corresponding **domains**).

$$I_{(iv)} \text{ with } \text{dom}(I) = \{+, -, \leq, \dots\}, \text{ e.g., } I(+) \in I(\text{Int}) \times I(\text{Int}) \rightarrow I(\text{Int})$$

- (v) **Set operations** similar:  $I_{(v)}$  with  $\text{dom}(I) = \{\text{isEmpty}, \dots\}$

- (vi) Equip each **expression** with a reasonable **interpretation**, i.e. define function

$$I_{(vi)}: \text{Expr} \times \Sigma_{\mathcal{S}}^{\mathcal{D}} \times (W \rightarrow I(\mathcal{T} \cup T_B \cup T_{\mathcal{E}})) \rightarrow I(\text{Bool})$$

...except for OCL being a **three-valued logic**, and the "iterate" expression.

## (i) Domains of Basic Types (of OCL)

### Recall:

- $T_B = \{\text{Bool}, \text{Int}, \text{String}\}$

### We set:

- $I_{(i)}(\text{Bool}) := \{\text{true}, \text{false}\} \cup \{\perp_{\text{Bool}}\}$
- $I(\text{Int}) := \mathbb{Z} \cup \{\perp_{\text{Int}}\}$
- $I(\text{String}) := \dots \cup \{\perp_{\text{String}}\}$

*finite sequences of characters*

We may omit index  $\tau$  of  $\perp_{\tau}$  if it is clear from context.

## (ii) Domains of Object and (iii) Set Types

- Now we need a structure  $\mathcal{D}$  of our signature  $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ .
- **Recall:**  $\mathcal{D}$  assigns an (infinite) domain  $\mathcal{D}(C)$  to each class  $C \in \mathcal{C}$ .

- Let  $\tau_C$  be an (OCL) **object type** for a class  $C \in \mathcal{C}$ .
- We set

$$I_{(ii)}(\tau_C) := \mathcal{D}(C) \dot{\cup} \{\perp_{\tau_C}\}$$

- Let  $\tau$  be a type from  $T_B \cup T_{\mathcal{C}}$ .
- We set

$$I_{(iii)}(Set(\tau)) := 2^{I(\tau)} \dot{\cup} \{\perp_{Set(\tau)}\}$$

**Note:** in the OCL standard, only **finite** subsets of  $I(\tau)$ .  
But infinity doesn't scare **us**, so we simply allow it.

## (iv) Interpretation of Arithmetic Operations

- **Literals** map to fixed values:
  - $I_{(iv)}(\text{true}) := \text{true}$ ,  $I_{(iv)}(\text{false}) := \text{false}$ ,  $I(\mathbf{0}) := \mathbf{0}$ ,  $I(\mathbf{1}) := \mathbf{1}, \dots$
  - $I(\text{OclUndefined}_\tau) := \perp_{\tau} \in I(\tau)$
- **Boolean operations** (defined point-wise for  $x_1, x_2 \in I(\tau)$ ):

$$I(=_{\tau})(x_1, x_2) := \begin{cases} \text{true} & \text{, if } x_1 \neq \perp_{\tau} \neq x_2 \text{ and } x_1 = x_2 \\ \text{false} & \text{, if } x_1 \neq \perp_{\tau} \neq x_2 \text{ and } x_1 \neq x_2 \\ \perp_{Bool} & \text{, otherwise} \end{cases}$$

- **Integer operations** (defined point-wise for  $x_1, x_2 \in I(Int)$ ):

$$I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & \text{, if } x_1 \neq \perp \neq x_2 \\ \perp & \text{, otherwise} \end{cases}$$

**Note:** There is a **common principle**.

Namely, the **interpretation** of an operation  $\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau$  is a function  $I_{(iv)}(\omega) : I(\tau_1) \times \dots \times I(\tau_n) \rightarrow I(\tau)$  on corresponding semantical domain(s).

### (iv) Interpretation of OclIsUndefined

- The **is-undefined** predicate (defined point-wise for  $x \in I(\tau)$ ):

$$I(\text{oclIsUndefined}_\tau)(x) := \begin{cases} true & , \text{ if } x = \perp_\tau \\ false & , \text{ otherwise} \end{cases}$$

### (v) Interpretation of Set Operations

Basically the same principle as with arithmetic operations...

Let  $\tau \in T_B \cup T_\emptyset$ .

$$\{x_1, \dots, x_n\} \subseteq \tau \cdot \tau \rightarrow \text{Set}(\tau) \\ = \{\}_{n}^\tau(x_1, \dots, x_n)$$

- Set comprehension** ( $x_1, \dots, x_n \in I(\tau)$ ):

$$(I(\{\}_{n}^\tau))(x_1, \dots, x_n) := \{x_1, \dots, x_n\}$$

for all  $n \in \mathbb{N}_0$

- Empty-ness check** ( $x \in I(\text{Set}(\tau))$ ):

$$I(\text{isEmpty}^\tau)(x) := \begin{cases} true & , \text{ if } x = \emptyset \\ \perp_{Bool} & , \text{ if } x = \perp_{\text{Set}(\tau)} \\ false & , \text{ otherwise} \end{cases}$$

- Counting** ( $x \in I(\text{Set}(\tau))$ ):

$$(I(\text{size}^\tau))(x) := |x| \text{ if } x \neq \perp_{\text{Set}(\tau)} \text{ and } \perp_{Int} \text{ otherwise}$$

*cardinality* ↙

## (vi) Putting It All Together

### OCL Syntax 1/4: Expressions

*expr* ::=

- $w$  :  $\tau(w)$
- $| expr_1 = expr_2$  :  $\tau \times \tau \rightarrow Bool$
- $| oclUndefined_\tau(expr_1)$  :  $\tau \rightarrow Bool$
- $| \{expr_1, \dots, expr_n\}$  :  $\tau \times \dots \times \tau \rightarrow Set(\tau)$
- $| isEmpty(expr_1)$  :  $Set(\tau) \rightarrow Bool$
- $| size(expr_1)$  :  $Set(\tau) \rightarrow Int$
- $| allInstances_C$  :  $Set(\tau_C)$
- $| v(expr_1)$  :  $\tau_C \rightarrow \tau(v)$
- $| r_1(expr_1)$  :  $\tau_C \rightarrow \tau_D$
- $| r_2(expr_1)$  :  $\tau_C \rightarrow Set(\tau_D)$

Where, given  $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ ,

- $W \supseteq \{self\}$  is a set of logical variables,  $w$  has
- $\tau$  is any type from  $\mathcal{T} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_\emptyset\}$
- $T_B$  is a set of basic types, the following we use  $T_B = \{Bool, Int, Str\}$
- $T_\emptyset = \{\tau_C \mid C \in \mathcal{C}\}$  set of object types,
- $Set(\tau_0)$  denotes the set-of- $\tau_0$  type for  $\tau_0 \in T_B \cup T_\emptyset$  (sufficient because of "flattening" (cf. star))
- $v : \tau(v) \in atr(C), \tau(v)$
- $r_1 : D_{0,1} \in atr(C),$
- $r_2 : D_* \in atr(C),$
- $C, D \in \mathcal{C}.$

### OCL Syntax 2/4: Constants, Arithmetical Operat

For example:

- $| true, false$  :  $Bool$
- $| expr_1 \{and, or, implies\} expr_2$  :  $Bool \times Bool \rightarrow Bool$
- $| not expr_1$  :  $Bool \rightarrow Bool$
- $| 0, -1, 1, -2, 2, \dots$  :  $Int$
- $| OclUndefined$  :  $\tau$
- $| expr_1 \{+, -, \dots\} expr_2$  :  $Int \times Int \rightarrow Int$
- $| expr_1 \{<, \leq, \dots\} expr_2$  :  $Int \times Int \rightarrow Bool$

Generalised notation:

$expr ::= \omega(expr_1, \dots, expr_n)$  :  $\tau_1 \times \dots \times \tau_n \rightarrow \tau$

with  $\omega \in \{+, -, \dots\}$

### OCL Syntax 3/4: Iterate

$expr ::= \dots \mid expr_1 \rightarrow iterate(w_1 : \tau_1 ; w_2 : \tau_2 = expr_2 \mid expr_3)$

or, with a little renaming,

$expr ::= \dots \mid expr_1 \rightarrow iterate(iter : \tau_1 ; result : \tau_2 = expr_2 \mid expr_3)$

### OCL Syntax 4/4: Context

$context ::= context w_1 : \tau_1, \dots, w_n : \tau_n \text{ inv } : expr$

where  $w \in W$  and  $\tau_i \in T_\emptyset, 1 \leq i \leq n, n \geq 0$ .

## Valuations of Logical Variables

- **Recall:** we have typed logical variables ( $w \in W$ ),  $\tau(w)$  is the type of  $w$ .
- By  $\beta$ , we denote a valuation of the logical variables, i.e. for each  $w \in W$ ,

$$\beta(w) \in I(\tau(w)).$$

$$\beta : W \longrightarrow \bigcup_{w \in W} I(\tau(w))$$

$$W = \{x : Int, self_C : \tau_C\}$$

$$\beta : W \rightarrow I(Int) \cup I(\tau_C)$$

Example:

$$\left| \begin{array}{l} \bullet \beta(x) = 27 \in I(Int) \\ \bullet \beta(self_C) = 1_C \in I(\tau_C) = \mathcal{D}(C) \cup \{1\} \end{array} \right| \left| \begin{array}{l} \bullet \beta_2(x) = \perp_{Int} \\ \bullet \beta_2(self) = \perp_{\tau_C} \end{array} \right.$$

$\approx \{self_C \mid C \in \mathcal{C}\}$

(vi) Putting It All Together...  $I : \text{OCL Expr} \times \sum_{\omega}^{\mathcal{D}} \times (\omega \rightarrow \bigcup_{u \in \mathcal{U}} I(\tau(u))) \rightarrow$

$$\text{expr} ::= w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)$$

$\left. \begin{array}{l} \text{true,} \\ \text{false,} \\ \text{...} \end{array} \right\}$

- $I[w](\sigma, \beta) := \beta(\omega)$
- $I[\omega(\text{expr}_1, \dots, \text{expr}_n)](\sigma, \beta) := I(\omega)(I[\text{expr}_1](\sigma, \beta), \dots, I[\text{expr}_n](\sigma, \beta))$
- $I[\text{allInstances}_C](\sigma, \beta) := \text{dom}(\sigma) \cap \mathcal{D}(C)$

**Note:** in the OCL standard,  $\text{dom}(\sigma)$  is assumed to be **finite**.  
Again: doesn't scare us.

$$\mathcal{D} = (\emptyset, \{C, D\}, \emptyset, \emptyset)$$

- $\sigma_1 = \{ 1_C \mapsto \emptyset, 3_C \mapsto \emptyset, 2_C \mapsto \emptyset, 5_D \mapsto \emptyset \}$
- $\omega = \{ x : \text{Int}, c : \tau_C \}$
- $\beta_1 = \{ x \mapsto 13, c \mapsto 3_C \}$  (\*\*)
- $I[\text{allInstances}_D](\sigma_1, \beta_1) = \text{dom}(\sigma_1) \cap \mathcal{D}(D) = \{5_D\}$  (\*)
- $I[\text{size}(\text{allInstances}_D)](\sigma_1, \beta_1) = (I(\text{size}))(I[\text{allInstances}_D](\sigma_1, \beta_1))$   
 $= (I(\text{size}))(\{5_D\}) = |\{5_D\}| = 1$  (\*\*\*)  
by def. of  $I(\text{size})$
- $I[x > \text{size}(\text{allInstances}_D)](\sigma_1, \beta_1) = (I(>))(I[x](\sigma_1, \beta_1), I[\text{size}(\text{allInstances}_D)](\sigma_1, \beta_1))$   
 $= (I(>))(\beta_1(x), 1) = (I(>))(13, 1)$  (\*\*\*)  
by def. of  $I(>)$
- $I[1 / (\text{size}(\text{allInstances}_D) - 1)](\sigma_1, \beta_1) = \perp_{\text{Int}}$   
by def. of  $I(\omega)$   
 $I[1](\sigma_1, \beta_1) = I(1) = 1 \in \text{Int}$

assuming  
 $I(1)(x_1, x_2) =$   
 $\begin{cases} x_1/x_2 & \text{if } x_1 \neq \perp \\ & \text{and } x_2 \neq \perp \\ & \text{and } x_2 \neq 0 \\ \perp_{\text{Int}} & \text{otherwise} \end{cases}$

(vi) Putting It All Together...

$$\text{expr} ::= w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)$$

Assume  $\text{expr}_1 : \tau_C$  for some  $C \in \mathcal{C}$ . Set  $u_1 := I[\text{expr}_1](\sigma, \beta) \in \mathcal{D}(\tau_C)$ .

- $I[v(\text{expr}_1)](\sigma, \beta) := \begin{cases} \sigma(u_1)(v) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp_{\tau} & , \text{ otherwise} \end{cases}$  assuming  $v : \tau$
  - $I[r_1(\text{expr}_1)](\sigma, \beta) := \begin{cases} u & , \text{ if } u_1 \in \text{dom}(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \perp & , \text{ otherwise} \end{cases}$   
 $r_1 : C_{0,1}$
  - $I[r_2(\text{expr}_1)](\sigma, \beta) := \begin{cases} \sigma(u_1)(r_2) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp_{\text{set}(\tau_2)} & , \text{ otherwise} \end{cases}$
- (Recall:  $\sigma$  evaluates  $r_2$  of type  $C_*$  to a set)

$$\mathcal{F} = \{ \{ \text{Int}, \text{Colour} \}, \{ C, D \}, \{ \text{ms} : C_{0,1}, \text{sl} : C_*, r : \text{Int}, c : \text{Colour} \}, \{ C \mapsto \{ \text{ms}, \text{sl}, r \}, D \mapsto \{ c \} \} \}$$

$$\mathcal{D}(\text{Int}) = \mathbb{Z}, \quad \mathcal{D}(\text{Colour}) = \{ \text{red}, \text{green}, \text{blue} \}$$

$$\sigma_2 = \{ 1_C \mapsto \{ \text{ms} \mapsto \emptyset, \text{sl} \mapsto \{ 2_C, 13_C \}, c \mapsto 9 \}, \\ 2_C \mapsto \{ \text{ms} \mapsto \{ 1_C \}, \text{sl} \mapsto \emptyset, r \mapsto 5 \}, \\ 3_C \mapsto \{ \text{ms} \mapsto \{ 4_C \}, \text{sl} \mapsto \emptyset, r \mapsto 3 \}, \\ 5_D \mapsto \{ c \mapsto \text{blue} \} \}$$

$$\beta_2 = \{ p : \tau_C, q : \tau_D, x : \text{Int}, d : \text{Colour}, m : \tau_C \}$$

$\xrightarrow{\text{sl}} 1_C$     $\xrightarrow{\text{ms}} 5_D$     $\xrightarrow{x} 4$     $\xrightarrow{d} \text{green}$     $\xrightarrow{m} 2_C$

- $I[C(c)](\sigma_2, \beta_2) = \sigma_2(5_D)(c) = \text{blue}, I[q](\sigma_2, \beta_2) = 5_D$
- $I[C(p)](\sigma_2, \beta_2) = \text{false}$
- $I[sl(p)](\sigma_2, \beta_2) = \{ 2_C, 13_C \}$
- $I[r(\text{ms}(m))](\sigma_2, \beta_2) = 9, I[\text{ms}(m)](\sigma_2, \beta_2) = 1_C$
- $I[r(\text{ms}(p))](\sigma_2, \beta_2) = \perp_{\text{Int}}, I[\text{ms}(p)](\sigma_2, \beta_2) = \perp_{\tau_C}$



(vi) Putting It All Together...

$expr ::= w \mid \omega(expr_1, \dots, expr_n) \mid \text{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)$

*assign to hlp the set denoted by expr<sub>1</sub> (i.e., v, r, ω, β)*

- $I[\text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)](\sigma, \beta)$ 
  - iterate* ↓
  - result* ↓
  - init. value expr.* ↓

*assign to v<sub>2</sub> the initial value as given by expr<sub>2</sub>*

*modification of β at hlp and v<sub>2</sub>*

$$:= \begin{cases} I[\text{expr}_2](\sigma, \beta) & , \text{ if } I[\text{expr}_1](\sigma, \beta) = \emptyset \\ \text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta') & , \text{ otherwise} \end{cases}$$

*hlp has more than one element left*

where  $\beta' = \beta[hlp \mapsto I[\text{expr}_1](\sigma, \beta), v_2 \mapsto I[\text{expr}_2](\sigma, \beta)]$  and

- $\text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta')$ 
  - $:= \begin{cases} I[\text{expr}_3](\sigma, \beta'[v_1 \mapsto x]) & , \text{ if } \beta'(hlp) = \{x\} \\ I[\text{expr}_3](\sigma, \beta'') & , \text{ if } \beta'(hlp) = X \cup \{x\} \text{ and } X \neq \emptyset \end{cases}$ 
    - new hlp is either hlp without x*

where  $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto \text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta'[hlp \mapsto X])]$

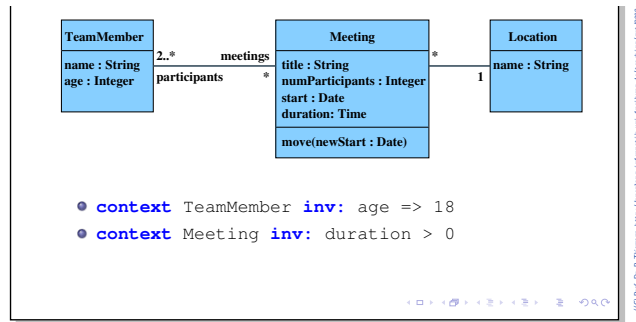
(vi) Putting It All Together...

$expr ::= w \mid \omega(expr_1, \dots, expr_n) \mid \text{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \mid r_2(expr_1) \mid expr_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)$

- $I[\text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)](\sigma, \beta)$ 
  - $:= \begin{cases} I[\text{expr}_2](\sigma, \beta) & , \text{ if } I[\text{expr}_1](\sigma, \beta) = \emptyset \\ \text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta') & , \text{ otherwise} \end{cases}$
  - where  $\beta' = \beta[hlp \mapsto I[\text{expr}_1](\sigma, \beta), v_2 \mapsto I[\text{expr}_2](\sigma, \beta)]$  and
  - $\text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta')$ 
    - $:= \begin{cases} I[\text{expr}_3](\sigma, \beta'[v_1 \mapsto x]) & , \text{ if } \beta'(hlp) = \{x\} \\ I[\text{expr}_3](\sigma, \beta'') & , \text{ if } \beta'(hlp) = X \cup \{x\} \text{ and } X \neq \emptyset \end{cases}$
    - where  $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto \text{iterate}(hlp, v_1, v_2, \text{expr}_3, \sigma, \beta'[hlp \mapsto X])]$

**Quiz:** Is (our)  $I$  a function?

## Example



## References

## References

---

- [Cabot and Clarisó, 2008] Cabot, J. and Clarisó, R. (2008). UML-OCL verification in practice. In Chaudron, M. R. V., editor, *MoDELS Workshops*, volume 5421 of *Lecture Notes in Computer Science*. Springer.
- [Cengarle and Knapp, 2001] Cengarle, M. V. and Knapp, A. (2001). On the expressive power of pure OCL. Technical Report 0101, Institut für Informatik, Ludwig-Maximilians-Universität München.
- [Cengarle and Knapp, 2002] Cengarle, M. V. and Knapp, A. (2002). Towards OCL/RT. In Eriksson, L.-H. and Lindsay, P. A., editors, *FME*, volume 2391 of *Lecture Notes in Computer Science*, pages 390–409. Springer-Verlag.
- [Flake and Müller, 2003] Flake, S. and Müller, W. (2003). Formal semantics of static and temporal state-oriented OCL constraints. *Software and Systems Modeling*, 2(3):164–186.
- [Jackson, 2002] Jackson, D. (2002). Alloy: A lightweight object modelling notation. *ACM Transactions on Software Engineering and Methodology*, 11(2):256–290.
- [OMG, 2006] OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.
- [OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.
- [OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.
- [Schumann et al., 2008] Schumann, M., Steinke, J., Deck, A., and Westphal, B. (2008)42/42  
Thesis on technical documentation, version 1.0. Technical Report, German Open-Source