

# Software Design, Modelling and Analysis in UML

## Lecture 04: OCL Cont'd, Object Diagrams

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### Contents & Goals

Last Lecture:

- OCL Syntax

This Lecture:

Educational Objectives: Capabilities for following tasks/questions.

- What is an object diagram? What are object diagrams good for?
- When is an object diagram called partial? What are partial ones good for?
- When is an object diagram an object diagram (vnr: what)?
- Is this an object diagram vnr. to that other thing?
- How are system states and object diagrams related?
- What does it mean that an OCL expression is satisfiable?
- When is a set of OCL constraints said to be consistent?
- Can you think of an object diagram which violates this OCL constraint?

Content:

- OCL Semantics
- Object Diagrams
- Example: Object Diagrams for Documentation
- OCL `and` and `or` and `not`

### OCL Semantics [OMG, 2006]

### The Task

OCL Syntax / OCL Expressions

When given  $\mathcal{S}$  =  $\langle T, \mathcal{O}, \mathcal{V}, \text{init} \rangle$ :

- If  $\mathcal{S}$  (init) is a set of typed literals
- If  $\mathcal{S}$  (init) is a set of typed literals and  $\mathcal{S}$  (ops) is a set of typed operators
- If  $\mathcal{S}$  (ops) is a set of typed operators and  $\mathcal{S}$  (vars) is a set of typed variables
- If  $\mathcal{S}$  (ops) is a set of typed operators,  $\mathcal{S}$  (vars) is a set of typed variables, and  $\mathcal{S}$  (inits) is a set of typed initial values

Let  $\mathcal{S}$  (ops) =  $\{ \text{op}_1, \dots, \text{op}_n \}$  and  $\mathcal{S}$  (vars) =  $\{ \text{var}_1, \dots, \text{var}_m \}$ .

Let  $\mathcal{S}$  (inits) =  $\{ \text{init}_1, \dots, \text{init}_m \}$  where  $\text{init}_i$  is a typed value of type  $\text{type}(\text{var}_i)$ .

Let  $\mathcal{S}$  (inits) =  $\{ \text{init}_1, \dots, \text{init}_m \}$  where  $\text{init}_i$  is a typed value of type  $\text{type}(\text{var}_i)$ .

Let  $\mathcal{S}$  (inits) =  $\{ \text{init}_1, \dots, \text{init}_m \}$  where  $\text{init}_i$  is a typed value of type  $\text{type}(\text{var}_i)$ .

- Given an OCL expression  $\text{expr}$ , a system state  $\sigma \in \Sigma_{\mathcal{S}}^{\mathcal{O}}$ , and a valuation of logical variables  $\beta$ , define  $\llbracket \text{expr} \rrbracket (\sigma, \beta) := (\text{true false}, \perp, \text{Bool}) = \perp$  ( $\text{Bool}$ ) such that  $\llbracket \text{expr} \rrbracket (\sigma, \beta) \in (\text{true false}, \perp, \text{Bool}) = \perp$  ( $\text{Bool}$ )

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### Basically: business as usual...

- (i) Equip each OCL (i) basic type with a reasonable domain, i.e. define function  $\llbracket \cdot \rrbracket$  with  $\text{dom}(f) = T_a$

- (ii) Equip each object type  $\text{rc}$  with a reasonable domain, i.e. define function  $\llbracket \cdot \rrbracket$  with  $\text{dom}(f) = \text{rc}$

(most reasonable  $\mathcal{S}(\mathcal{O})$  determined by structure  $\mathcal{S}$  of  $\mathcal{S}$ )

- (iii) Equip each set type  $\text{Set}(\mathcal{O})$  with reasonable domain, i.e. define function  $\llbracket \cdot \rrbracket$  with  $\text{dom}(f) = \{ \text{Set}(\mathcal{O}) \mid \mathcal{O} \in T_{\mathcal{S}} \cup T_{\mathcal{V}} \}$

- (iv) Equip each arithmetical operation with a reasonable interpretation (that is, with a function operating on the corresponding domains)

- (v) Set operations similar:  $\llbracket \cdot \rrbracket$  with  $\text{dom}(f) = \{ \text{Empy}, \dots \}$

- (vi) Equip each expression with a reasonable interpretation, i.e. define function  $\llbracket \cdot \rrbracket : \Sigma_{\mathcal{S}}^{\mathcal{O}} \times \Sigma_{\mathcal{S}}^{\mathcal{V}} \times (TV \cup T_{\mathcal{S}} \cup T_{\mathcal{V}}) \rightarrow (I, \text{Bool})$

...except for OCL being a three-valued logic, and the "iterate" expression

### (i) Domains of Basic Types (cf. OCL)

Recall:

- $T_{\text{Bool}}$  = {Bool, Init, String}

We set:

- $\llbracket \text{Bool} \rrbracket := \{ \text{true, false} \} \cup \{ \perp, \text{Bool} \}$
- $\llbracket \text{Init} \rrbracket := \mathbb{Z} \cup \{ \perp, \text{Init} \}$
- $\llbracket \text{String} \rrbracket := \dots \cup \{ \perp, \text{String} \}$

We may omit index  $\tau$  of  $\llbracket \cdot \rrbracket$  if it is clear from context.

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(ii) Domains of Object and (iii) Set Types

- Now we need a structure  $\mathcal{D}$  of our signature  $\mathcal{S} = (\mathcal{F}, \mathcal{C}, \mathcal{V}, \text{arity})$ .
  - Recall:  $\mathcal{D}$  assigns an (infinite) domain  $\mathcal{D}(C)$  to each class  $C \in \mathcal{C}$ .
  - Let  $\tau_C$  be an (OCL) object type for a class  $C \in \mathcal{C}$ .
  - We set  $\mathcal{D}(\tau_C) := \mathcal{D}(C) \cup \{\perp_{\tau_C}\}$
  - Let  $\tau$  be a type from  $T_D \cup T_E$ .  $\tau$  is a subset of  $A$
  - We set  $\mathcal{D}(\tau) := \mathcal{D}(\tau_C) \cup \{\perp_{\tau}\}$
- Note: in the OCL standard, only finite subsets of  $I(\tau)$ . But infinity doesn't scare us, so we simply allow it.

(iv) Interpretation of Arithmetic Operations

- Literal map to fixed values
  - Boolean operations (defined pointwise for  $x_1, x_2 \in I(\tau)$ ):
  - Integer operations (defined pointwise for  $x_1, x_2 \in I(\text{int})$ ):
- Note: There is a common principle. Namely, the interpretation of an operation  $o$  is a function  $\mathcal{D}(o) : \mathcal{D}(T_1) \times \dots \times \mathcal{D}(T_n) \rightarrow \mathcal{D}(T)$  on corresponding semantic domain(s).

(v) Interpretation of Set Operations

- Basically the same principle as with arithmetic operations...
- Let  $\tau \in T_D \cup T_E$ .
- Set comprehension  $(\tau_1, \dots, \tau_n \in I(\tau))$ :  $\{x_1, \dots, x_n \mid c, x \in \tau \rightarrow \mathcal{D}(c)\}$
  - Empty-set check  $(x \in I(\mathcal{S}et(\tau)))$ :  $\begin{cases} true & \text{if } x = \emptyset \\ \perp_{\text{bool}} & \text{if } x \neq \perp_{\mathcal{S}et(\tau)} \\ false & \text{otherwise} \end{cases}$
  - Counting  $(x \in I(\mathcal{S}et(\tau)))$ :  $\begin{cases} \text{cardinality} & \text{if } x \neq \perp_{\mathcal{S}et(\tau)} \\ \perp_{\text{int}} & \text{otherwise} \end{cases}$

OCL Syntax 3.2: Expressions	When given $\mathcal{D} = (\mathcal{S}, \mathcal{D})$
$\text{true}$	$\perp_{\text{bool}}$
$\text{false}$	$\perp_{\text{bool}}$
$\text{int}$	$\perp_{\text{int}}$
$\text{float}$	$\perp_{\text{float}}$
$\text{string}$	$\perp_{\text{string}}$
$\text{set}$	$\perp_{\text{set}}$
$\text{array}$	$\perp_{\text{array}}$
$\text{enum}$	$\perp_{\text{enum}}$
$\text{class}$	$\perp_{\text{class}}$
$\text{union}$	$\perp_{\text{union}}$
$\text{intersection}$	$\perp_{\text{intersection}}$
$\text{difference}$	$\perp_{\text{difference}}$
$\text{complement}$	$\perp_{\text{complement}}$
$\text{cardinality}$	$\perp_{\text{int}}$
$\text{count}$	$\perp_{\text{int}}$
$\text{is-empty}$	$\perp_{\text{bool}}$
$\text{is-not-empty}$	$\perp_{\text{bool}}$
$\text{is-subset}$	$\perp_{\text{bool}}$
$\text{is-superset}$	$\perp_{\text{bool}}$
$\text{is-equal}$	$\perp_{\text{bool}}$
$\text{is-not-equal}$	$\perp_{\text{bool}}$
$\text{is-less-than}$	$\perp_{\text{bool}}$
$\text{is-greater-than}$	$\perp_{\text{bool}}$
$\text{is-less-than-or-equal}$	$\perp_{\text{bool}}$
$\text{is-greater-than-or-equal}$	$\perp_{\text{bool}}$
$\text{is-between}$	$\perp_{\text{bool}}$
$\text{is-not-between}$	$\perp_{\text{bool}}$
$\text{is-in}$	$\perp_{\text{bool}}$
$\text{is-not-in}$	$\perp_{\text{bool}}$
$\text{is-member-of}$	$\perp_{\text{bool}}$
$\text{is-not-member-of}$	$\perp_{\text{bool}}$
$\text{is-kind-of}$	$\perp_{\text{bool}}$
$\text{is-not-kind-of}$	$\perp_{\text{bool}}$
$\text{is-a}$	$\perp_{\text{bool}}$
$\text{is-not-a}$	$\perp_{\text{bool}}$
$\text{is-ancestor-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-of}$	$\perp_{\text{bool}}$
$\text{is-parent-of}$	$\perp_{\text{bool}}$
$\text{is-not-parent-of}$	$\perp_{\text{bool}}$
$\text{is-child-of}$	$\perp_{\text{bool}}$
$\text{is-not-child-of}$	$\perp_{\text{bool}}$
$\text{is-sibling-of}$	$\perp_{\text{bool}}$
$\text{is-not-sibling-of}$	$\perp_{\text{bool}}$
$\text{is-cousin-of}$	$\perp_{\text{bool}}$
$\text{is-not-cousin-of}$	$\perp_{\text{bool}}$
$\text{is-uncle-of}$	$\perp_{\text{bool}}$
$\text{is-not-uncle-of}$	$\perp_{\text{bool}}$
$\text{is-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-not-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-not-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-not-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-cousin-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-cousin-of}$	$\perp_{\text{bool}}$
$\text{is-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-not-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-grandparent-or-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-not-grandparent-or-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-grandchild-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-grandchild-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-cousin-or-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-cousin-or-uncle-or-aunt-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-or-great-grandchild-or-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$
$\text{is-not-ancestor-or-cousin-or-uncle-or-aunt-or-grandparent-or-great-grandparent-or-great-grandchild-or-great-grandparent-or-great-grandchild-of}$	$\perp_{\text{bool}}$

(vi) Interpretation of OCLUndefined

- The is-undefined predicate (defined pointwise for  $x \in I(\tau)$ ):

Valuations of Logical Variables

- Recall: we have typed logical variables  $(w \in \{V, \tau(w)\})$  is the type of  $w$ .
  - By  $\beta$ , we denote a valuation of the logical variables, i.e. for each  $w \in W$ ,
- Example:
- $$\beta: W \rightarrow \bigcup_{w \in W} I(\tau(w))$$
- $$W = \{x, w, c, w', c'\}$$
- $$\beta: W \rightarrow I(\text{bool}) \cup I(\text{int})$$
- $$\beta(x) = \perp_{\text{bool}}, \beta(w) = \perp_{\text{int}}, \beta(c) = \perp_{\text{bool}}, \beta(w') = \perp_{\text{int}}, \beta(c') = \perp_{\text{bool}}$$

(vi) Putting It All Together...  $\Gamma : \mathcal{O}(\text{Exp}_n \times \mathcal{S}) \times \mathcal{S} \rightarrow \mathcal{O}(\text{Exp}_n) \rightarrow \mathcal{S}$

$$\text{expr} ::= w \mid \text{let}(c_1, \dots, c_n; e) \mid \text{allinstancec} \mid \text{v}(c_1) \mid r(c_1)$$

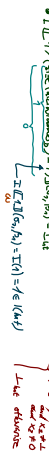
- $\llbracket \text{let}(c, \beta) \rrbracket(\alpha, \beta) := \beta(c)$
- $\llbracket \text{let}(c_1, \dots, c_n; e) \rrbracket(\alpha, \beta) := \llbracket e \rrbracket(\alpha, \beta)$
- $\llbracket \text{allinstancec} \rrbracket(\alpha, \beta) := \text{dom}(c) \cap \beta(C)$

Note: in the OCL standard,  $\text{dom}(c)$  is assumed to be finite. Again, doesn't scare us.

*Handwritten notes: "let", "allinstance", "v", "r"*

$\mathcal{S} = \{\emptyset, \{c, D\}, \emptyset, \emptyset\}$

- $\sigma_1 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\sigma_2 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\sigma_3 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\sigma_4 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\llbracket \text{let}(c, \beta) \rrbracket(\alpha, \beta) = \text{dom}(c) \cap \beta(C) = \emptyset$
- $\llbracket \text{allinstancec} \rrbracket(\alpha, \beta) = \{c, D\} \cap \beta(C) = \{c, D\}$
- $\llbracket v(c) \rrbracket(\alpha, \beta) = \{c\}$
- $\llbracket r(c) \rrbracket(\alpha, \beta) = \{c, D\} \cap \beta(C) = \{c, D\}$
- $\llbracket \Gamma(\sigma) \rrbracket(\alpha, \beta) = \text{dom}(c) \cap \beta(C) = \{c, D\}$



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(vi) Putting It All Together...

$$\text{expr} ::= w \mid \text{let}(c_1, \dots, c_n; e) \mid \text{allinstancec} \mid \text{v}(c_1) \mid r(c_1)$$

Handwritten notes explaining the evaluation process. It details the function  $\llbracket e \rrbracket(\alpha, \beta)$  and its interaction with the environment  $(\alpha, \beta)$ . It notes that the result is the intersection of the domain of  $c$  and  $\beta(C)$ .

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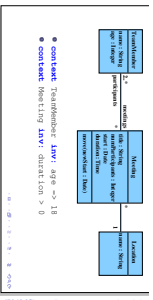
$\mathcal{S} = \{\emptyset, \{c, D\}, \emptyset, \emptyset\}$

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- $\sigma_2 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\sigma_3 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\sigma_4 = \{c \mapsto D, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset, \emptyset \mapsto \emptyset\}$
- $\llbracket \text{let}(c, \beta) \rrbracket(\alpha, \beta) = \text{dom}(c) \cap \beta(C) = \emptyset$
- $\llbracket \text{allinstancec} \rrbracket(\alpha, \beta) = \{c, D\} \cap \beta(C) = \{c, D\}$
- $\llbracket v(c) \rrbracket(\alpha, \beta) = \{c\}$
- $\llbracket r(c) \rrbracket(\alpha, \beta) = \{c, D\} \cap \beta(C) = \{c, D\}$
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