

(ii) Domains of Object and (iii) Set Types

- Now we need a structure \mathcal{D} of our signature $\mathcal{S} = (\mathcal{F}, \mathcal{C}, \mathcal{V}, \text{arity})$.
- Recall: \mathcal{D} assigns an (infinite) domain $\mathcal{D}(C)$ to each class $C \in \mathcal{C}$.
- Let τ_C be an (OCL) object type for a class $C \in \mathcal{C}$.
- We set $\mathcal{D}(\tau_C) := \mathcal{D}(C) \cup \{\perp_{\tau_C}\}$

- Let τ be a type from $T_D \cup T_{\mathcal{C}}$. τ is a subset of A
- We set $\mathcal{D}(\tau) := \mathcal{D}(\tau_C) \cup \{\perp_{\tau}\}$

Note: in the OCL standard, only finite subsets of $I(\tau)$. But infinity doesn't scare us, so we simply allow it.

(iv) Interpretation of Arithmetic Operations

- Literal map to fixed values
- Boolean operations (defined pointwise for $x_1, x_2 \in I(\tau)$):
- Integer operations (defined pointwise for $x_1, x_2 \in I(\text{int})$):

$$I(\text{true}) := \text{true}, I(\text{false}) := \text{false}, I(0) := 0, I(1) := 1, \dots$$

$$I(\text{OrUsedAnd}) := \text{true} \vee \text{false} = \text{true}$$

$$I(\text{And}) := \begin{cases} \text{true} & \text{if } x_1 \neq \perp_{\tau} \text{ and } x_2 = x_2 \\ \text{false} & \text{if } x_1 \neq \perp_{\tau} \text{ and } x_2 \neq x_2 \\ \perp_{\text{And}} & \text{otherwise} \end{cases}$$

$$I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & \text{if } x_1 \neq \perp_{\tau} \text{ and } x_2 \neq \perp_{\tau} \\ \perp_{+} & \text{otherwise} \end{cases}$$

Note: There is a common principle. Namely, the interpretation of an operation $op: T_1 \times \dots \times T_n \rightarrow T$ is a function $\mathcal{D}(op): \mathcal{D}(T_1) \times \dots \times \mathcal{D}(T_n) \rightarrow \mathcal{D}(T)$ on corresponding semantic domains!

(v) Interpretation of OCLUndefined

- The is-undefined predicate (defined pointwise for $x \in I(\tau)$):

$$I(\text{isUndefined})(x) := \begin{cases} \text{true} & \text{if } x = \perp_{\tau} \\ \text{false} & \text{otherwise} \end{cases}$$

(v) Interpretation of Set Operations

- Basically the same principle as with arithmetic operations...
- Let $\tau \in T_D \cup T_{\mathcal{C}}$.
- Set comprehension $(\tau_1, \dots, \tau_n \in I(\tau))$: $\{x_1, \dots, x_n\} \text{ with } x_i \in \tau_i \rightarrow \mathcal{D}(\tau)$
- Empty-set check $(x \in I(\text{Set}(\tau)))$: true if $x = \perp_{\text{Set}(\tau)}$, otherwise false

- Counting $(x \in I(\text{Set}(\tau)))$: cardinality

(vi) Putting It All Together

OCL Syntax 3.2: Expressions	OCL Syntax 3.2: Constants, Arithmetic Operat
<p>When given $\mathcal{D} = (\mathcal{S}, \mathcal{D})$</p> <p>$\tau \in T_D \cup T_{\mathcal{C}}$ is a set of legal values, τ is a set of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p>	<p>For example:</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p> <p>$\tau_1, \dots, \tau_n \in T_D \cup T_{\mathcal{C}}$ are sets of legal values</p>
OCL Syntax 3.4: Infix	OCL Syntax 3.4: Constant

Valuations of Logical Variables

- Recall: we have typed logical variables $(w \in W, \tau(w))$ is the type of w .
- By β , we denote a valuation of the logical variables, i.e. for each $w \in W$,

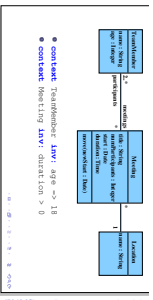
$$\beta: W \rightarrow \bigcup_{w \in W} I(\tau(w))$$

$$W = \{x, w, c, w', c'\}$$

$$\beta: W \rightarrow I(\text{int}) \cup I(\text{set})$$

Example:

$$\beta(x) = 1, \beta(w) = \{1, 2, 3\}, \beta(c) = \{1, 2\}, \beta(w') = 1, \beta(c') = 1$$



References

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