

Software Design, Modelling and Analysis in UML

Lecture 04: OCL Cont'd, Object Diagrams

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:

- OCL Syntax

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
 - What is an object diagram? What are object diagrams good for?
 - When is an object diagram called partial? What are partial ones good for?
 - When is an object diagram an object diagram (wrt. what)?
 - Is this an object diagram wrt. to that other thing?
 - How are system states and object diagrams related?
 - What does it mean that an OCL expression is satisfiable?
 - When is a set of OCL constraints said to be consistent?
 - Can you think of an object diagram which violates this OCL constraint?
- **Content:**
 - OCL Semantics
 - Object Diagrams
 - Example: Object Diagrams for Documentation
 - OCL : consistency, satisfiability

OCL Semantics [OMG, 2006]

The Task

OCL Syntax 1/4: Expressions

$expr ::=$

w	$: \tau(w)$
$ expr_1 =_{\tau} expr_2$	$: \tau \times \tau \rightarrow Bool$
$ oclIsUndefined_{\tau}(expr_1)$	$: \tau \rightarrow Bool$
$ \{expr_1, \dots, expr_n\}$	$: \tau \times \dots \times \tau \rightarrow Set(\tau)$
$ isEmpty(expr_1)$	$: Set(\tau) \rightarrow Bool$
$ size(expr_1)$	$: Set(\tau) \rightarrow Int$
$ allInstances_C$	$: Set(\tau_C)$
$ v(expr_1)$	$: \tau_C \rightarrow \tau(v)$
$ r_1(expr_1)$	$: \tau_C \rightarrow \tau_D$
$ r_2(expr_1)$	$: \tau_C \rightarrow Set(\tau_D)$

– 03 – 2010-10-27 – Soclsyn –

Where, given $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$,

- $W \supseteq \{self\}$ is a set of typed **logical variables**, w has type $\tau(w)$
- τ is any type from $\mathcal{T} \cup T_B \cup T_{\mathcal{C}}$
 $\cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$
 - T_B is a set of **basic types**, in the following we use
 $T_B = \{Bool, Int, String\}$
 - $T_{\mathcal{C}} = \{\tau_C \mid C \in \mathcal{C}\}$ is the set of **object types**,
 - $Set(\tau_0)$ denotes the **set-of- τ_0** type for
 $\tau_0 \in T_B \cup T_{\mathcal{C}}$
(sufficient because of “flattening” (cf. standard))
- $v : \tau(v) \in atr(C), \tau(v) \in \mathcal{T}$,
- $r_1 : D_{0,1} \in atr(C)$,
- $r_2 : D_* \in atr(C)$,
- $C, D \in \mathcal{C}$.

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- Given an OCL expression $expr$, a system state $\sigma \in \Sigma_{\mathcal{S}}$, and a valuation of logical variables β , define

$$I[\![\cdot]\!](\cdot, \cdot) : OCLExpressions(\mathcal{S}) \times \Sigma_{\mathcal{S}} \times (W \rightarrow I(\mathcal{T} \cup T_B \cup T_{\mathcal{C}})) \rightarrow I(Bool)$$

such that

$$I[\![expr]\!](\sigma, \beta) \in \{true, false, \perp_{Bool}\}. = I(Bool)$$

Basically business as usual...

- (i) Equip each OCL (!) **basic type** with a reasonable **domain**, i.e. define function

$$I_{(i)} \text{ with } \text{dom}(I) = T_B$$

- (ii) Equip each **object type** τ_C with a reasonable **domain**, i.e. define function

$$I_{(ii)} \text{ with } \text{dom}(I) = \tau_C$$

(most reasonable: $\mathcal{D}(C)$ determined by structure \mathcal{D} of \mathcal{S}).

- (iii) Equip each **set type** $Set(\tau_0)$ with reasonable **domain**, i.e. define function

$$I_{(iii)} \text{ with } \text{dom}(I) = \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$$

- (iv) Equip each **arithmetical operation** with a reasonable **interpretation** (that is, with a **function** operating on the corresponding **domains**).

$$I_{(iv)} \text{ with } \text{dom}(I) = \{+, -, \leq, \dots\}, \text{ e.g., } I(+) \in I(Int) \times I(Int) \rightarrow I(Int)$$

- (v) **Set operations** similar: $I_{(v)} \text{ with } \text{dom}(I) = \{\text{isEmpty}, \dots\}$

- (vi) Equip each **expression** with a reasonable **interpretation**, i.e. define function

$$I_{(vi)}; Expr \times \Sigma_{\mathcal{D}} \times (W \rightarrow I(\mathcal{T} \cup T_B \cup T_{\mathcal{C}})) \rightarrow I(Bool)$$

...except for OCL being a **three-valued logic**, and the “iterate” expression.

(i) Domains of Basic Types (of $\alpha\mathcal{L}$)

Recall:

- $T_B = \{Bool, Int, String\}$

We set:

- $I_{\tau}(Bool) := \{true, false\} \cup \{\perp_{Bool}\}$
- $I(Int) := \mathbb{Z} \cup \{\perp_{Int}\}$
- $I(String) := \dots \cup \{\perp_{String}\}$

finite sequences of characters

We may omit index τ of \perp_{τ} if it is clear from context.

assume both sets disjoint
"undefined"

(ii) Domains of Object and (iii) Set Types

- Now we need a structure \mathcal{D} of our signature $\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr)$.
- **Recall:** \mathcal{D} assigns an (infinite) domain $\mathcal{D}(C)$ to each class $C \in \mathcal{C}$.

- Let τ_C be an (OCL) **object type** for a class $C \in \mathcal{C}$.
- We set

$$I_{(ii)}(\tau_C) := \mathcal{D}(C) \dot{\cup} \{\perp_{\tau_C}\}$$

- Let τ be a type from $T_B \cup T_{\mathcal{C}}$.
- We set

$$I_{(iii)}(Set(\tau)) := 2^{I(\tau)} \dot{\cup} \{\perp_{Set(\tau)}\}$$

2^A is
powerset of A

Note: in the OCL standard, only **finite** subsets of $I(\tau)$.
But infinity doesn't scare **us**, so we simply allow it.

(iv) Interpretation of Arithmetic Operations

- **Literals** map to fixed values:

$$I_{(iv)}(\mathbf{true}) := \mathbf{true}, \quad I(\mathbf{false}) := \mathbf{false}, \quad I(\mathbf{0}) := \mathbf{0}, \quad I(\mathbf{1}) := \mathbf{1}, \dots$$

$$I(\mathbf{OclUndefined}_\tau) := \perp_\tau \in \mathcal{I}(\tau)$$

- **Boolean operations** (defined point-wise for $x_1, x_2 \in I(\tau)$):

$$I(=_\tau)(x_1, x_2) := \begin{cases} \mathbf{true} & , \text{ if } x_1 \neq \perp_\tau \neq x_2 \text{ and } x_1 = x_2 \\ \mathbf{false} & , \text{ if } x_1 \neq \perp_\tau \neq x_2 \text{ and } x_1 \neq x_2 \\ \perp_{Bool} & , \text{ otherwise} \end{cases}$$

- **Integer operations** (defined point-wise for $x_1, x_2 \in I(Int)$):

$$I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & , \text{ if } x_1 \neq \perp \neq x_2 \\ \perp & , \text{ otherwise} \end{cases}$$

Note: There is a **common principle**.

Namely, the **interpretation** of an operation $\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau$

is a function $I_{(iv)}(\omega) : I(\tau_1) \times \dots \times I(\tau_n) \rightarrow I(\tau)$ on corresponding semantical domain(s).

(iv) Interpretation of OclIsUndefined

- The **is-undefined** predicate (defined point-wise for $x \in I(\tau)$):

$$I(\mathbf{oclIsUndefined}_\tau)(x) := \begin{cases} true & , \text{ if } x = \perp_\tau \\ false & , \text{ otherwise} \end{cases}$$

(v) Interpretation of Set Operations

Basically the same principle as with arithmetic operations...

Let $\tau \in T_B \cup T_{\mathcal{C}}$.

$$\{x_1, \dots, x_n\} \quad \tau \times \dots \times \tau \rightarrow \text{Set}(\tau) \\ \equiv \{\}_n^{\tau}(x_1, \dots, x_n)$$

- **Set comprehension** ($x_1, \dots, x_n \in I(\tau)$):

$$(I(\{\}_n^{\tau}))(x_1, \dots, x_n) := \{x_1, \dots, x_n\}$$

for all $n \in \mathbb{N}_0$

- **Empty-ness check** ($x \in I(\text{Set}(\tau))$):

$$I(\text{isEmpty}^{\tau})(x) := \begin{cases} true & , \text{ if } x = \emptyset \\ \perp_{Bool} & , \text{ if } x = \perp_{Set(\tau)} \\ false & , \text{ otherwise} \end{cases}$$

- **Counting** ($x \in I(\text{Set}(\tau))$): *cardinality*

$$(I(\text{size}^{\tau}))(x) := |x| \text{ if } x \neq \perp_{Set(\tau)} \text{ and } \perp_{Int} \text{ otherwise}$$

(vi) Putting It All Together

OCL Syntax 1/4: Expressions

$expr ::=$

w $: \tau(w)$

$| expr_1 =_{\tau} expr_2$ ✓ $: \tau \times \tau \rightarrow Bool$

$| oclIsUndefined_{\tau}(expr_1)$ ✓ $: \tau \rightarrow Bool$

$| \{expr_1, \dots, expr_n\}$ ✓ $: \tau \times \dots \times \tau \rightarrow Set(\tau)$

$| isEmpty(expr_1)$ ✓ $: Set(\tau) \rightarrow Bool$

$| size(expr_1)$ ✓ $: Set(\tau) \rightarrow Int$

$| allInstances_C$ $: Set(\tau_C)$

$| v(expr_1)$ $: \tau_C \rightarrow \tau(v)$

$| r_1(expr_1)$ $: \tau_C \rightarrow \tau_D$

$| r_2(expr_1)$ $: \tau_C \rightarrow Set(\tau_D)$

Where, given $\mathcal{S} = (\mathcal{T}, \mathcal{C})$,

- $W \supseteq \{self\}$ is a set of **logical variables**, w has
- τ is any type from $\mathcal{T} \cup \mathcal{C} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$
- T_B is a set of **basic types**; for the following we use $T_B = \{Bool, Int, String\}$
- $T_{\mathcal{C}} = \{\tau_C \mid C \in \mathcal{C}\}$ is a set of **object types**,
- $Set(\tau_0)$ denotes the **set-of- τ_0 type** for $\tau_0 \in T_B \cup T_{\mathcal{C}}$ (sufficient because of “flattening” (cf. standard OCL))
- $v : \tau(v) \in atr(C), \tau(v)$
- $r_1 : D_{0,1} \in atr(C),$
- $r_2 : D_* \in atr(C),$
- $C, D \in \mathcal{C}.$

OCL Syntax 2/4: Constants, Arithmetical Operators

For example:

$expr ::= \dots$

$| true, false$ ✓ $: Bool$

$| expr_1 \{and, or, implies\} expr_2$ ✓ $: Bool \times Bool \rightarrow Bool$

$| not expr_1$ ✓ $: Bool \rightarrow Bool$

$| 0, -1, 1, -2, 2, \dots$ ✓ $: Int$

$| OclUndefined$ ✓ $: \tau$

$| expr_1 \{+, -, \dots\} expr_2$ ✓ $: Int \times Int \rightarrow Int$

$| expr_1 \{<, \leq, \dots\} expr_2$ ✓ $: Int \times Int \rightarrow Bool$

Generalised notation:

$expr ::= \omega(expr_1, \dots, expr_n) \quad : \tau_1 \times \dots \times \tau_n \rightarrow \tau$

with $\omega \in \{+, -, \dots\}$

OCL Syntax 3/4: Iterate

$expr ::= \dots \mid expr_1 \rightarrow iterate(w_1 : \tau_1 ; w_2 : \tau_2 = expr_2 \mid expr_3)$

or, with a little renaming,

$expr ::= \dots \mid expr_1 \rightarrow iterate(iter : \tau_1 ; result : \tau_2 = expr_2 \mid expr_3)$

OCL Syntax 4/4: Context

$context ::= context w_1 : \tau_1, \dots, w_n : \tau_n \text{ inv} : expr$

where $w \in W$ and $\tau_i \in T_{\mathcal{C}}, 1 \leq i \leq n, n \geq 0.$

Valuations of Logical Variables

$\{self_c \mid c \in \mathcal{C}\}$

- **Recall:** we have typed logical variables ($w \in$) W , $\tau(w)$ is the type of w .
- By β , we denote a valuation of the logical variables, i.e. for each $w \in W$,

$$\beta(w) \in I(\tau(w)).$$

$$\beta: W \longrightarrow \bigcup_{w \in W} I(\tau(w))$$

$$W = \{x: \text{Int}, self_c: \tau_c\}$$

$$\beta: W \longrightarrow I(\text{Int}) \cup I(\tau_c)$$

Example:

$$\bullet \beta(x) = 27 \in I(\text{Int})$$

$$\bullet \beta(self_c) = 1_c \in I(\tau_c) = \mathcal{D}(c) \cup \{\perp\}$$

$$\bullet \beta_2(x) = \perp_{\text{Int}}$$

$$\bullet \beta_2(self) = \perp_{\tau_c}$$

(vi) Putting It All Together...

$$I : \text{OCL Expr} \times \Sigma_{\mathcal{D}}^{\mathcal{D}} \times (\omega \rightarrow \bigcup_{u \in \omega} I(\tau(u))) \rightarrow$$

{true,
false,
⊥_bool}

$$\begin{aligned} \text{expr} ::= & w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \\ & \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3) \end{aligned}$$

- $I[[w]](\sigma, \beta) := \beta(w)$
- $I[[\omega(\text{expr}_1, \dots, \text{expr}_n)]](\sigma, \beta) := I(\omega)(I[[\text{expr}_1]](\sigma, \beta), \dots, I[[\text{expr}_n]](\sigma, \beta))$
- $I[[\text{allInstances}_C]](\sigma, \beta) := \text{dom}(\sigma) \cap \mathcal{D}(C)$

Note: in the OCL standard, $\text{dom}(\sigma)$ is assumed to be **finite**.

Again: doesn't scare us.

$$\mathcal{F} = (\emptyset, \{C, D\}, \emptyset, \emptyset)$$

$$\bullet \sigma_1 = \{ 1_C \mapsto \emptyset, 3_C \mapsto \emptyset, 27_C \mapsto \emptyset, 5_D \mapsto \emptyset \}$$

$$\bullet W = \{ x: \text{Int}, c: \tau_c \}$$

$$\bullet \beta_1 = \{ x \mapsto 13, c \mapsto 3_C \} \quad (**)$$

$$\bullet I[\llbracket \text{all instances}_D \rrbracket](\sigma_1, \beta_1) = \text{dom}(\sigma_1) \cap \mathcal{D}(D) = \{5_D\} \quad (*)$$

$$\bullet I[\llbracket \text{size}(\text{all instances}_D) \rrbracket](\sigma_1, \beta_1) = (I(\text{size})) (I[\llbracket \text{all instances}_D \rrbracket](\sigma_1, \beta_1))$$

$$= (I(\text{size})) (\{5_D\}) = |\{5_D\}| = 1 \quad (***)$$

\uparrow by (*) \nwarrow by def. of I(size)

$$\bullet I[\llbracket x > \text{size}(\text{all instances}_D) \rrbracket](\sigma_1, \beta_1) = (I(>)) (I[x](\sigma_1, \beta_1), I[\llbracket \text{size}(\text{all instances}_D) \rrbracket](\sigma_1, \beta_1))$$

$$= (I(>)) (\beta_1(x), 1) = (I(>)) (13, 1)$$

$$= \text{true} \quad (***)$$

\uparrow by def. of I(>) \nwarrow by def. I(>)

$$\bullet I[\llbracket 1 / (\text{size}(\text{all instances}_D) - 1) \rrbracket](\sigma_1, \beta_1) = \perp_{\text{Int}}$$

$$\uparrow$$

$$I[\llbracket 1 \rrbracket](\sigma_1, \beta_1) = I(1) = 1 \in \text{Int}$$

assuming

$$I(1)(x_1, x_2) = \begin{cases} x_1/x_2 & \text{if } x_1 \neq \perp \\ & \text{and } x_2 \neq \perp \\ & \text{and } x_2 \neq 0 \\ \perp_{\text{Int}} & \text{otherwise} \end{cases}$$

(vi) Putting It All Together...

$$\begin{aligned} \text{expr} ::= & w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \\ & \mid r_2(\text{expr}_1) \mid \text{expr}_1 \text{->iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3) \end{aligned}$$

Assume $\text{expr}_1 : \tau_C$ for some $C \in \mathcal{C}$. Set $\underline{u_1 := I[\text{expr}_1](\sigma, \beta) \in \mathcal{D}(\tau_C)}$.

- $$I[v(\text{expr}_1)](\sigma, \beta) := \begin{cases} \sigma(u_1)(v) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp_{\tau} & , \text{ otherwise} \end{cases} \quad \text{assuming } v : \tau$$
- $$I[r_1(\text{expr}_1)](\sigma, \beta) := \begin{cases} u & , \text{ if } u_1 \in \text{dom}(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \perp & , \text{ otherwise} \end{cases}$$

$r_1 : C_{0,1}$
- $$I[r_2(\text{expr}_1)](\sigma, \beta) := \begin{cases} \sigma(u_1)(r_2) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp_{\text{set}(\tau_c)} & , \text{ otherwise} \end{cases}$$

(Recall: σ evaluates r_2 of type C_* to a set)

$$\mathcal{Y} = (\{ \text{Int}, \text{Colour} \}, \{ C, D \}, \{ ms: C_{\text{in}}, sl: C_{\text{in}}, r: \text{Int}, c: \text{Colour} \}, \{ C \mapsto \{ms, sl, r\}, D \mapsto \{c\} \})$$

$$\mathcal{D}(\text{Int}) = \mathbb{Z}, \mathcal{D}(\text{Colour}) = \{ \text{red}, \text{green}, \text{blue} \}$$

$$\sigma_2 = \{ \begin{array}{l} 1_C \mapsto \{ ms \mapsto \emptyset, sl \mapsto \{2_C, 13_C\}, r \mapsto 9 \}, \\ 2_C \mapsto \{ ms \mapsto \{1_C\}, sl \mapsto \emptyset, r \mapsto 5 \}, \\ 3_C \mapsto \{ ms \mapsto \{4_C\}, sl \mapsto \emptyset, r \mapsto 3 \}, \\ 5_D \mapsto \{ c \mapsto \text{blue} \} \end{array}$$

$$\beta_2 = \{ p: \tau_C, q: \tau_D, x: \text{Int}, d: \text{Colour}, m: \tau_C \}$$

$\begin{array}{ccccccc} \nearrow 1_C & \searrow 5_D & \nearrow 4 & \searrow \text{green} & \nearrow 2_C & & \\ & & & & & & \end{array}$

- $\mathbb{I} \llbracket c(q) \rrbracket (\sigma_2, \beta_2) = \sigma_2(5_D)(c) = \text{blue}, \mathbb{I} \llbracket q \rrbracket (\sigma_2, \beta_2) = 5_D$
- $\mathbb{I} \llbracket c(q) = d \rrbracket (\sigma_2, \beta_2) = \text{false}$
- $\mathbb{I} \llbracket sl(p) \rrbracket (\sigma_2, \beta_2) = \{2_C, 13_C\}$
- $\mathbb{I} \llbracket r(ms(m)) \rrbracket (\sigma_2, \beta_2) = 9, \mathbb{I} \llbracket ms(m) \rrbracket (\sigma_2, \beta_2) = 1_C$
- $\mathbb{I} \llbracket r(ms(p)) \rrbracket (\sigma_2, \beta_2) = \perp_{\text{Int}}, \mathbb{I} \llbracket ms(p) \rrbracket (\sigma_2, \beta_2) = \perp_{\tau_C}$

(vi) Putting It All Together...

assign to hlp the set denoted by $expr_1$ (i.e., under β)

$$\begin{aligned}
 expr ::= & w \mid \omega(expr_1, \dots, expr_n) \mid \text{allInstances}_C \mid v(expr_1) \mid r_1(expr_1) \\
 & \mid r_2(expr_1) \mid expr_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)
 \end{aligned}$$

iterate result init. value expr.

assign to v_2 the initial value as given by $expr_2$

- $I[expr_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = expr_2 \mid expr_3)](\sigma, \beta)$

modification of β at hlp and v_2

$$:= \begin{cases} I[expr_2](\sigma, \beta) & , \text{ if } I[expr_1](\sigma, \beta) = \emptyset \\ \text{iterate}(hlp, v_1, v_2, expr_3, \sigma, \beta') & , \text{ otherwise} \end{cases}$$

where $\beta' = \beta[hlp \mapsto I[expr_1](\sigma, \beta), v_2 \mapsto I[expr_2](\sigma, \beta)]$ and

- $\text{iterate}(hlp, v_1, v_2, expr_3, \sigma, \beta')$

hlp has more than one element left

$$:= \begin{cases} I[expr_3](\sigma, \beta'[v_1 \mapsto x]) & , \text{ if } \beta'(hlp) = \{x\} \\ I[expr_3](\sigma, \beta'') & , \text{ if } \beta'(hlp) = X \dot{\cup} \{x\} \text{ and } X \neq \emptyset \end{cases}$$

where $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto \text{iterate}(hlp, v_1, v_2, expr_3, \sigma, \beta'[hlp \mapsto X])]$

new hlp is either hlp without x

(vi) Putting It All Together...

$$\begin{aligned} \text{expr} ::= & w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \\ & \mid r_2(\text{expr}_1) \mid \text{expr}_1 \text{->iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3) \end{aligned}$$

- $I[\text{expr}_1 \text{->iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)](\sigma, \beta)$

$$:= \begin{cases} I[\text{expr}_2](\sigma, \beta) & , \text{ if } I[\text{expr}_1](\sigma, \beta) = \emptyset \\ \text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta') & , \text{ otherwise} \end{cases}$$

where $\beta' = \beta[\text{hlp} \mapsto I[\text{expr}_1](\sigma, \beta), v_2 \mapsto I[\text{expr}_2](\sigma, \beta)]$ and

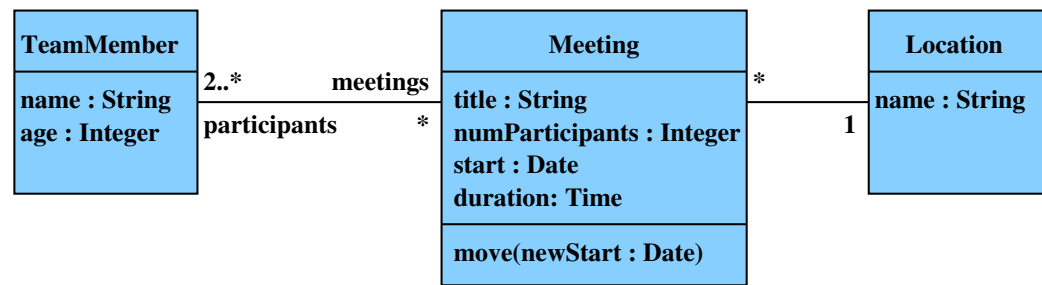
- $\text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta')$

$$:= \begin{cases} I[\text{expr}_3](\sigma, \beta'[v_1 \mapsto x]) & , \text{ if } \beta'(\text{hlp}) = \{x\} \\ I[\text{expr}_3](\sigma, \beta'') & , \text{ if } \beta'(\text{hlp}) = X \dot{\cup} \{x\} \text{ and } X \neq \emptyset \end{cases}$$

where $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto \text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta'[\text{hlp} \mapsto X])]$

Quiz: Is (our) I a function?

Example



- **context** TeamMember **inv:** age => 18
- **context** Meeting **inv:** duration > 0



References

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