Software Design, Modelling and Analysis in UML

Lecture 05: Object Diagrams, OCL Consistency

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Contents & Goals

Last Lecture:
- OCL Semantics

This Lecture:
- Educational Objectives: Capabilities for following tasks/questions.
  - What is an object diagram? What are object diagrams good for?
  - When is an object diagram called partial? What are partial ones good for?
  - When is an object diagram an object diagram (wrt. what)?
  - Is this an object diagram wrt. to that other thing?
  - How are system states and object diagrams related?
  - What does it mean that an OCL expression is satisfiable?
  - When is a set of OCL constraints said to be consistent?
  - Can you think of an object diagram which violates this OCL constraint?

- Content:
  - Object Diagrams
  - Example: Object Diagrams for Documentation
  - OCL: consistency, satisfiability
Where Are We?

You Are Here.

\[
\begin{align*}
\mathcal{S} &= (\mathcal{F}, \mathcal{E}, V, atr), \ SM \\
M &= (\Sigma \mathcal{F}, A_{\mathcal{F}}, \neg_{\mathcal{SM}}) \\
\pi &= (\sigma_0, \epsilon_0) \\
\mathcal{G} &= (N, E, f) \\
\mathcal{B} &= (Q_{SD}, q_0, A_{\mathcal{F}}, \neg_{SD}, F_{SD}) \\
\mathcal{W} &= ((\sigma_i, \cons_i, \Snd_i))_{i \in \mathbb{N}}
\end{align*}
\]
**Graph**

Definition. A node labelled graph is a triple

\[ G = (N, E, f) \]

consisting of
- vertexes \( N \),
- edges \( E \),
- node labeling \( f : N \to X \), where \( X \) is some label domain,
**Object Diagrams**

**Definition.** Let \( \mathcal{D} \) be a structure of signature \( \mathcal{S} = (\mathcal{T}, \mathcal{C}, V, \text{atr}) \) and \( \sigma \in \Sigma_{\mathcal{D}} \) a system state.

Then any graph \( G = (N, E, f) \) where

- nodes are identities (not necessarily alive), i.e. \( N \subset \mathcal{D}(\mathcal{C}) \) finite,
- edges correspond to "links" of objects, i.e. \( E \subseteq N \times \{ v : \tau \in V \mid \tau \in \{ C_0, C_*, \mathcal{C} \} \} \times N \),
- objects are labelled with attribute valuations and non-alive identities marked with "X", i.e. \( X = \{ \text{X} \} \cdot \mathcal{D}(\mathcal{C}) \) \( \cup \mathcal{D}(\mathcal{C}) \cup \mathcal{D}(\mathcal{C}) \cup \mathcal{D}(\mathcal{C}) \)
- \( \forall u \in N \cap \text{dom}(\sigma) : f(u) \subseteq \sigma(u) \) or \( f(u) \subseteq \{ \text{X} \} \).

is called object diagram of \( \sigma \).

**Graphical Representation of Object Diagrams**

- Assume \( \mathcal{S} = (\{ \text{Int} \}, \{ C \}, \{ v_1 : \text{Int}, v_2 : \text{Int}, r : C_* \}, \{ C \rightarrow \{ v_1, v_2, r \} \}) \).
- Consider \( \sigma = \{ u_1 \mapsto \{ v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{ u_2 \} \}, u_2 \mapsto \{ v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset \} \} \)
- Then \( G = (N, E, f) \) is an object diagram of \( \sigma \) wrt. \( \mathcal{S} \) and any \( \mathcal{D} \) with \( \mathcal{D}(\text{Int}) \supseteq \{ 1, 2, 3, 4 \} \).

\( G = (\{ u_1, u_2 \}, \{ (u_1, r, u_2), (u_1 \mapsto \{ v_1 \mapsto 1, v_2 \mapsto 2 \}, u_2 \mapsto \{ v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset \} \}) \)
Graphical Representation of Object Diagrams

\[ N \subset \mathcal{P}(C) \text{ finite, } E \subset N \times V_{0,1} \times N, \quad X = \{X\} \cup (V \mapsto (\mathcal{P}(E) \cup \mathcal{P}(N))) \]
\[ u_1 \in \text{dom}(\sigma) \land u_2 \in \sigma(u_1)(r), \quad f(u) \subseteq \sigma(u) \lor f(u) = \{X\} \]

- Assume \( \mathcal{P} = \{\text{Int}, \{C\}, \{v_1 : \text{Int}, v_2 : \text{Int}, r : C\}, \{C \mapsto \{v_1, v_2, r\}\}\)).
- Consider \( \sigma = \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{u_2\}\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset\}\}\)
- Then \( G = (N, E, f) \)
  \[ = (\{u_1, u_2\}, \{(u_1, r, u_2)\}, \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4\}\}, \]
  is an object diagram of \( \sigma \) wrt. \( \mathcal{P} \) and any \( \mathcal{P} \) with \( \mathcal{P}(\text{Int}) \supseteq \{1, 2, 3, 4\}\).
- We may equivalently (!) represent \( G \) graphically as follows:

UML Notation for Object Diagrams

- We assume different "boxes" represent different objects.
Object Diagrams: More Examples

\[ N \subset \mathcal{P}(\mathcal{E}) \text{ finite, } E \subset N \times V_{0,1}^* \times N, \quad X = \{X\} \cup (V \rightarrow (\mathcal{P}(\mathcal{F}) \cup \mathcal{P}(\mathcal{E}))) \]

\[ \sigma = \{1C \mapsto \{p \mapsto \emptyset, n \mapsto \{5C\}\}, 5C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1D \mapsto \{x \mapsto 23\}\} \]

vs.

\[ \sigma = \{1C \mapsto \{p \mapsto \emptyset, n \mapsto \{5C\}\}, 5C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1D \mapsto \{x \mapsto 23\}\} \]

Definition. Let \( G = (N, E, f) \) be an object diagram of system state \( \sigma \in \Sigma_{\mathcal{S}}^{\mathcal{P}} \).

We call \( G \) complete wrt. \( \sigma \) if and only if

- \( G \) is object complete, i.e.
  - \( G \) consists of all alive objects, i.e. \( N = \text{dom}(\sigma) \),

- \( G \) is attribute complete, i.e.
  - \( G \) comprises all "links" between alive objects, i.e.
    - if \( u_2 \in \sigma(u_1)(r) \) for some \( u_1, u_2 \in \text{dom}(\sigma) \) and \( r \in V \),
      then \( (u_1, r, u_2) \in E \), and
    - each node is labelled with the values of all \( \mathcal{F} \)-typed attributes, i.e. for each \( u \in \text{dom}(\sigma) \),
      \[ f(u) = \sigma(u)|_{V_{\mathcal{F}}} \cup \{r \mapsto (\sigma(u)(r)) \mid r \in V : \sigma(u)(r) \setminus N \neq \emptyset\} \]
      where \( V_{\mathcal{F}} := \{v : \tau \in V \mid \tau \in \mathcal{F}\} \).

Otherwise we call \( G \) partial.
Complete vs. Partial Examples

- $N = \text{dom}(\sigma)$, if $u_2 \in \sigma(u_1)(r)$, then $(u_1, r, u_2) \in E$,
- $f(u) = \sigma(u)_{\sigma} \cup \{r \mapsto (\sigma(u)(r) \setminus N) \mid \sigma(u)(r) \setminus N\}$

Complete or partial?

$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$

Complete/Partial is Relative

- Claim:
  - Each finite system state has exactly one complete object diagram.
  - A finite system state can have many partial object diagrams.

- Each object diagram $G$ represents a set of system states, namely
  $$G^{-1} := \{\sigma \in \Sigma^\sigma \mid G \text{ is an object diagram of } \sigma\}$$

- Observation: If somebody tells us, that a given (consistent) object diagram $G$ is complete, we can uniquely reconstruct the corresponding system state.
  In other words: $G^{-1}$ is then a singleton.
Closed Object Diagrams vs. Dangling References

Find the 10 differences! (Both diagrams shall be complete.)

Definition. Let \( \sigma \) be a system state. We say attribute \( v \in V_{0,1,\sigma} \) has a dangling reference in object \( u \in \text{dom}(\sigma) \) if and only if the attribute's value comprises an object which is not alive in \( \sigma \), i.e. if

\[
\sigma(u)(v) \not\in \text{dom}(\sigma).
\]

We call \( \sigma \) closed if and only if no attribute has a dangling reference in any object alive in \( \sigma \).
Closed Object Diagrams vs. Dangling References

Find the 10 differences! (Both diagrams shall be complete.)

Definition. Let \( \sigma \) be a system state. We say attribute \( v \in V_{0,1,*} \) has a dangling reference in object \( u \in \text{dom}(\sigma) \) if and only if the attribute’s value comprises an object which is not alive in \( \sigma \), i.e. if

\[
\sigma(u)(v) \not\subset \text{dom}(\sigma).
\]

We call \( \sigma \) closed if and only if no attribute has a dangling reference in any object alive in \( \sigma \).

Observation: Let \( G \) be the (!) complete object diagram of a closed system state \( \sigma \). Then the nodes in \( G \) are labelled with \( \mathcal{S} \)-typed attribute/value pairs only.

Special Notation

- \( \mathcal{S} = (\{\text{Int}\}, \{C\}, \{n, p : C_{*}\}, \{C \mapsto \{n, p\}\}) \).

- Instead of

  \[
  \begin{array}{c}
  \text{1o} : C \\
  \ \ \ \ \ \ n \\
  \text{2o} : C \\
  \end{array}
  \]

  we want to write

  \[
  \begin{array}{c}
  \text{1o} : C \\
  \ \ \ \ \ \ n \\
  \text{2o} : C \\
  \end{array}
  \]

  \[
  p = \emptyset
  \]

  or

  \[
  \begin{array}{c}
  \text{1o} : C \\
  \ \ \ \ \ \ n \\
  \text{2o} : C \\
  \end{array}
  \]

  to explicitly indicate that attribute \( p : C_{*} \) has value \( \emptyset \) (also for \( p : C_{0,1} \)).
Aftermath

We slightly deviate from the standard (for reasons):

- In the course, $C_{0,1}$ and $C_\ast$-typed attributes only have sets as values. UML also considers multisets, that is, they can have

  \[
  u_1 : C \quad \sim \quad n \quad u_2 : C
  \]

  (This is not an object diagram in the sense of our definition because of the requirement on the edges $E$. Extension is straightforward but tedious.)

- We allow to give the valuation of $C_{0,1}$- or $C_\ast$-typed attributes in the values compartment.
  - Allows us to indicate that a certain $r$ is not referring to another object.
  - Allows us to represent “dangling references”, i.e. references to objects which are not alive in the current system state.

- We introduce a graphical representation of $\emptyset$ values.

The Other Way Round
The Other Way Round

- If we only have a picture as below, we typically assume that it’s meant to be an object diagram wrt. some signature and structure.

\[ u_1 : C \quad z \quad u_2 : C \quad p \quad u_3 : D \quad z = 0 \]

- In the example, we can conclude (by “good will”) that the author is referring to some signature \( \mathcal{S} = (\mathcal{F}, \mathcal{E}, V, atr) \) with at least
  - \( \{u_1, u_2\} \subseteq \mathcal{C} \)
  - \( \tau \in \mathcal{F} \)
  - \( \{x : \mathcal{C}, \mathcal{T}, p : \mathcal{C}\} \)
  - \( \{x\} \subseteq atr(\mathcal{C}) \)
  - \( \{p, z\} \subseteq atr(\mathcal{D}) \)
and a structure with
  - \( \{u_1, u_2\} \subseteq \mathcal{D} \mathcal{C} \mathcal{C} \)
  - \( \nu_3 \in \mathcal{D}(\mathcal{D}) \)
  - \( 0 \in \mathcal{D}(\mathcal{T}) \)

Example: Object Diagrams for Documentation
Example: Data Structure [Schumann et al., 2008]

```
BaseNode
  +parent: BaseNode*
  +prevSibling: BaseNode*
  +nextSibling: BaseNode*
  +firstChild: BaseNode*
  +lastChild: BaseNode*

Node
 <data T>
  +Node(data: T)

Iterator
  +operator++() -> node
  +operator--() -> node
  +operator*() -> node

Forest
  +appendTopLevel(data: T)
  +appendChild(parent: Iterator&, data: T)
  +remove(it: Iterator&)
  +depth(it: Iterator&)
  +end()
  +begin()
  +empty()
  +size()
```

Example: Illustrative Object Diagram [Schumann et al., 2008]
**OCL Consistency**

**OCL Satisfaction Relation**

In the following, $S$ denotes a signature and $D$ a structure of $S$.

**Definition (Satisfaction Relation).** Let $\varphi$ be an OCL constraint over $S$ and $\sigma \in \Sigma^D$ a system state. We write

- $\sigma \models \varphi$ if and only if $I_{\varphi}(\sigma, \emptyset) = \text{true}$.
- $\sigma \not\models \varphi$ if and only if $I_{\varphi}(\sigma, \emptyset) = \text{false}$.

**Note:** In general we can’t conclude from $\neg(\sigma \models \varphi)$ to $\sigma \not\models \varphi$ or vice versa.
Object Diagrams and OCL

- Let $G$ be an object diagram of signature $\mathcal{S}$ wrt. structure $\mathcal{B}$.
  Let $expr$ be an OCL expression over $\mathcal{S}$.
  We say $G$ satisfies $expr$, denoted by $G \models expr$, if and only if
  $$\forall \sigma \in G^{-1} : \sigma \models expr.$$  
- If $G$ is complete, we can also talk about “$\not\models$”.
  (Otherwise better not to avoid confusion: $G^{-1}$ could comprise different system
  states in which $expr$ evaluates to true, false, and $\perp$.)

Example: (complete — what if not complete wrt. object/attribute/both?)

- context $C$  $	ext{inv : } n \to \text{isEmpty()}$
- context $C$  $	ext{inv : } p \cdot n \to \text{isEmpty()}$
- context $D$  $	ext{inv : } x \neq 0$

OCL Consistency

Definition (Consistency). A set $\text{inv} = \{\varphi_1, \ldots, \varphi_n\}$ of OCL
constraints over $\mathcal{S}$ is called consistent (or satisfiable) if and only if
there exists a system state of $\mathcal{S}$ wrt. $\mathcal{B}$ which satisfies all of them,
i.e. if
$$\exists \sigma \in \Sigma^\mathcal{S} : \sigma \models \varphi_1 \land \ldots \land \sigma \models \varphi_n$$
and inconsistent (or unrealizable) otherwise.
OCL Inconsistency Example

Deciding OCL Consistency

- Whether a set of OCL constraints is satisfiable or not is in general not as obvious as in the made-up example.
- Wanted: A procedure which decides the OCL satisfiability problem.
- Unfortunately: in general undecidable.

Otherwise we could, for instance, solve diophantine equations

\[ c_1 x_1^{n_1} + \cdots + c_m x_m^{n_m} = d. \]

Encoding in OCL:

\[ \text{allInstances}_C \rightarrow \exists (w : C \mid c_1 * w.x_1^{n_1} + \cdots + c_m * w.x_m^{n_m} = d). \]

- And now? Options: [Cabot and Clarisó, 2008]
  - Constrain OCL, use a less rich fragment of OCL.
  - Revert to finite domains — basic types vs. number of objects.
OCL Critique

- **Expressive Power:**
  - “Pure OCL expressions only compute primitive recursive functions, but not recursive functions in general.” [Cengarle and Knapp, 2001]

- **Evolution over Time:** “finally self.x > 0”
  - Proposals for fixes e.g. [Flake and Müller, 2003]. (Or: sequence diagrams.)

- **Real-Time:** “Objects respond within 10s”
  - Proposals for fixes e.g. [Cengarle and Knapp, 2002]

- **Reachability:** “After insert operation, node shall be reachable.”
  - Fix: add transitive closure.

- **Concrete Syntax**
  - “The syntax of OCL has been criticized – e.g., by the authors of Catalysis […] – for being hard to read and write.
  - OCL’s expressions are stacked in the style of Smalltalk, which makes it hard to see the scope of quantified variables.
  - Navigations are applied to atoms and not sets of atoms, although there is a collect operation that maps a function over a set.
  - Attributes, […], are partial functions in OCL, and result in expressions with undefined value.” [Jackson, 2002]
References


