

Software Design, Modelling and Analysis in UML

Lecture 07: A Type System for Visibility

2013-11-18

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Contents & Goals

Last Lecture:

- Representing class diagrams as (extended) signatures — for the moment without associations (see Lecture 08).
- **And:** in Lecture 03, implicit assumption of well-typedness of OCL expressions.

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
 - Is this OCL expression well-typed or not? Why?
 - How/in what form did we define well-definedness?
 - What is visibility good for?
- **Content:**
 - Recall: type theory/static type systems.
 - Well-typedness for OCL expression.
 - Visibility as a matter of well-typedness.

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Recall: From Class Boxes to Extended Signatures

Extended Classes

From now on, we assume that each class $C \in \mathcal{C}$ has:

- a finite (possibly empty) set S_C of **stereotypes**,
- a boolean flag $a \in \mathbb{B}$ indicating whether C is **abstract**,
- a boolean flag $t \in \mathbb{B}$ indicating whether C is **active**.

We use $S_{\mathcal{C}}$ to denote the set $\bigcup_{C \in \mathcal{C}} S_C$ of stereotypes in \mathcal{S} .

(Alternatively, we could add a set St as 5-th component to \mathcal{S} to provide the stereotypes (names of stereotypes) to choose from. But: too unimportant to care.)

Convention:

- We write

$$\langle C, S_C, a, t \rangle \in \mathcal{C}$$

when we want to refer to all aspects of C .

- If the new aspects are irrelevant (for a given context), we simply write $C \in \mathcal{C}$ i.e. old definitions are still valid.

Extended Attributes

- From now on, we assume that each attribute $v \in V$ has (in addition to the type):

- a **visibility**

$$\xi \in \underbrace{\{\text{public}\}}_{:=+}, \underbrace{\{\text{private}\}}_{:= -}, \underbrace{\{\text{protected}\}}_{:=\#}, \underbrace{\{\text{package}\}}_{:=\sim}$$

- an **initial value** $expr_0$ given as a word from **language for initial values**, e.g. OCL expressions.
(If using Java as **action language** (later) Java expressions would be fine.)
- a finite (possibly empty) set of **properties** P_v .
We define P_v analogously to stereotypes.

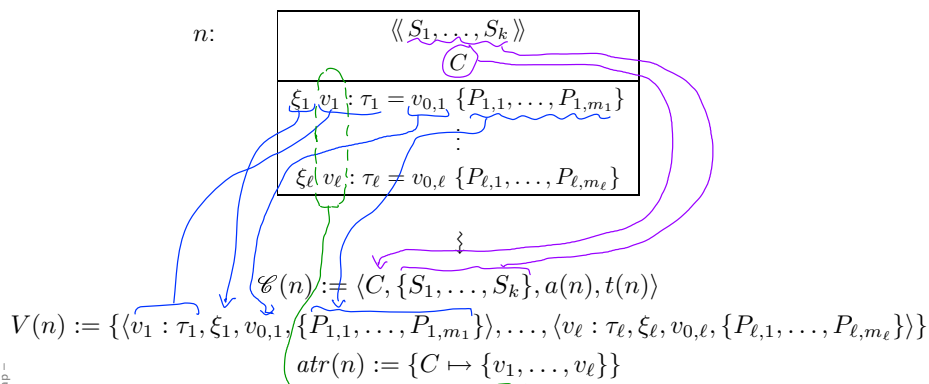
Convention:

- We write $\langle v : \tau, \xi, expr_0, P_v \rangle \in V$ when we want to refer to all aspects of v .
- Write only $v : \tau$ or v if details are irrelevant.

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From Class Boxes to Extended Signatures

A class box n **induces** an (extended) signature class as follows:



where

- “abstract” is determined by the font:
- “active” is determined by the frame:

$$a(n) = \begin{cases} true & \text{, if } n = \boxed{C} \text{ or } n = \boxed{C \{A\}} \\ false & \text{, otherwise} \end{cases} \quad t(n) = \begin{cases} true & \text{, if } n = \boxed{C} \text{ or } n = \boxed{C} \\ false & \text{, otherwise} \end{cases}$$

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Excursus: Type Theory (cf. Thiemann, 2008)

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Type Theory

Recall: In lecture 03, we introduced OCL expressions with **types**, for instance:

$expr ::= w$	$: \tau$... logical variable w
true false	$: Bool$... constants
0 -1 1 ...	$: Int$... constants
$expr_1 + expr_2$	$: Int \times Int \rightarrow Int$... operation
size ($expr_1$)	$: Set(\tau) \rightarrow Int$	
not $expr$	$: Bool \rightarrow Bool$	

Wanted: A procedure to tell **well-typed**, such as $(w : Bool)$
not w

from **not well-typed**, such as,

size(w).

Approach: Derivation System, that is, a finite set of derivation rules.

We then say $expr$ is **well-typed** if and only if we can derive

$A, C \vdash expr : \tau$ (read: "expression $expr$ has type τ ")

for some OCL type τ , i.e. $\tau \in T_B \cup T_{\mathcal{E}} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{E}}\}$, $C \in \mathcal{C}$.

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A Type System for OCL

A Type System for OCL

We will give a finite set of **type rules** (a **type system**) of the form

$$\text{("name")} \frac{\text{"premises"}}{\text{"conclusion"}} \text{"side condition"}$$

These rules will establish well-typedness statements (**type sentences**) of three different **qualities**:

(i) Universal well-typedness:

$$\begin{aligned} &\vdash \text{expr} : \tau \\ &\vdash 1 + 2 : \text{Int} \end{aligned}$$

(ii) Well-typedness in a **type environment** A : (for logical variables)

$$\begin{aligned} &A \vdash \text{expr} : \tau \\ &\text{self} : \tau_C \vdash \text{self}.v : \text{Int} \end{aligned}$$

(iii) Well-typedness in type environment A and **context** B : (for visibility)

$$\begin{aligned} &A, B \vdash \text{expr} : \tau \\ &\text{self} : \tau_C, C \vdash \text{self}.r.v : \text{Int} \end{aligned}$$

Constants and Operations

- If $expr$ is a **boolean constant**, then $expr$ is of type $Bool$:

$$(BOOL) \frac{}{\vdash B : Bool}, \quad B \in \{true, false\}$$

- If $expr$ is an **integer constant**, then $expr$ is of type Int :

$$(INT) \frac{}{\vdash N : Int}, \quad N \in \{0, 1, -1, \dots\}$$

- If $expr$ is the application of **operation** $\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau$ to expressions $expr_1, \dots, expr_n$ which are of type τ_1, \dots, τ_n , then $expr$ is of type τ :

$$(Fun_0) \frac{\vdash expr_1 : \tau_1 \dots \vdash expr_n : \tau_n}{\vdash \omega(expr_1, \dots, expr_n) : \tau}, \quad \omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, \\ n \geq 1, \omega \notin atr(\mathcal{C})$$

(Note: this rule also covers '=' _{τ} ', 'isEmpty', and 'size'.)

Constants and Operations Example

(BOOL)	$\frac{}{\vdash B : Bool},$	$B \in \{true, false\}$
(INT)	$\frac{}{\vdash N : Int},$	$N \in \{0, 1, -1, \dots\}$
(Fun ₀)	$\frac{\vdash expr_1 : \tau_1 \dots \vdash expr_n : \tau_n}{\vdash \omega(expr_1, \dots, expr_n) : \tau},$	$\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau,$ $n \geq 1, \omega \notin atr(\mathcal{C})$

Example:

- not true

$$(Fun_0) \frac{\frac{(BOOL) \frac{}{\vdash true : Bool}}{\vdash not(true) : Bool}}{not : Bool \rightarrow Bool}}$$

- $true + 3$
 - got stuck - we cannot derive this from the rules

$$(Fun_0) \frac{\frac{\vdash true : Int \quad \frac{(INT) \frac{}{\vdash 3 : Int}}{+ : Int \times Int \rightarrow Int}}{\vdash true + 3 : Int}}$$

② $\hookrightarrow true + 3$ is not well-typed

Type Environment

- **Problem:** Whether

$$w + 3$$

is well-typed or not depends on the type of logical variable $w \in W$.

- **Approach:** Type Environments

Definition. A **type environment** is a (possibly empty) finite sequence of type declarations.

The set of type environments for a given set W of logical variables and types T is defined by the grammar

$$A ::= \emptyset \mid A, w : \tau$$

where $w \in W, \tau \in T$.

Clear: We use this definition for the set of OCL logical variables W and the types $T = T_B \cup T_{\mathcal{C}} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$.

Environment Introduction and Logical Variables

- If $expr$ is of type τ , then it is of type τ **in any** type environment:

$$(EnvIntro) \quad \frac{\vdash expr : \tau}{A \vdash expr : \tau}$$

- Care for logical variables in **sub-expressions** of operator application:

$$(Fun_1) \quad \frac{A \vdash expr_1 : \tau_1 \dots A \vdash expr_n : \tau_n}{A \vdash \omega(expr_1, \dots, expr_n) : \tau}, \quad \begin{array}{l} \omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, \\ n \geq 1, \omega \notin atr(\mathcal{C}) \end{array}$$

- If $expr$ is a **logical variable** such that $w : \tau$ occurs in A , then we say w is of type τ ,

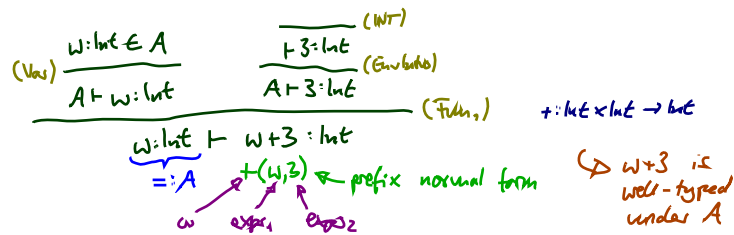
$$(Var) \quad \frac{w : \tau \in A}{A \vdash w : \tau}$$

Type Environment Example

(EnvIntro)	$\frac{\vdash \text{expr} : \tau}{A \vdash \text{expr} : \tau}$	
(Fun ₁)	$\frac{A \vdash \text{expr}_1 : \tau_1 \dots A \vdash \text{expr}_n : \tau_n}{A \vdash \omega(\text{expr}_1, \dots, \text{expr}_n) : \tau}$	$\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, n \geq 1, \omega \notin \text{atr}(\mathcal{C})$
(Var)	$\frac{w : \tau \in A}{A \vdash w : \tau}$	

Example:

- $w + 3, A = w : \text{Int}$



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All Instances and Attributes in Type Environment

- If expr refers to **all instances** of class C , then it is of type $\text{Set}(\tau_C)$,

$$(\text{AllInst}) \quad \frac{}{\vdash \text{allInstances}_C : \text{Set}(\tau_C)}$$

- If expr is an **attribute access** of an attribute of type τ for an object of C as denoted by expr_1 , then the premise is that expr_1 is of type τ_C :

$$(\text{Attr}_0) \quad \frac{A \vdash \text{expr}_1 : \tau_C}{A \vdash v(\text{expr}_1) : \tau}, \quad v : \tau \in \text{atr}(C), \tau \in \mathcal{T}$$

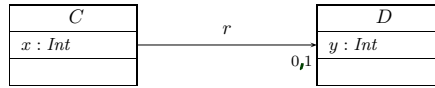
$$(\text{Attr}_0^{0,1}) \quad \frac{A \vdash \text{expr}_1 : \tau_C}{A \vdash r_1(\text{expr}_1) : \tau_D}, \quad r_1 : D_{0,1} \in \text{atr}(C)$$

$$(\text{Attr}_0^*) \quad \frac{A \vdash \text{expr}_1 : \tau_C}{A \vdash r_2(\text{expr}_1) : \text{Set}(\tau_D)}, \quad r_2 : D_* \in \text{atr}(C)$$

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Attributes in Type Environment Example

$(Attr_0)$	$\frac{A \vdash expr_1 : \tau_C}{A \vdash v(expr_1) : \tau}$	$v : \tau \in atr(C), \tau \in \mathcal{T}$
$(Attr_{0,1}^{0,1})$	$\frac{A \vdash expr_1 : \tau_C}{A \vdash r_1(expr_1) : \tau_D}$	$r_1 : D_{0,1} \in atr(C)$
$(Attr_0^*)$	$\frac{A \vdash expr_1 : \tau_C}{A \vdash r_2(expr_1) : Set(\tau_D)}$	$r_2 : D_* \in atr(C)$



$V = \{x: Int, r: D_{0,1}, y: Int\}$
 $atr(C) = \{x, r\}$

$self : \tau_C \vdash self : \tau_C$

$self : \tau_C \vdash self.y : Int$

derivable but not useful
 needed
 ↳ get stuck but not
 ↳ not well-f. derivable

- $self : \tau_C \vdash self.y : Int$
- $self : \tau_C \vdash self.x : Int$ well-typed by $(Attr_0), (Var)$
- $self : \tau_C \vdash self.r : \tau_D$ well-typed $(Attr_0^*), (Var)$
- $self : \tau_C \vdash self.r.x : Int$ not well-typed, get stuck after applying $(Attr_0^*)$
- $self : \tau_C \vdash self.f.y : Int$ well-typed by $(Attr_0^*), (Attr_0), (Var)$

Iterate

- If $expr$ is an **iterate expression**, then
 - the iterator variable has to be type consistent with the base set, and
 - initial and update expressions have to be consistent with the result variable:

$$(Iter) \frac{A \vdash expr : Set(\tau_1) \quad Set(\tau_1) \quad A \vdash expr_2 : \tau_2 \quad A' \vdash expr_3 : \tau_2}{A \vdash expr_1 \rightarrow iterate(w_1 : \tau_1 ; w_2 : \tau_2 = expr_2 \mid expr_3) : \tau_2}$$

well-typedness of $expr_2$ depends on outer scope inner scope

where $A' = A \oplus (w_1 : \tau_1) \oplus (w_2 : \tau_2)$.

override typing of w_1 and w_2 in A
 (" $w_1 : \tau_1, w_2 : \tau_2$ hide outer scope")

outer scope
 $all\ list_c \rightarrow iterate(i \dots 1)$
 $if \rightarrow iterate(i \dots 1 \dots)$
 inner scope

Casting in the Type System

One Possible Extension: Implicit Casts

- We **may wish** to have

$$\vdash 1 \text{ and } \textit{false} : \textit{Bool} \quad (*)$$

In other words: We may wish that the type system allows to use $0, 1 : \textit{Int}$ instead of *true* and *false* without breaking well-typedness.

- Then just have a rule:

$$(\textit{Cast}) \quad \frac{A \vdash \textit{expr} : \textit{Int}}{A \vdash \textit{expr} : \textit{Bool}}$$

- With (Cast) (and (Int), and (Bool), and (Fun₀)), we can derive the sentence (*), thus conclude well-typedness.
- **But:** that's only half of the story — the definition of the interpretation function *I* that we have is not prepared, it doesn't tell us what (*) means...

Implicit Casts Cont'd

So, why isn't there an interpretation for (1 and false)?

- First of all, we have (syntax)

$$expr_1 \text{ and } expr_2 : Bool \times Bool \rightarrow Bool$$

- Thus,

$$I(\text{and}) : I(Bool) \times I(Bool) \rightarrow I(Bool)$$

where $I(Bool) = \{true, false\} \cup \{\perp_{Bool}\}$.

- By definition,

$$I[\![1 \text{ and } false]\!](\sigma, \beta) = I(\text{and})(I[\![1]\!](\sigma, \beta), I[\![false]\!](\sigma, \beta)),$$

and **there we're stuck**.

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Implicit Casts: Quickfix

- Explicitly define

$$I[\![\text{and}(expr_1, expr_2)]\!](\sigma, \beta) := \begin{cases} b_1 \wedge b_2 & , \text{ if } b_1 \neq \perp_{Bool} \neq b_2 \\ \perp_{Bool} & , \text{ otherwise} \end{cases}$$

where

- $b_1 := toBool(I[\![expr_1]\!](\sigma, \beta))$,
- $b_2 := toBool(I[\![expr_2]\!](\sigma, \beta))$,

and where

$$toBool : I(Int) \cup I(Bool) \rightarrow I(Bool)$$

$$x \mapsto \begin{cases} true & , \text{ if } x \in \{true\} \cup I(Int) \setminus \{0, \perp_{Int}\} \\ false & , \text{ if } x \in \{false, 0\} \\ \perp_{Bool} & , \text{ otherwise} \end{cases}$$

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Bottomline

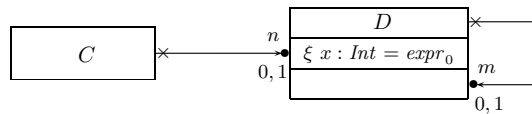
- There are **wishes** for the type-system which require changes in both, the definition of *I* **and** the type system.
In most cases not difficult, but tedious.
- **Note:** the extension is still a basic type system.
- **Note:** OCL has a far more elaborate type system which in particular addresses the relation between *Bool* and *Int* (cf. [OMG, 2006]).

Visibility in the Type System

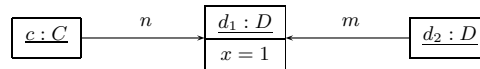
Visibility — The Intuition

$$\mathcal{S} = (\{Int\}, \{C, D\}, \{n : D_{0,1}, m : D_{0,1}, \langle x : Int, \xi, expr_0, \emptyset \rangle\}, \{C \mapsto \{n\}, D \mapsto \{x, m\}\})$$

Let's study an **Example**:



and



Assume $w_1 : \tau_C$ and $w_2 : \tau_D$ are logical variables. Which of the following syntactically correct (?) OCL expressions shall we consider to be well-typed?

ξ of x :	public	private	protected	package
$w_1 . n . x = 0$	✓	✓ -	later	not
	✗	✗ w_1		
	?	? rest		
$w_2 . m . x = 0$	✓	✓ w_2	later	not
$x(m(w_2)) = 0$	✗	✗ w_2		
	?	? rest		

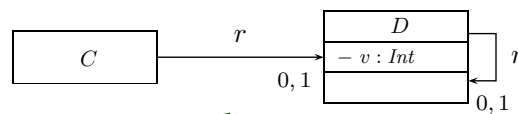
Handwritten notes: "privateness is by class, not by object" with arrows pointing to the private and protected cells in the first two rows.

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Context

$$\mathcal{S} = (\{Int\}, \{C, D\}, \{r : D_{0,1}, v : Int\}, \{C \mapsto \{r\}, D \mapsto \{r, v\}\})$$

- Example:** A problem?



$$self : \tau_D \vdash self . r . v > 0 \quad \checkmark$$

$$self : \tau_C \not\vdash self . r . v > 0 \quad \times$$

Handwritten annotations: Arrows point from τ_D to the first expression and from τ_C to the second expression.

- That is, whether an expression involving attributes with visibility is well-typed **depends** on the class of objects for which it is evaluated.
- Therefore:** well-typedness in type environment A and **context** $B \in \mathcal{C}$:

$$A, B \vdash expr : \tau$$

- In particular: prepare to treat "protected" later (when doing inheritance).

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Attribute Access in Context

- If $expr$ is of type τ in a type environment, then it is in **any context**:

$$(Context) \frac{A \vdash expr : \tau}{A \vdash expr : \tau}$$

- Accessing attribute** v of a C -object via logical variable w is well-typed if
 - ~~v is public, or~~ w is of type τ_B

$$(Attr_1) \frac{A \vdash w : \tau_B}{A, B \vdash v(w) : \tau}, \quad \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(B)$$

- Accessing attribute** v of a C -object of via expression $expr_1$ is well-typed **in context** B if
 - v is public, or $expr_1$ denotes an object of class B :

$$(Attr_2) \frac{A, B \vdash expr_1 : \tau_C}{A, B \vdash v(expr_1) : \tau}, \quad \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(C), \quad \xi = +, \text{ or } C = B$$

- Accessing $C_{0,1}$ - or C_* -typed attributes: similar.

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Context in Operator Application

- Operator Application:

$$(Fun_2) \frac{A, B \vdash expr_1 : \tau_1 \dots A, B \vdash expr_n : \tau_n}{A, B \vdash \omega(expr_1, \dots, expr_n) : \tau}, \quad \omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau, \quad n \geq 1, \omega \notin atr(\mathcal{C})$$

- Iterate:

$$(Iter_1) \frac{A, B \vdash expr_1 : Set(\tau_1) \quad A', B \vdash expr_2 : \tau_2 \quad A', B \vdash expr_3 : \tau_2}{A, B \vdash expr_1 \rightarrow iterate(w_1 : \tau_1 ; w_2 : \tau_2 = expr_2 \mid expr_3) : \tau_2}$$

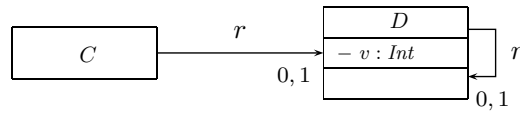
where $A' = A \oplus (w_1 : \tau_1) \oplus (w_2 : \tau_2)$.

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Attribute Access in Context Example

$$\begin{array}{c}
 \text{(Context)} \quad \frac{A \vdash \text{expr} : \tau}{A \vdash \text{expr} : \tau} \\
 \text{(Attr}_1\text{)} \quad \frac{A, B \vdash \text{expr}_1 : \tau_C}{A, B \vdash v(\text{expr}_1) : \tau}, \quad \langle v : \tau, \xi, \text{expr}_0, P_\xi \rangle \in \text{atr}(C), \\
 \xi = +, \text{ or } \xi = - \text{ and } C = B
 \end{array}$$



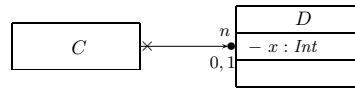
Example:

$$self : \tau_C \quad \vdash self . r . v > 0$$

The Semantics of Visibility

- **Observation:**
 - Whether an expression **does** or **does not** respect visibility is a matter of well-typedness **only**.
 - We only evaluate (= apply I to) **well-typed** expressions.
- We **need not** adjust the interpretation function I to support visibility.

What is Visibility Good For?



- Visibility is a property of attributes — is it useful to consider it in OCL?
- In other words: given the picture above, **is it useful** to state the following invariant (even though x is private in D)

context C inv : $n..x > 0$?

- **It depends.** (cf. [OMG, 2006], Sect. 12 and 9.2.2)

- **Constraints and pre/post conditions:**

- Visibility is **sometimes** not taken into account. To state “global” requirements, it may be adequate to have a “global view”, be able to look into all objects.
- But: visibility supports “narrow interfaces”, “information hiding”, and similar good design practices. To be more robust against changes, try to state requirements only in the terms which are visible to a class.

Rule-of-thumb: if attributes are important to state requirements on design models, leave them public or provide get-methods (later).

- **Guards and operation bodies:**

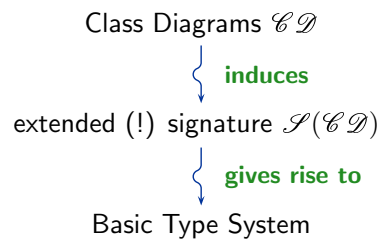
If in doubt, **yes** (= do take visibility into account).

Any so-called **action language** typically takes visibility into account.

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Recapitulation

Recapitulation



- We extended the type system for
 - **casts** (requires change of I) and \triangleleft *see earlier slides*
 - **visibility** (no change of I).
- **Later: navigability** of associations.

Good: well-typedness is decidable for these type-systems. That is, we can have automatic tools that check, whether OCL expressions in a model are well-typed.

References

References

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- [OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.