

Software Design, Modelling and Analysis in UML

Lecture 14: Core State Machines IV

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Contents & Goals

Last Lecture:

- System configuration
- Transformer
- Action language: skip, update, send

This Lecture:

- Educational Objectives: Capabilities for following tasks/questions:
 - What does this State Machine mean? What happens if I inject this event?
 - Can you please model the following behaviour.
 - What is: Signal, Event, Ether, Transformer, Step, RTC.

- Content:
 - Transformers for Action Language
 - Run-to-completion Step
 - Putting It All Together

Transformer Cont'd

Transformer: Create

abstract syntax	create(C, expr, v)
concrete syntax	expr.v := new C
intuitive semantics	Create an object of class C and assign it to attribute v of the object denoted by expression expr.
well-typedness	expr : T _D , v ∈ attr(D), attr(C) = {{v1 : T1, v2 : T2} 1 ≤ i ≤ n}
semantics	...
observables	...
(error) conditions	[[expr]](C, β) not defined

* We use an "and assign" action for simplicity — it doesn't add or remove expressive power, but moving creation to the expression language raises all kinds of other problems such as order of evaluation (and thus creation)!

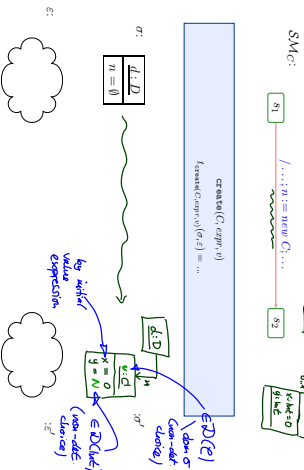
* Also for simplicity: no parameters to construction (= parameters of constructor). Adding them is straightforward (but somewhat tedious).

(4) 20 Apr: x := (new C) * + (new C) y

if needed

obj := new C
 x := obj * x
 obj := new C
 x := obj * x
 obj := new C
 x := obj * x

Create Transformer Example



How To Choose New Identities?

- Re-use: choose any identity that is not alive now, i.e. not in dom(σ). *our choice*
- Doesn't depend on history.
- May "undangle" dangling references — may happen on some platforms.
- Fresh: choose any identity that has not been alive ever, i.e. not in dom(σ) and any predecessor in current run.
- Depends on history.
- Dangling references remain dangling — could make "dirty" effects of platform.

Transformer: Create

abstract syntax Create(C, expr, v)	concrete syntax
inlative semantics Create an object of class C and assign it to attribute v of the object denoted by expression expr .	
well-synedness $\text{expr} : \tau_C, v \in \text{attr}(D), \text{attr}(C) = \{(v_1 : \tau_1, \text{expr}_1^i) \mid 1 \leq i \leq n\}$	
semantics $\llbracket \text{Create} \rrbracket(\sigma, \varepsilon) = \{v\}$ if $v \in \text{attr}(D)$ and $\text{attr}(C) = \{(v_1 : \tau_1, \text{expr}_1^i) \mid 1 \leq i \leq n\}$. if $\sigma' = \sigma \uparrow v_0 \rightarrow \sigma \uparrow \text{dom}(v)$ and $v_0 \in \text{dom}(\sigma)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$. if $\sigma' = \llbracket \text{expr} \rrbracket(\sigma, \varepsilon)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$ if $\text{expr}^i \neq \text{fresh}$ and arbitrary value from $\mathcal{D}(\tau_i)$ otherwise.	<p><i>similar to variables</i></p> <p><i>if $\sigma' = \sigma \uparrow v_0 \rightarrow \sigma \uparrow \text{dom}(v)$ and $v_0 \in \text{dom}(\sigma)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$. make</i></p> <p><i>if $\sigma' = \llbracket \text{expr} \rrbracket(\sigma, \varepsilon)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$ if $\text{expr}^i \neq \text{fresh}$ and arbitrary value from $\mathcal{D}(\tau_i)$ otherwise.</i></p> <p><i>if $\sigma' = \llbracket \text{expr} \rrbracket(\sigma, \varepsilon)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$ if $\text{expr}^i \neq \text{fresh}$ and arbitrary value from $\mathcal{D}(\tau_i)$ otherwise.</i></p> <p><i>if $\sigma' = \llbracket \text{expr} \rrbracket(\sigma, \varepsilon)$, then $\llbracket \text{Create} \rrbracket(\sigma', \varepsilon) = \{v\}$ if $\text{expr}^i \neq \text{fresh}$ and arbitrary value from $\mathcal{D}(\tau_i)$ otherwise.</i></p>
observables $\text{Obs}_{\text{Create}}(\llbracket v \rrbracket) = \{(v_0, \perp, (\sigma_0, \theta))\}$	
(error) conditions $\llbracket \text{Create} \rrbracket(\sigma)$ not defined.	

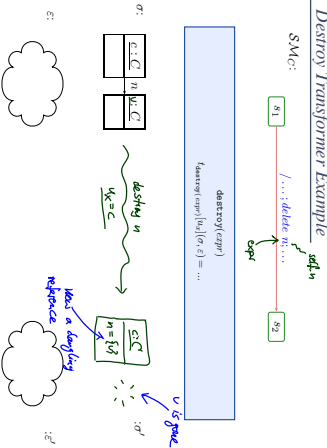
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Transformer: Destroy

abstract syntax destroy(expr)	concrete syntax delete expr
inlative semantics Destroy the object denoted by expression expr .	
well-synedness $\text{expr} : \tau_C, C \in \mathcal{C}$	
semantics ...	
observables $\text{Obs}_{\text{destroy}}(\llbracket v \rrbracket) = \{(v_0, \perp, (+, \emptyset), v)\}$	
(error) conditions $\llbracket \text{destroy} \rrbracket(\sigma, \mathcal{D})$ not defined.	

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Destroy Transformer Example



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What to Do With the Remaining Objects?

- Assume object v_0 is destroyed...
 - object v_1 may still refer to it via association r :
 - allow dangling references?
 - or remove v_0 from $\sigma(v_1, r)$?
 - object v_0 may have been the last one linking to object v_2 :
 - leave v_2 alone?
 - or remove v_2 also? (**garbage collection!**)
 - Plus: (temporal extensions of) OCL may have dangling references.
- Our choice:** Dangling references and no garbage collection! This is in line with "expect the worst", because there are target platforms which don't provide garbage collection — and models shall (in general) be correct without assumptions on target platform.
- But:** the more "dirty" effects we see in the model, the more expensive it often is to analyse. Valid proposal for simple analysis: monotone frame semantics, no destruction at all.

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Transformer: Destroy

abstract syntax destroy(expr)	concrete syntax delete expr
inlative semantics Destroy the object denoted by expression expr .	
well-synedness $\text{expr} : \tau_C, C \in \mathcal{C}$	
semantics $\llbracket \text{destroy} \rrbracket(\sigma, \varepsilon) = \sigma \uparrow \text{dom}(\sigma) \setminus \{v\}$ with $v = \llbracket \text{expr} \rrbracket(\sigma, \varepsilon)$.	
observables $\text{Obs}_{\text{destroy}}(\llbracket v \rrbracket) = \{(v_0, \perp, (+, \emptyset), v)\}$	
(error) conditions $\llbracket \text{destroy} \rrbracket(\sigma, \mathcal{D})$ not defined.	

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Sequential Composition of Transformers

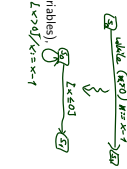
- Sequential composition** $t_1 \circ t_2$ of transformers t_1 and t_2 is canonically defined as $\llbracket t_1 \circ t_2 \rrbracket(\sigma, \varepsilon) = \llbracket t_2 \rrbracket(\llbracket t_1 \rrbracket(\sigma, \varepsilon))$ with observation $\text{Obs}_{t_1 \circ t_2}(\llbracket v \rrbracket) = \text{Obs}_{t_1}(\llbracket v \rrbracket) \cup \text{Obs}_{t_2}(\llbracket v \rrbracket)$ ($t_2 \circ t_1$)
 - Clear:** not defined if one of the two intermediate "micro steps" is not defined.
- $\sigma \times \sigma' \rightarrow \sigma \uparrow \text{dom}(\sigma')$ **seq. composition of observables***
- $\llbracket \text{seq} \rrbracket(\sigma, \varepsilon) = \llbracket \text{seq} \rrbracket(\sigma, \varepsilon)$*

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Observation: our transformers are in principle the **denotational semantics** of the actions/ action sequences. The trivial case, to be precise.

Note: with the previous examples, we can capture

- empty statements, skips.
- assignments,
- conditionals (by normalisation and auxiliary variables),
- create/destroy,



but not possibly diverging loops.

Our (Simple) Approach: if the action language is, e.g., Java, then (syntactically) forbid loops and calls of recursive functions.

Other Approach: use full blown denotational semantics.

No show-stopper, because loops in the action annotation can be converted into transition cycles in the state machine.

Step and Run-to-completion Step

Definition. Let A be a set of actions and S a (not necessarily finite) set of states.

We call

$$\rightarrow \subseteq S \times A \times S$$

a (labelled) transition relation.

Let $S_0 \subseteq S$ be a set of initial states. A sequence

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

with $s_i \in S, a_i \in A$ is called **computation of the labelled transition system** (S, \rightarrow, S_0) if and only if

- **initiation:** $s_0 \in S_0$
- **consecution:** $(s_i, a_i, s_{i+1}) \in \rightarrow$ for $i \in \mathbb{N}_0$.

Note: for simplicity, we only consider infinite runs.

Active vs. Passive Classes/Objects

Note: From now on, assume that all classes are **active** for simplicity.

We'll later briefly discuss the Rhapsody framework which proposes a way how to integrate non-active objects.

Note: The following RT-C "algorithm" follows [Harel and Gery, 1997] (i.e. the one realised by the Rhapsody code generation) **where** the standard is ambiguous or leaves choices.

From Core State Machines to LTS

Definition. Let $\mathcal{S}_0 = (\mathcal{S}_0, \mathcal{K}_0, \gamma_0, \text{act}, \sigma, \mathcal{E})$ be a signature with signals (all classes **active**) \mathcal{S}_0 , a structure of \mathcal{S}_0 , and $(EBM, \text{ready}, \Phi, \Theta, \Gamma)$ an ether over \mathcal{S}_0 and \mathcal{S}_0 . Assume there is one core state machine M_C per class $C \in \mathcal{E}$.

We say, the state machines **induce** the following labelled transition relation on states

$$S := (\mathcal{S}_0 \cup \{\#\}) \times EBM \text{ with actions } A := (\mathcal{E}^{\text{in}} \cup \{\text{error}\} \cup \{\text{consum}\}) \times \mathcal{E}^{\text{out}}$$

- $(\sigma, \varepsilon) \xrightarrow{\text{consum}, \text{Send}} (\sigma', \varepsilon')$ if and only if $\sigma \xrightarrow{\text{error}} \sigma'$
- $(\sigma, \varepsilon) \xrightarrow{\text{consum}, \text{Send}} (\sigma', \varepsilon')$ if an event with destination u is discarded.
- (ii) an event is dispatched to u, i i.e. stable object processes an event, or runs-to-completion processing by u commences.
- (iii) the object u is not stable and continues to process an event.
- (iv) the environment interacts with object u .
- $\# \xrightarrow{\text{consum}, \#} \#$ if and only if $\#$ and $\text{consum} = \emptyset$, or an error condition occurs during consumption of consum .

(i) Discarding An Event

if

$$(\sigma, \varepsilon) \xrightarrow{\text{consum}, \text{Send}} (\sigma', \varepsilon')$$

• an E -event (instance of signal E) is ready in ε for object u of a class \mathcal{C} , i.e. if

$$u \in \text{dom}(\sigma) \cap \mathcal{D}(C) \wedge \exists u_E \in \mathcal{D}(E); u_E \in \text{ready}(C, u)$$

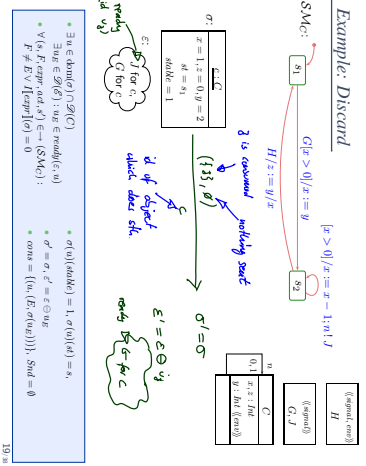
• u is stable and in state machine state s_i , i.e. $\sigma(u)(\text{stable}) = 1$ and $\sigma(u)(s) = s_i$, but there is no corresponding transition enabled (all transitions incident with current state of u either have other triggers or the guard is not satisfied)

$$\forall (s, F, \text{app}, \text{act}, s') \in \text{--}(SM_C); F \neq E \vee \llbracket \text{app} \rrbracket (s) = 0$$

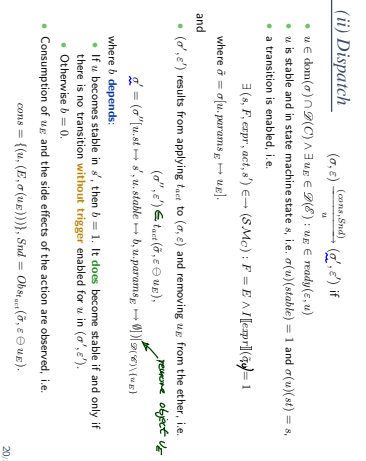
and

- the system configuration doesn't change, i.e. $\sigma' = \sigma$
- the event u_E is removed from the ether, i.e. $\varepsilon' = \varepsilon \ominus u_E$
- consumption of u_E is observed, i.e. $\text{consum} = ((u, E, \sigma'(u_E)))$; $\text{Send} = \emptyset$

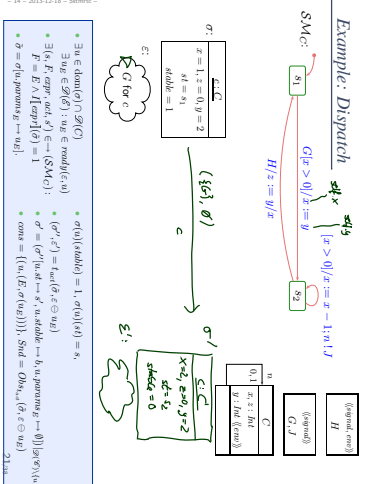
Example: Discard



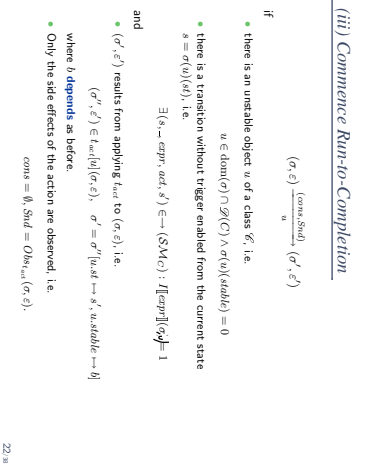
(ii) Dispatch



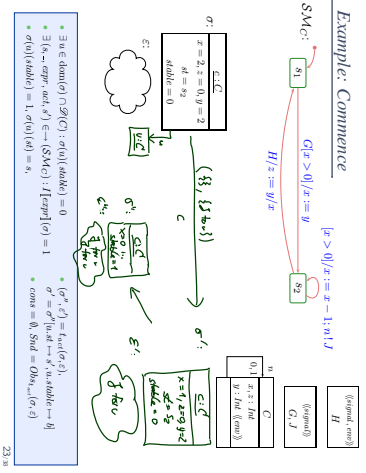
Example: Dispatch



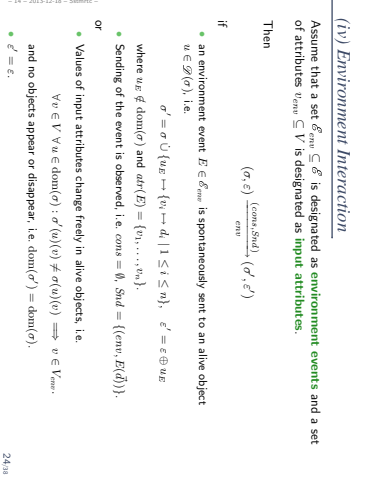
(iii) Commence Run-to-Completion

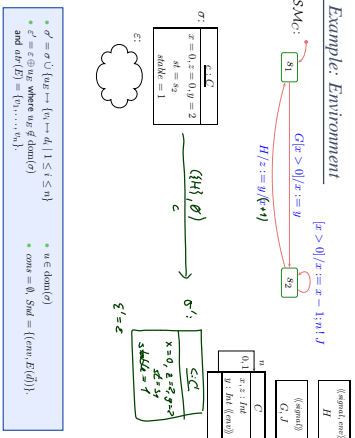


Example: Commence



(iv) Environment Interaction





Notions of Steps: The Step

Note: we call one evolution $(\sigma, \varepsilon) \xrightarrow{\text{Comm, Sml}} (\sigma', \varepsilon')$ a **step**.
 Thus in our setting, a **step directly corresponds** to **one object** (namely u) takes a **single transition** between regular states.
 (We have to extend the concept of "single transition" for hierarchical state machines)
 That is: We're going for an interleaving semantics without true parallelism.
Remark: With only methods (later), the notion of step is not so clear.
 For example, consider

- c_1 calls $f(O)$ at c_2 , which calls $g(O)$ at c_1 which in turn calls $h(O)$ for c_2 .
- Is the completion of $h(O)$ a step?
- Or the completion of $f(O)$?
- Or doesn't it play a role?

It does play a role, because **constraints/invariants** are typically (= by convention) assumed to be evaluated at step boundaries, and sometimes the convention is meant to admit (temporary) violation in between steps.

(v) Error Conditions

if, in (ii) or (iii),

- $\llbracket \text{Error} \rrbracket$ is not defined for σ , or
- Error is not defined for (σ, ε) ,

and

- consumption is observed according to (ii) or (iii), but $\text{Sml} = \emptyset$.

Examples

$s_1 \xrightarrow{E[err]/x := x/0} s_2$

$s_1 \xrightarrow{E[err]/x := x/0} s_2$

$\text{Error} = \{ \text{err} \}$

Notions of Steps: The Run-to-Completion Step

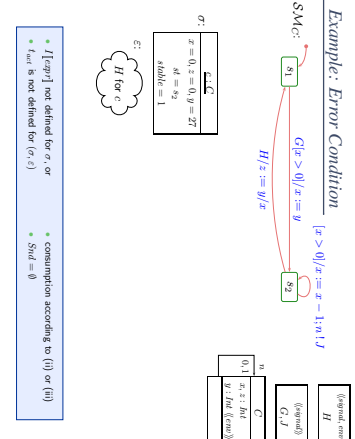
What is a **run-to-completion step**...?

- Intuition:** a maximal sequence of steps, where the first step is a **dispatch** step and all later steps are **commence** steps.
- Note:** one step corresponds to one transition in the state machine.

A run-to-completion step is in general not syntactically definable — one transition may be taken multiple times during an RTC-step.

Example:

$s_1 \xrightarrow{E[x > 0] / x := x - 1} s_2$



Notions of Steps: The Run-to-Completion Step Cont'd

Proposal: Let

$$(\sigma_0, \varepsilon_0) \xrightarrow{\text{Comm, Sml}} \dots \xrightarrow{\text{Comm, Sml}} (\sigma_n, \varepsilon_n), \quad n > 0,$$

be a finite (i), non-empty, maximal, consecutive sequence such that

- object u is alive in σ_0 ,
- $u_0 = u$ and $(\text{Comm}, \text{Sml}, u)$ indicates dispatching to u , i.e. $\text{Comm} = \{(c_1, \vec{r} \mapsto \vec{d})\}$,
- there are no receptors by u in between, i.e. $\text{Comm} \cap \{u\} \times \text{Env}(\varepsilon', \mathcal{D}) = \emptyset, \forall i > 1$.

$\sigma_{k+1} = u$ and u is stable only in σ_0 and σ_n , i.e.

$$\sigma_0(u) \text{ (stable)} = \sigma_1(u) \text{ (stable)} = 1 \text{ and } \sigma_i(u) \text{ (stable)} = 0 \text{ for } 0 < i < n,$$

Let $0 = k_1 < k_2 < \dots < k_N = n$ be the maximal sequence of indices such that $u_{k_i} = u$ for $1 \leq i \leq N$. Then we call the sequence

$$(\sigma_0(u) \Rightarrow) \sigma_{k_1}(u), \sigma_{k_2}(u), \dots, \sigma_{k_N}(u) \quad (= \sigma_{k_i-1}(u))$$

a (i) **run-to-completion computation** of u (from (local) configuration $\sigma_0(u)$)

We say, object u can diverge on reception *cons* from (local) configuration $\sigma_0(u)$ if and only if there is an infinite, consecutive sequence

$$(\sigma_0, \varepsilon_0) \xrightarrow{(\text{cons}, \text{Send}_0)} (\sigma_1, \varepsilon_1) \xrightarrow{(\text{cons}, \text{Send}_1)} \dots$$

such that u doesn't become stable again.

- Note: disappearance of object not considered in the definitions. By the current definitions, it's neither divergence nor an RTC-step.

What people may **dislike** on our definition of RTC-step is that it takes a **global** and **non-compositional** view. That is:

- in the projection onto a single object we still see the effect of interaction with other objects
 - Adding classes (or even objects) may change the divergence behaviour of existing ones.
 - Compositional would be: the behaviour of a set of objects is determined by the behaviour of each object "in isolation".
- Our semantics and notion of RTC-step doesn't have this (often desired) property.

Can we give (syntactical) criteria such that any global run-to-completion step is an interleaving of local ones?

Maybe: **Strict interfaces**. (Proof left as exercise...)

- (A): Refer to private features only via "self".
- (B): Let objects only communicate by events, i.e. don't let them modify each other's local state via links at all.

Putting It All Together

The Missing Piece: Initial States

Recall: a labelled transition system is (S, \rightarrow, S_0) . We have

- S : system configurations (σ, ε)
- \rightarrow : labelled transition relation $(\sigma, \varepsilon) \xrightarrow{(\text{cons}, \text{Send}_0)} (\sigma', \varepsilon')$.
- Wanted: initial states S_0 .

Proposal:

Require a (finite) set of **object diagrams** $OD \in \mathcal{O}^{\mathcal{D}_0, \varepsilon}$ empty}.

And set

$$S_0 = \{(\sigma, \varepsilon) \mid \sigma \in G^{-1}(OD), OD \in \mathcal{O}^{\mathcal{D}_0, \varepsilon} \text{ empty}\}$$

Other Approach: (used by Rhapsody tool) multiplicity of classes. We can read that as an abbreviation for an object diagram.

Semantics of UML Model — So Far

The semantics of the UML model

$$\mathcal{M} = (\mathcal{C}_0, \mathcal{K}, \mathcal{O}_0)$$

where

- some classes in \mathcal{C}_0 are stereotyped as "signal" (standard), some signals and attributes are stereotyped as "external" (non-standard).
 - there is a 1-to-1 relation between classes and state machines
 - \mathcal{O}_0 is a set of object diagrams over \mathcal{C}_0 .
- is the **transition system** (S, \rightarrow, S_0) constructed on the previous slide.

The computations of \mathcal{M} are the computations of (S, \rightarrow, S_0) .

OCCL Constrains and Behaviour

- Let $\mathcal{M} = (\mathcal{C}_0, \mathcal{K}, \mathcal{O}_0)$ be a UML model.
- We call \mathcal{M} **consistent** iff for each OCCL constraint $\text{expr} \in \text{Inv}(\mathcal{C}_0)$, $\sigma \models \text{expr}$ for each "reasonable point" (σ, ε) of computations of \mathcal{M} . (Cf. exercises and tutorial for discussion of "reasonable point".)

Note: we could define $\text{Inv}(\mathcal{K})$ similar to $\text{Inv}(\mathcal{C}_0)$.

Pragmatics:

- In **UML-as-blueprint** mode, if \mathcal{K} doesn't exist yet, then $\mathcal{M} = (\mathcal{C}_0, \emptyset, \mathcal{O}_0)$ is typically asking the developer to provide \mathcal{K}' such that $\mathcal{M}' = (\mathcal{C}_0, \mathcal{K}', \mathcal{O}_0)$ is consistent.
- If the developer makes a mistake, then \mathcal{M}' is inconsistent.
- Not common**: if \mathcal{K}' is given, then constraints are also considered when choosing transitions in the RTC-algorithm. In other words: even in presence of mistakes, the \mathcal{K}' never move to inconsistent configurations.

References

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