Software Design, Modelling and Analysis in UML

Lecture 21: Inheritance

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Contents & Goals

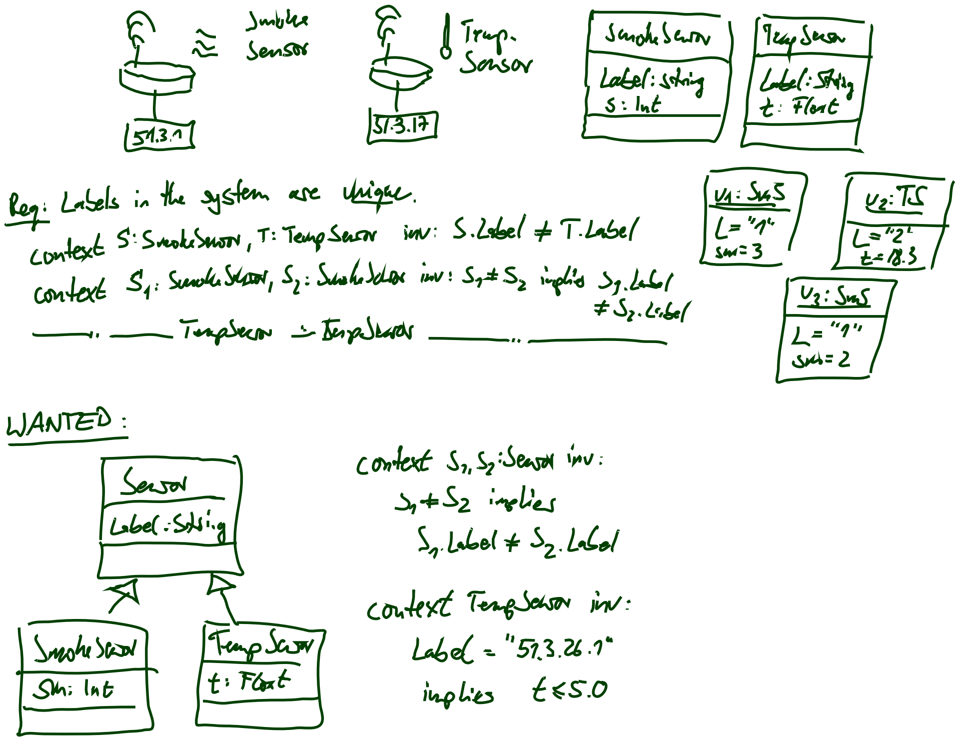
Last Lecture:

- Live Sequence Charts Semantics

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What's the Liskov Substitution Principle?
  - What is late/early binding?
  - What is the subset, what the uplink semantics of inheritance?
  - What's the effect of inheritance on LSCs, State Machines, System States?

- **Content:**
  - Inheritance in UML: concrete syntax
  - Liskov Substitution Principle — desired semantics
  - Two approaches to obtain desired semantics



Motivations for Generalisation

- **Re-use**,
- **Sharing**,
- **Avoiding Redundancy**,
- **Modularisation**,
- **Separation of Concerns**,
- **Abstraction**,
- **Extensibility**,
- ...

→ See textbooks on object-oriented analysis, development, programming.

Inheritance: Syntax

Abstract Syntax

method

behavioural features

Recall: a signature (with signals) is a tuple $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$.

method mapping

Now (finally): extend to

inheritance relation

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E}, F, mth, \lhd)$$

where F/mth are methods, analogously to attributes and

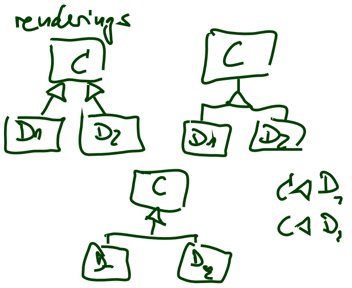
"don't mix signals and non-signals"

$$\lhd \ \subseteq ((\mathscr{C} \setminus \mathscr{E}) \times (\mathscr{C} \setminus \mathscr{E})) \cup (\mathscr{E} \times \mathscr{E})$$

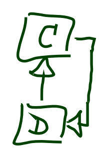
is a **generalisation** relation such that $C \lhd^+ C$ for **no** $C \in \mathscr{C}$ ("acyclic").

$C \lhd D$ reads as

- C is a generalisation of D,
- D is a specialisation of C,
- D inherits from C,
- D is a sub-class of C,
- C is a super-class of D,
- ...



NOT:

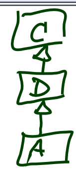


Reflexive, Transitive Closure of Generalisation

Definition. Given classes $C_0, C_1, D \in \mathscr{C}$, we say D inherits from C_0 **via** C_1 if and only if there are $C_0^1, \dots C_0^n, C_1^1, \dots C_1^m \in \mathscr{C}$ such that

$$C_0 \ \lhd C_0^1 \lhd \dots C_0^n \lhd \ C_1 \ \lhd C_1^1 \lhd \dots C_1^m \lhd \ D.$$

We use '$\preceq$' to denote the reflexive, transitive closure of '$\lhd$'.



$C \lhd D$
$D \lhd A$
$C \preceq A$

In the following, we assume

- that all attribute (method) names are of the form

$$C::v, \quad C \in \mathscr{C} \cup \mathscr{E} \qquad (C::f, \quad C \in \mathscr{C}),$$

- that we have $C::v \in atr(C)$ resp. $C::f \in mth(C)$ **if and only if** v (f) appears in an attribute (method) compartment of C in a class diagram.

Extend Typing Rules

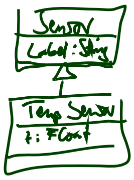
Well-Typedness with Inheritance

Recall: With extension for visibility we obtained

$$\begin{array}{lll} v(w) & : \tau_C \to \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathscr{C}} \rangle \in atr(C),\ w : \tau_C \\ v(expr_1(w)) & : \tau_{C_2} \to \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathscr{C}} \rangle \in atr(C_2), \\ & & expr_1(w) : \tau_{C_2},\ w : \tau_{C_1},\ \text{and } C_1 = C_2 \text{ or } \xi = + \end{array}$$

Now:

$$\begin{array}{lll} v(w) & : \tau_C \to \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathscr{C}} \rangle \in atr(C), \\ & & w : \tau_{C_1},\ \tau_C \preceq \tau_{C_1} \\ v(expr_1(w)) & : \tau_{C_2} \to \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathscr{C}} \rangle \in atr(C_2), \\ & & expr_1(w) : \tau_{C_2},\ w : \tau_{C_1}, \\ & & \text{and } (C_1 = C_2 \text{ or } \xi = + \text{ or } (C_2 \preceq C_1 \text{ and } \xi = \#)) \end{array}$$



context self : TempSensor :
self . Label = "54.3.26.1"
implies
self . t ≤ 5.0

Inheritance: System States

System States

Wanted: a formal representation of "if $C \preceq D$ then D instance '**is a**' C instance", i.e.,

(i) D has the same attributes as C, and

(ii) D objects (identities)
can be used in any context where C objects can be used.

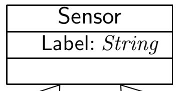
We'll discuss **two approaches** to semantics:

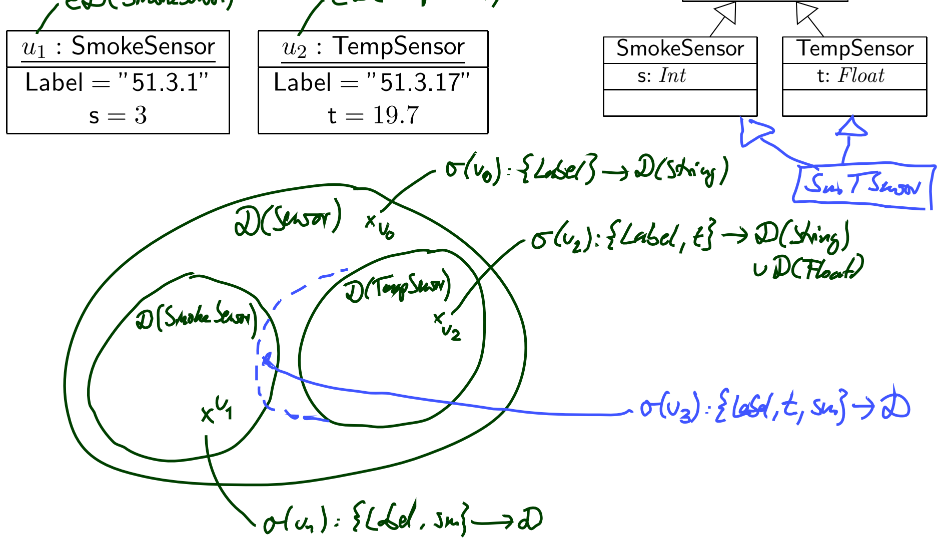
- **Domain-inclusion** Semantics (more **theoretical**)
- **Uplink** Semantics (more **technical**)

Domain Inclusion Semantics

Domain Inclusion Semantics: Idea

context s_1, s_2 : Sensor inv : ~~false~~ $s_1 \neq s_2$ implies s_1.Label $\neq s_2$.Label





Domain Inclusion Structure

Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E}, F, mth, \lhd)$ be a signature.

Now a **structure** $\mathscr{D}$

- [**as before**] maps types, classes, associations to domains,
- [**for completeness**] methods to transformers,
- [**as before**] indentities of instances of classes not (transitively) related by generalisation are disjoint,
- [**changed**] the indentities of a super-class comprise all identities of sub-classes, i.e.

$$\forall\, C \in \mathscr{C} : \mathscr{D}(C) \supsetneq \bigcup_{C \lhd D} \mathscr{D}(D).$$

Note: the old setting coincides with the special case $\lhd\ = \emptyset$.

Domain Inclusion System States

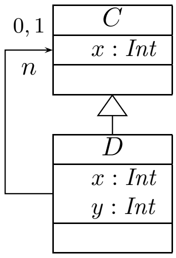
Now: a **system state** of $\mathscr{S}$ wrt. $\mathscr{D}$ is a **type-consistent** mapping

$$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_{0,1}) \cup \mathscr{D}(\mathscr{C}_*)))$$

that is, for all $u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(C)$,

- [**as before**] $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau$, $\tau \in \mathscr{T}$ or $\tau \in \{C_*, C_{0,1}\}$.
- [**changed**] $\mathrm{dom}(\sigma(u)) = \bigcup_{C_0 \preceq C} atr(C_0)$,

Example:



Note: the old setting still coincides with the special case $\lhd \; = \emptyset$.

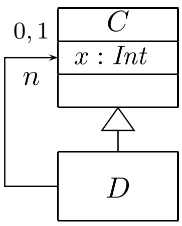
Satisfying OCL Constraints (Domain Inclusion)

- Let $\mathcal{M} = (\mathscr{CD}, \mathscr{OD}, \mathscr{SM}, \mathscr{I})$ be a UML model, and $\mathscr{D}$ a structure.
- We (**continue to**) say $\mathcal{M} \models expr$ for $\underbrace{\text{context } C \text{ inv} : expr_0}_{=expr} \in Inv(\mathcal{M})$ iff

$$\forall \pi = (\sigma_i, \varepsilon_i)_{i \in \mathbb{N}} \in [\![\mathcal{M}]\!] \quad \forall i \in \mathbb{N} \quad \forall u \in \mathrm{dom}(\sigma_i) \cap \mathscr{D}(C) :$$
$$I[\![expr_0]\!](\sigma_i, \{self \mapsto u\}) = 1.$$

- $\mathcal{M}$ is (still) consistent if and only if it satisfies all constraints in $Inv(\mathcal{M})$.
- **Example**: σ: $\boxed{\underline{v_1 : SmS}}$ $\boxed{\underline{v_2 : TS}}$ $\in \mathscr{D}(TempSensor) \subsetneq \mathscr{D}(Sensor)$

$\mathrm{dom}(\sigma) \cap \mathscr{D}(Sensor) = \{v_1, v_2\}$



Transformers (Domain Inclusion)

- Transformers also remain **the same**, e.g. [VL 12, p. 18]

$$update(expr_1, v, expr_2) : (\sigma, \varepsilon) \mapsto (\sigma', \varepsilon)$$

with

$$\sigma' = \sigma[u \mapsto \sigma(u)[v \mapsto I[\![expr_2]\!](\sigma)]]$$

where $u = I[\![expr_1]\!](\sigma)$.

Inheritance and State Machines: Triggers

- **Wanted**: triggers shall also be sensitive for inherited events, sub-class shall execute super-class' state-machine (unless overridden).

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon') \text{ if}$$

- $\exists\, u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(C)\ \exists\, u_E \in \mathscr{D}(\mathscr{E}) : u_E \in ready(\varepsilon, u)$
- u is stable and in state machine state s, i.e. $\sigma(u)(stable) = 1$ and $\sigma(u)(st) = s$,
- a transition is enabled, i.e.

$$\exists\, (s, F, expr, act, s') \in \to (\mathcal{SM}_C) : F = E \wedge I[\![expr]\!](\tilde{\sigma}) = 1$$

  where $\tilde{\sigma} = \sigma[u.params_E \mapsto u_e]$.

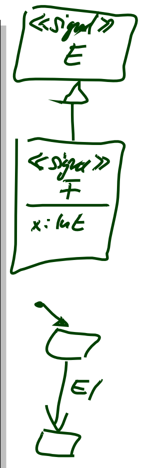
and

- (σ', ε') results from applying t_{act} to (σ, ε) and removing u_E from the ether, i.e.

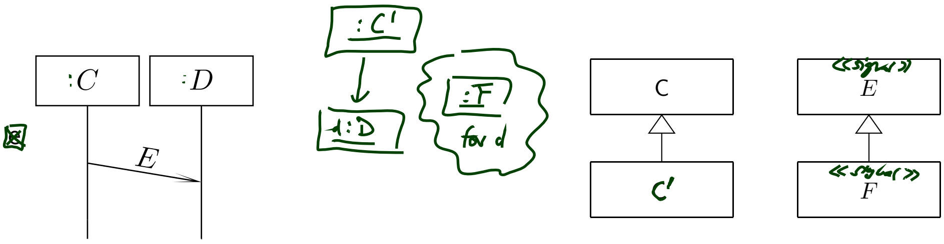
$$(\sigma'', \varepsilon') = t_{act}(\tilde{\sigma}, \varepsilon \ominus u_E),$$
$$\sigma' = (\sigma''[u.st \mapsto s', u.stable \mapsto b, u.params_E \mapsto \emptyset])|_{\mathscr{D}(\mathscr{C}) \setminus \{u_E\}}$$

  where b **depends**:

  - If u becomes stable in s', then $b = 1$. It **does** become stable if and only if there is no transition **without trigger** enabled for u in (σ', ε').
  - Otherwise $b = 0$.



Domain Inclusion and Interactions



- Similar to satisfaction of OCL expressions above:
  - An instance line stands for all instances of C (exact or inheriting).
  - Satisfaction of event observation has to take inheritance into account, too, so we have to **fix**, e.g.

    $$\sigma, cons, Snd \models_\beta E^!_{x,y}$$

    if and only if

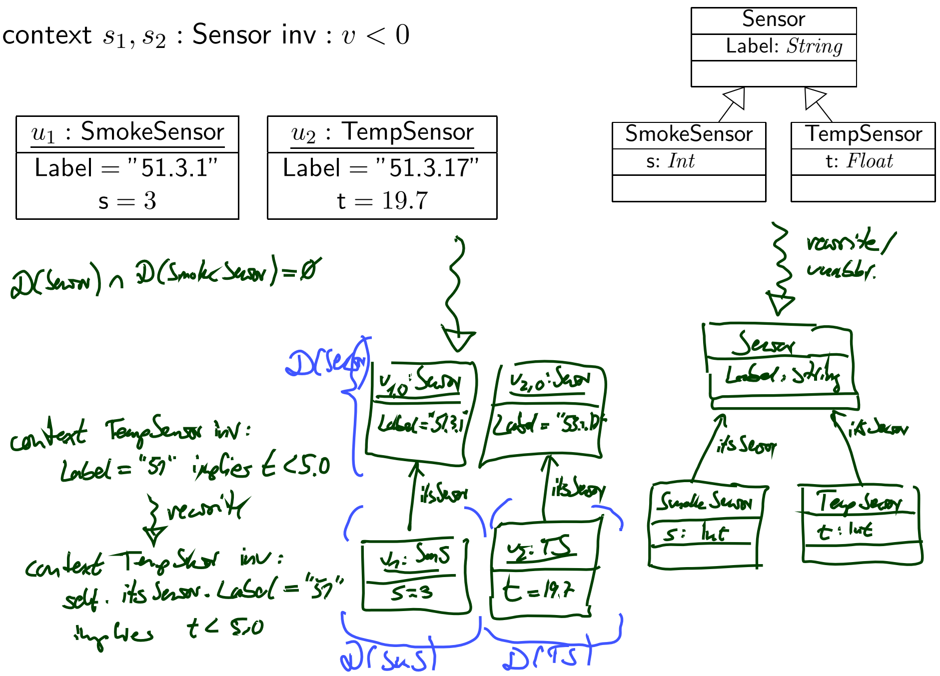
    $$\beta(x) \text{ sends an } F\text{-event to } \beta(y) \text{ where } E \preceq F.$$

  - C-instance line also binds to C'-objects.

Uplink Semantics

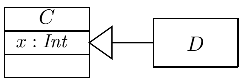
Uplink Semantics: Idea

context s_1, s_2 : Sensor inv : $v < 0$



Uplink Semantics

- **Idea**:
  - Continue with the existing definition of **structure**, i.e. disjoint domains for identities.
  - Have an **implicit association** from the child to each parent part (similar to the implicit attribute for stability).



  - Apply (a different) pre-processing to make appropriate use of that association, e.g. rewrite (C++)

$$x = 0;$$

in D to

$$\texttt{uplink}_C \texttt{ -> } x = 0;$$

Pre-Processing for the Uplink Semantics

- For each pair $C \lhd D$, extend D by a (fresh) association

$$uplink_C : C \text{ with } \mu = [1,1],\ \xi = +$$

  (**Exercise**: public necessary?)
- Given expression v (or f) in the **context** of class D,
  - let C be the **smallest** class wrt. "$\preceq$" such that
    - $C \preceq D$, and
    - $C{::}v \in atr(D)$
  - then there exists (by definition) $C \lhd C_1 \lhd \ldots \lhd C_n \lhd D$,
  - **normalise** v to (= replace by)

$$uplink_{C_n} \text{ -> } \cdots \text{ -> } uplink_{C_1}.C{::}v$$

- If no (unique) smallest class exists,
  the model is considered **not well-formed**; the expression is ambiguous.

Uplink Structure, System State, Typing

- Definition of structure remains **unchanged**.
- Definition of system state remains **unchanged**.
- Typing and transformers remain **unchanged** —
  the preprocessing has put everything in shape.

Satisfying OCL Constraints (Uplink)

- Let $\mathcal{M} = (\mathscr{CD}, \mathscr{OD}, \mathscr{SM}, \mathscr{I})$ be a UML model, and $\mathscr{D}$ a structure.
- We (**continue to**) say

$$\mathcal{M} \models expr$$

for

$$\underbrace{\text{context } C \text{ inv} : expr_0}_{=expr} \in \mathit{Inv}(\mathcal{M})$$

if and only if

$$\begin{aligned} &\forall\, \pi = (\sigma_i)_{i \in \mathbb{N}} \in [\![\mathcal{M}]\!] \\ &\quad \forall\, i \in \mathbb{N} \\ &\qquad \forall\, u \in \mathrm{dom}(\sigma_i) \cap \mathscr{D}(C) : \\ &\qquad\quad I[\![expr_0]\!](\sigma_i, \{self \mapsto u\}) = 1. \end{aligned}$$

- $\mathcal{M}$ is (still) consistent if and only if it satisfies all constraints in $\mathit{Inv}(\mathcal{M})$.

Transformers (Uplink)

- What **has to change** is the **create** transformer:

$$create(C, expr, v)$$

- Assume, C's inheritance relations are as follows.

$$\begin{gathered} C_{1,1} \lhd \ldots \lhd C_{1,n_1} \lhd C, \\ \ldots \\ C_{m,1} \lhd \ldots \lhd C_{m,n_m} \lhd C. \end{gathered}$$

- Then, we have to
  - create one fresh object for each part, e.g.

$$u_{1,1}, \ldots, u_{1,n_1}, \ldots, u_{m,1}, \ldots, u_{m,n_m},$$

  - set up the uplinks recursively, e.g.

$$\sigma(u_{1,2})(uplink_{C_{1,1}}) = u_{1,1}.$$

  - And, if we had constructors, be careful with their order.

Domain Inclusion vs. Uplink Semantics

Cast-Transformers

- `C c;`
- `D d;`

- **Identity upcast** (C++):
  - `C* cp = &d;` // *assign address of 'd' to pointer 'cp'*

- **Identity downcast** (C++):
  - `D* dp = (D*)cp;` // *assign address of 'd' to pointer 'dp'*

- **Value upcast** (C++):
  - `*c = *d;` // *copy attribute values of 'd' into 'c', or,*
    // *more precise, the values of the C-part of 'd'*

Casts in Domain Inclusion and Uplink Semantics

	Domain Inclusion	**Uplink**
$\mathtt{C}*\ \mathtt{cp} = \&\mathtt{d};$	**easy**: immediately compatible (in underlying system state) because $\&\mathtt{d}$ yields an identity from $\mathscr{D}(D) \subset \mathscr{D}(C)$.	**easy**: By pre-processing, $\mathtt{C}*\ \mathtt{cp} = \mathtt{d.uplink}_C;$
$\mathtt{D}*\ \mathtt{dp} = (\mathtt{D}*)\mathtt{cp};$	**easy**: the value of cp is in $\mathscr{D}(D) \cap \mathscr{D}(C)$ because the pointed-to object is a D. Otherwise, error condition.	**difficult**: we need the identity of the D whose C-slice is denoted by cp. (See next slide.)
$\mathtt{c} = \mathtt{d};$	**bit difficult**: set (for all $C \preceq D$) $(C)(\,\cdot\,,\cdot\,) : \tau_D \times \Sigma \to \Sigma\vert_{atr(C)}$ $(u, \sigma) \mapsto \sigma(u)\vert_{atr(C)}$ Note: $\sigma' = \sigma[u_C \mapsto \sigma(u_D)]$ is not type-compatible!	**easy**: By pre-processing, $\mathtt{c} = *(\mathtt{d.uplink}_C);$

Identity Downcast with Uplink Semantics

- **Recall** (C++): $\mathtt{D\ d;}\quad \mathtt{C}*\ \mathtt{cp} = \&\mathtt{d};\quad \mathtt{D}*\ \mathtt{dp} = (\mathtt{D}*)\mathtt{cp};$
- **Problem**: we need the identity of the D whose C-slice is denoted by cp.
- **One technical solution**:
  - Give up disjointness of domains for **one additional type** comprising all identities, i.e. have

$$\mathtt{all} \in \mathscr{T}, \qquad \mathscr{D}(\mathtt{all}) = \bigcup_{C \in \mathscr{C}} \mathscr{D}(C)$$

  - In each $\preceq$-**minimal class** have associations "mostspec" pointing to **most specialised** slices, plus information of which type that slice is.
  - Then **downcast** means, depending on the mostspec type (only finitely many possibilities), **going down and then up** as necessary, e.g.

$$
\begin{array}{l}
\mathtt{switch(mostspec\_type)\{} \\
\quad \mathtt{case}\ C : \\
\qquad \mathtt{dp} = \mathtt{cp} \mathbin{\texttt{->}} \mathtt{mostspec} \mathbin{\texttt{->}} \mathtt{uplink}_{D_n} \mathbin{\texttt{->}} \ldots \mathbin{\texttt{->}} \mathtt{uplink}_{D_1} \mathbin{\texttt{->}} \mathtt{uplink}_D; \\
\ldots
\end{array}
$$

Domain Inclusion vs. Uplink: Differences

- **Note**: The uplink semantics views inheritance as an abbreviation:
  - We only need to touch transformers (create) — and if we had constructors, we didn't even needed that (we could encode the recursive construction of the upper slices by a transformation of the existing constructors.)
- **So**:
  - Inheritance **doesn't add** expressive power.
  - And it also **doesn't improve** conciseness **soo dramatically**.

As long as we're "**early binding**", that is...

Domain Inclusion vs. Uplink: Motivations

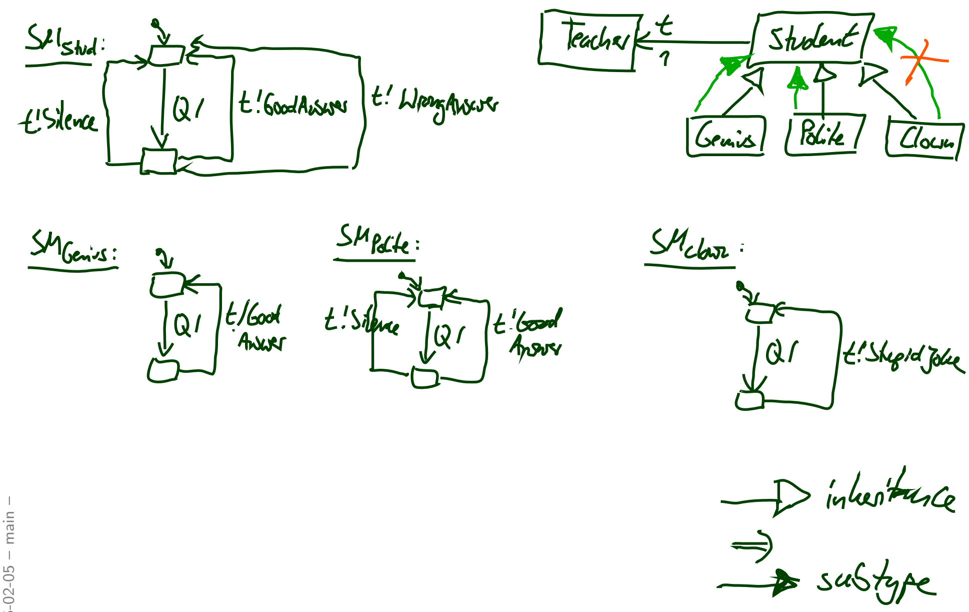
- **Exercise**:

What's the point of

- having the **tedious** adjustments of the **theory**
  if it can be approached **technically**?
- having the **tedious** technical **pre-processing**
  if it can be approached **cleanly** in the **theory**?

More Interesting: Behaviour

Example: Behaviour of Kinds of Students



Desired Semantics of Specialisation: Subtyping

There is a classical description of what one **expects** from **sub-types**, which in the OO domain is closely related to inheritance:

The principle of type substitutability [Liskov, 1988, Liskov and Wing, 1994].
(**Liskov Substitution Principle** (LSP).)

> "If for each object o_1 of type S there is an object o_2 of type T such that for all programs P defined in terms of T,
> **the behavior of P is unchanged** when o_1 is substituted for o_2
> then S is a **subtype** of T."

In other words: [Fischer and Wehrheim, 2000]

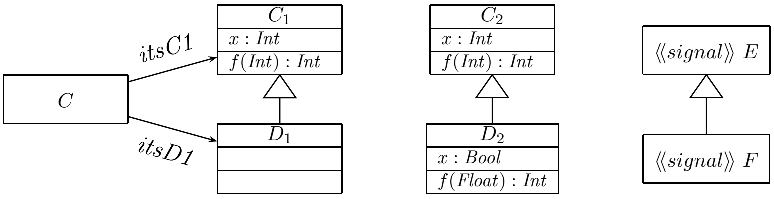
> "An instance of the **sub-type** shall be **usable** whenever an instance of the supertype was expected,
> **without a client being able to tell the difference**."

So, what's "**usable**"? Who's a "**client**"? And what's a "**difference**"?

"...shall be usable..." for UML

Easy: Static Typing for Attributes

Given:



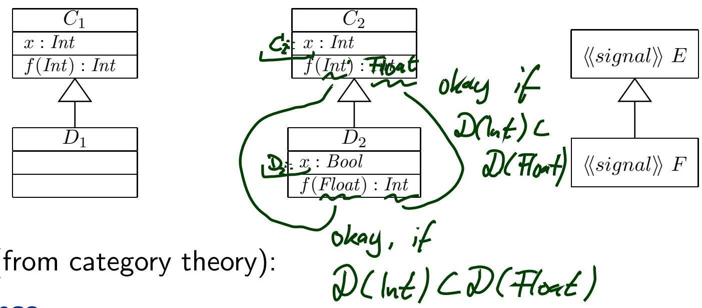
Wanted:

- $x > 0$ also **well-typed** for D_1
- assignment $itsC1 := itsD1$ being **well-typed**
- $itsC1.x = 0$, $itsC1.f(0)$, $itsC1 \,!\, F$
  being well-typed (and doing the right thing).

Approach:

- Simply define it as being well-typed,
  adjust system state definition to do the right thing.

Static Typing for Methods



Notions (from category theory):

- **invariance**,
- **covariance**,
- **contravariance**.

We could call, e.g. a method, **sub-type preserving**, if and only if it

- accepts **more general** types as input (**contravariant**),
- provides a **more specialised** type as output (**covariant**).

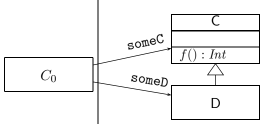
This is a notion used by many programming languages — and easily type-checked.

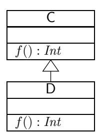
Excursus: Late Binding of Behavioural Features

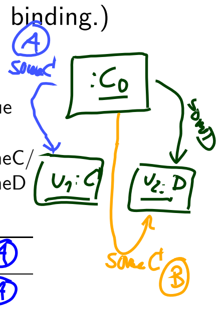
Late Binding

What transformer applies in what situation? (Early (compile time) binding.)

the type of the link determines which impl. is used at runtime







	f not overridden in D	f overridden in D	value of someC/ someD
someC -> f()	C::f()	C::f()	A
someD -> f()	C::f()	D::f()	A
someC -> f()	C::f()	$\boxed{\text{C::f()}}$	B

What one could want is something different: (Late binding.)

type of object (at runtime) determines which impl. is used

someC -> f()	〃	〃	A
someD -> f()	〃	〃	A
someC -> f()	〃	$\boxed{\text{D::f()}}$	B

Late Binding in the Standard and in Prog. Lang.

- In **the standard**, Section 11.3.10, "CallOperationAction":

  > "**Semantic Variation Points**
  > The mechanism for determining the method to be invoked as a result of a call operation is unspecified." [OMG, 2007b, 247]

- In **C++**,
  - methods are by default "(early) compile time binding",
  - can be declared to be "late binding" by keyword "`virtual`",
  - the declaration applies to all inheriting classes.
- In **Java**,
  - methods are "late binding";
  - there are patterns to imitate the effect of "early binding"

Exercise: What could be the rationale of the designers of C++?

Note: late binding typically applies only to **methods**, **not** to **attributes**.
(But: getter/setter methods have been invented recently.)

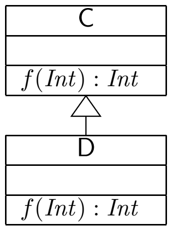
Back to the Main Track: "...tell the difference..." for UML

With Only Early Binding...

- ...we're **done** (if we realise it correctly in the framework).
- Then
  - if we're calling method f of an object u,
  - which is an instance of D with $C \preceq D$
  - via a C-link, use the transformer provided by C
  - then we (by definition) only see and change the C-part.
  - We cannot tell whether u is a C or an D instance.

  So we immediately also have behavioural/dynamic subtyping.

Difficult: Dynamic Subtyping



- $C::f$ and $D::f$ are **type compatible**, but D is **not necessarily** a **sub-type** of C.
- **Examples**: (C++)

```
int C::f(int) {
   return 0;
};
```

vs.

```
int D::f(int) {
   return 1;
};
```

```
int C::f(int) {
   return (rand() %
2);
};
```

vs.

```
int D::f(int x) {
   return (x % 2);
};
```

Sub-Typing Principles Cont'd

- In the standard, Section 7.3.36, "**Operation**":

  > "**Semantic Variation Points**
  >
  > [...] When operations are redefined in a specialization, rules regarding **invariance**, **covariance**, or **contravariance** of types and preconditions determine whether the specialized classifier is substitutable for its more general parent. Such rules constitute semantic variation points with respect to redefinition of operations." [OMG, 2007a, 106]

- So, better: call a method **sub-type preserving**, if and only if it

  (i) accepts **more input values** (**contravariant**),

  (ii) on the **old values**, has **fewer behaviour** (**covariant**).

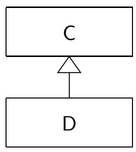
  Note: ~~This~~ (ii) is no longer a matter of simple type-checking!

- And not necessarily the end of the story:
  - One could, e.g. want to consider execution time.
  - Or, like [Fischer and Wehrheim, 2000], relax to "fewer observable behaviour", thus admitting the sub-type to do more work on inputs.

  Note: "testing" differences depends on the **granularity** of the semantics.

- **Related**: "has a weaker pre-condition," (**contravariant**),
  "has a stronger post-condition." (**covariant**).

Ensuring Sub-Typing for State Machines



- In the CASE tool we consider, multiple classes in an inheritance hierarchy can have state machines.
- But the state machine of a sub-class **cannot** be drawn from scratch.
- Instead, the state machine of a sub-class can only be obtained by applying actions from a **restricted** set to a copy of the original one.

  Roughly (cf. User Guide, p. 760, for details),
  - add things into (hierarchical) states,
  - add more states,
  - attach a transition to a different target (limited).

- They **ensure**, that the sub-class is a **behavioural sub-type** of the super class. (But method implementations can still destroy that property.)

- Technically, the idea is that (by late binding) only the state machine of the most specialised classes are running.

  By knowledge of the framework, the (code for) state machines of super-classes is still accessible — but using it is hardly a good idea...

References

[Fischer and Wehrheim, 2000] Fischer, C. and Wehrheim, H. (2000). Behavioural subtyping relations for object-oriented formalisms. In Rus, T., editor, AMAST, number 1816 in Lecture Notes in Computer Science. Springer-Verlag.

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[Liskov and Wing, 1994] Liskov, B. H. and Wing, J. M. (1994). A behavioral notion of subtyping. ACM Transactions on Programming Languages and Systems (TOPLAS), 16(6):1811–1841.

[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

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