

Software Design, Modelling and Analysis in UML

Lecture 21: Inheritance

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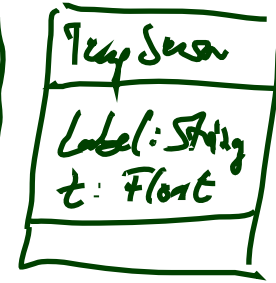
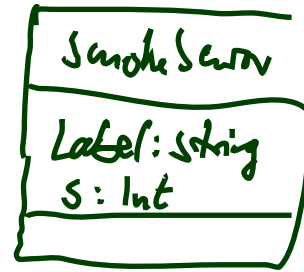
Contents & Goals

Last Lecture:

- Live Sequence Charts Semantics

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
 - What's the Liskov Substitution Principle?
 - What is late/early binding?
 - What is the subset, what the uplink semantics of inheritance?
 - What's the effect of inheritance on LSCs, State Machines, System States?
- **Content:**
 - Inheritance in UML: concrete syntax
 - Liskov Substitution Principle — desired semantics
 - Two approaches to obtain desired semantics

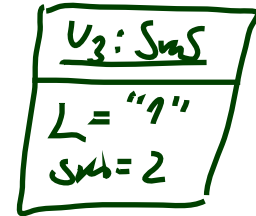
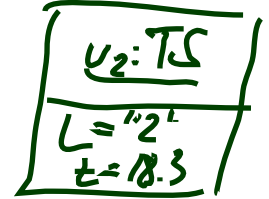
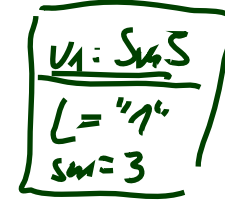


Req: Labels in the system are unique.

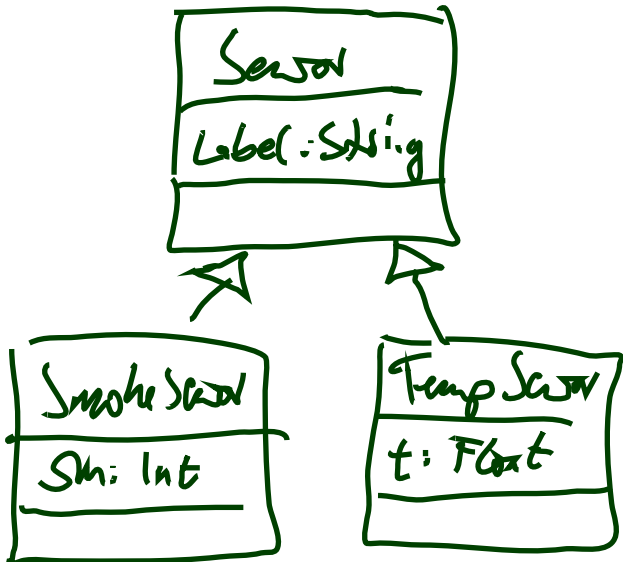
context S: SmokeSensor, T: TempSensor inv: S.Label ≠ T.Label

context S₁: SmokeSensor, S₂: SmokeSensor inv: S₁ ≠ S₂ implies S₁.Label ≠ S₂.Label

TempSensor ≡ TempSensor



WANTED:



context S₁, S₂: Sensor inv:

S₁ ≠ S₂ implies

S₁.Label ≠ S₂.Label

context TempSensor inv:

Label = "51.3.26.1"

implies t ≤ 5.0

Motivations for Generalisation

- **Re-use,**
- **Sharing,**
- **Avoiding Redundancy,**
- **Modularisation,**
- **Separation of Concerns,**
- **Abstraction,**
- **Extensibility,**
- ...

→ See textbooks on object-oriented analysis, development, programming.

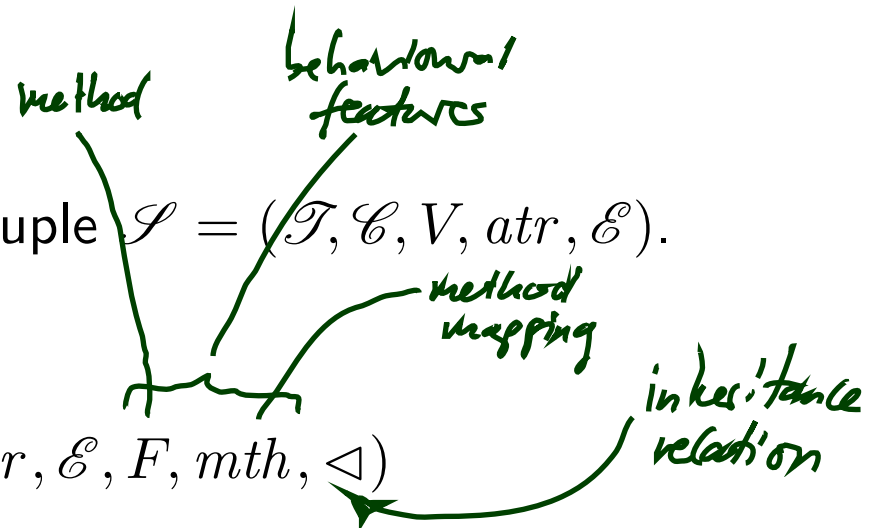
Inheritance: Syntax

Abstract Syntax

Recall: a signature (with signals) is a tuple $\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr, \mathcal{E})$.

Now (finally): extend to

$$\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr, \mathcal{E}, F, mth, \triangleleft)$$



where F/mth are methods, analogously to attributes and

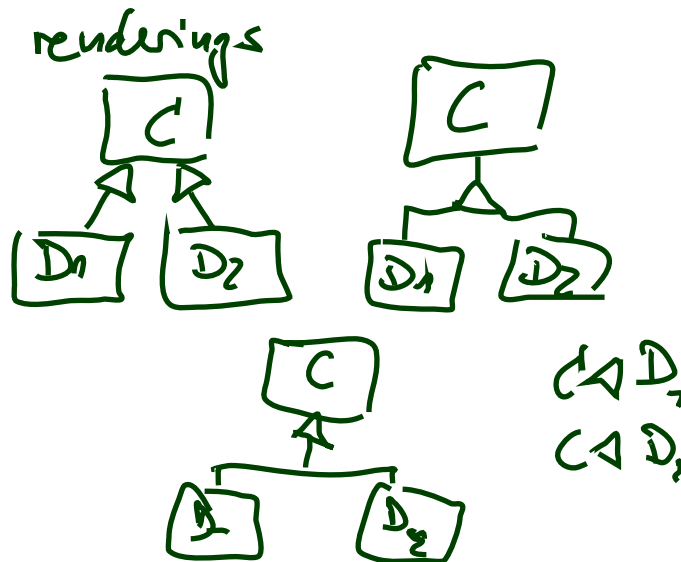
$$\triangleleft \subseteq ((\mathcal{C} \setminus \mathcal{E}) \times (\mathcal{C} \setminus \mathcal{E})) \cup (\mathcal{E} \times \mathcal{E})$$

"don't mix signals and non-signals"

is a **generalisation** relation such that $C \triangleleft^+ C$ for **no** $C \in \mathcal{C}$ ("acyclic").

$C \triangleleft D$ reads as

- C is a generalisation of D ,
- D is a specialisation of C ,
- D inherits from C ,
- D is a sub-class of C ,
- C is a super-class of D ,
- ...



NOT:



Reflexive, Transitive Closure of Generalisation

Definition. Given classes $C_0, C_1, D \in \mathcal{C}$, we say D inherits from C_0 **via** C_1 if and only if there are $C_0^1, \dots, C_0^n, C_1^1, \dots, C_1^m \in \mathcal{C}$ such that

$$C_0 \triangleleft C_0^1 \triangleleft \dots \triangleleft C_0^n \triangleleft C_1 \triangleleft C_1^1 \triangleleft \dots \triangleleft C_1^m \triangleleft D.$$

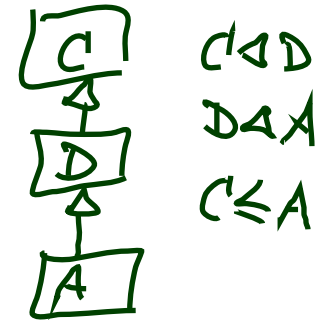
We use ' \preceq ' to denote the reflexive, transitive closure of ' \triangleleft '.

In the following, we assume

- that all attribute (method) names are of the form

$$C::v, \quad C \in \mathcal{C} \cup \mathcal{E} \quad (C::f, \quad C \in \mathcal{C}),$$

- that we have $C::v \in \text{atr}(C)$ resp. $C::f \in \text{mth}(C)$ **if and only if** v (f) appears in an attribute (method) compartment of C in a class diagram.



Extend Typing Rules

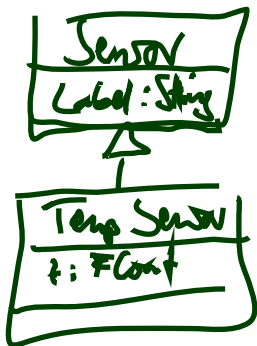
Well-Typedness with Inheritance

Recall: With extension for visibility we obtained

$$\begin{array}{ll} v(w) & : \tau_C \rightarrow \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(C), w : \tau_C \\ v(expr_1(w)) & : \tau_{C_2} \rightarrow \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(C_2), \\ & & expr_1(w) : \tau_{C_2}, w : \tau_{C_1}, \text{ and } C_1 = C_2 \text{ or } \xi = + \end{array}$$

Now:

$$\begin{array}{ll} v(w) & : \tau_C \rightarrow \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(C), \\ & & w : \tau_{C_1}, \tau_C \preceq \tau_{C_1} \\ v(expr_1(w)) & : \tau_{C_2} \rightarrow \tau(v) & \langle v : \tau, \xi, expr_0, P_{\mathcal{E}} \rangle \in atr(C_2), \\ & & expr_1(w) : \tau_{C_2}, w : \tau_{C_1}, \\ & & \text{and } (C_1 = C_2 \text{ or } \xi = + \text{ or } (C_2 \preceq C_1 \text{ and } \xi = \#)) \end{array}$$



context self: TempSensor:
self.Label = "59.3.26.1"
implies
self.t < 5.0

Inheritance: System States

System States

Wanted: a formal representation of “if $C \preceq D$ then D **is a** C ”, i.e.,

instance *instance*

- (i) D has the same attributes as C , and
- (ii) D objects (identities)
can be used in any context where C objects can be used.

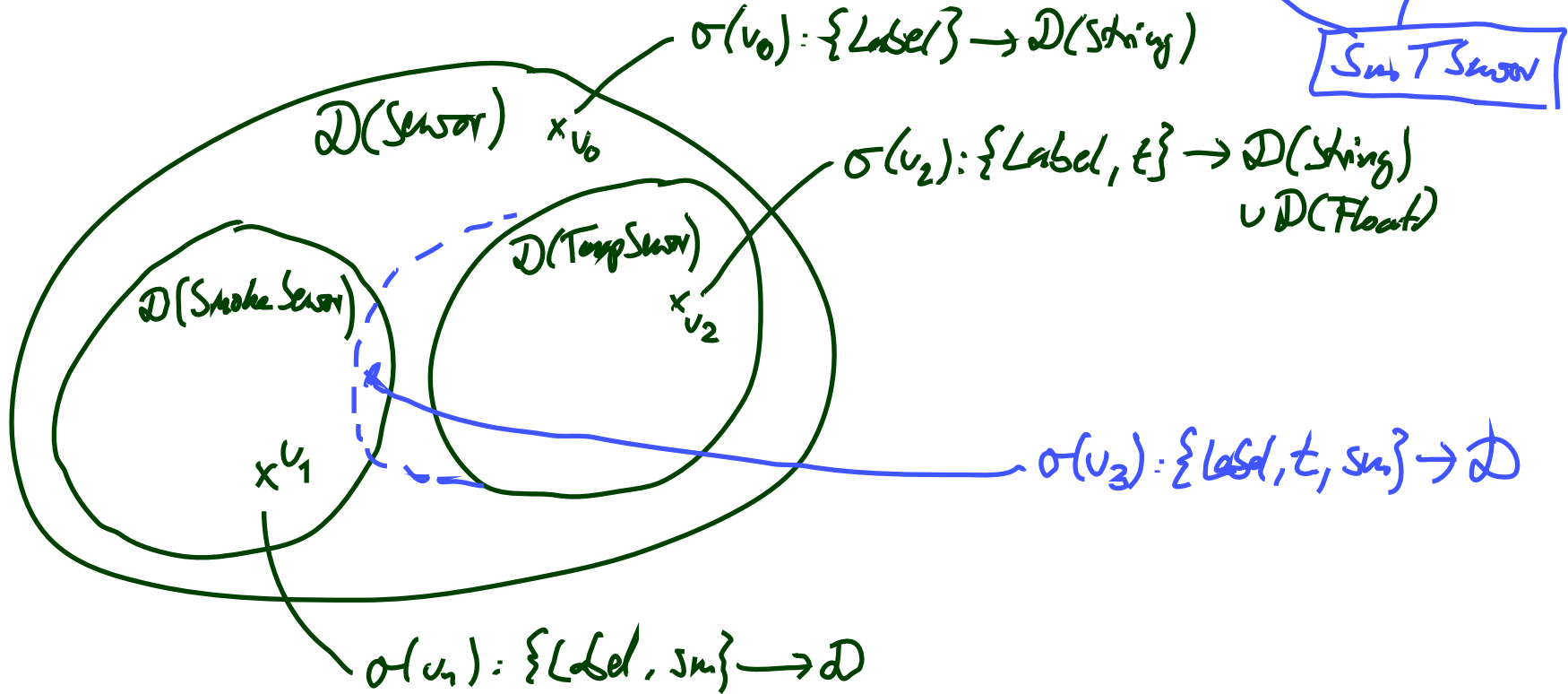
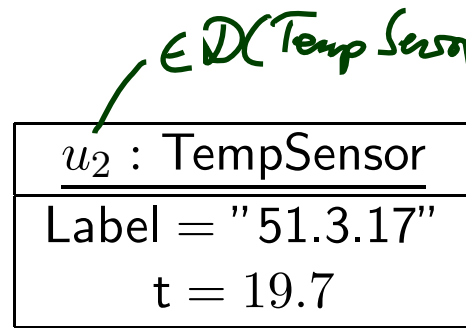
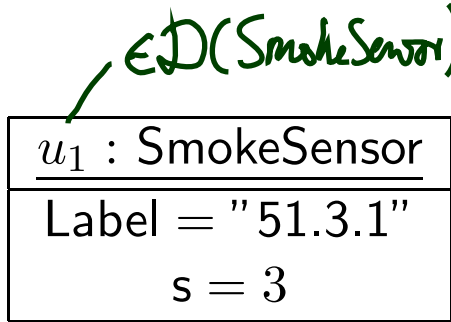
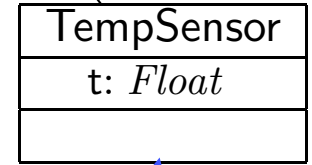
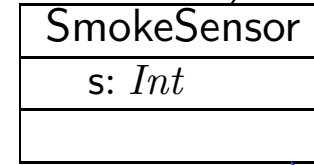
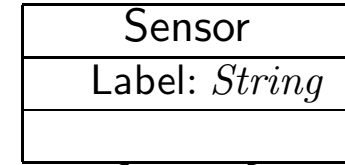
We'll discuss **two approaches** to semantics:

- **Domain-inclusion** Semantics (more **theoretical**)
- **Uplink** Semantics (more **technical**)

Domain Inclusion Semantics

Domain Inclusion Semantics: Idea

context $s_1, s_2 : \text{Sensor}$ inv : ~~$s_1 \neq s_2$~~ $s_1 \neq s_2$ implies $s_1.\text{Label} \neq s_2.\text{Label}$



Domain Inclusion Structure

Let $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr, \mathcal{E}, F, mth, \triangleleft)$ be a signature.

Now a **structure** \mathcal{D}

- [as before] maps types, classes, associations to domains,
- [for completeness] methods to transformers,
- [as before] indentities of instances of classes not (transitively) related by generalisation are disjoint,
- [changed] the indentities of a super-class comprise all indentities of sub-classes, i.e.

$$\forall C \in \mathcal{C} : \mathcal{D}(C) \supseteq \bigcup_{C \triangleleft D} \mathcal{D}(D).$$

Note: the old setting coincides with the special case $\triangleleft = \emptyset$.

Domain Inclusion System States

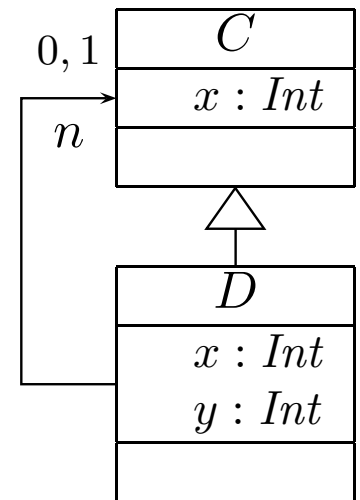
Now: a **system state** of \mathcal{S} wrt. \mathcal{D} is a **type-consistent** mapping

$$\sigma : \mathcal{D}(\mathcal{C}) \mapsto (V \mapsto (\mathcal{D}(\mathcal{T}) \cup \mathcal{D}(\mathcal{C}_{0,1}) \cup \mathcal{D}(\mathcal{C}_*)))$$

that is, for all $u \in \text{dom}(\sigma) \cap \mathcal{D}(C)$,

- **[as before]** $\sigma(u)(v) \in \mathcal{D}(\tau)$ if $v : \tau$, $\tau \in \mathcal{T}$ or $\tau \in \{C_*, C_{0,1}\}$.
- **[changed]** $\text{dom}(\sigma(u)) = \bigcup_{C_0 \preceq C} \text{atr}(C_0)$,

Example:



Note: the old setting still coincides with the special case $\triangleleft = \emptyset$.

Satisfying OCL Constraints (Domain Inclusion)

- Let $\mathcal{M} = (\mathcal{CD}, \mathcal{OD}, \mathcal{SM}, \mathcal{I})$ be a UML model, and \mathcal{D} a structure.
- We (**continue to**) say $\mathcal{M} \models \text{expr}$ for context C $\text{inv} : \text{expr}_0 \in \text{Inv}(\mathcal{M})$ iff

$$\underbrace{\text{inv} : \text{expr}_0}_{=\text{expr}}$$

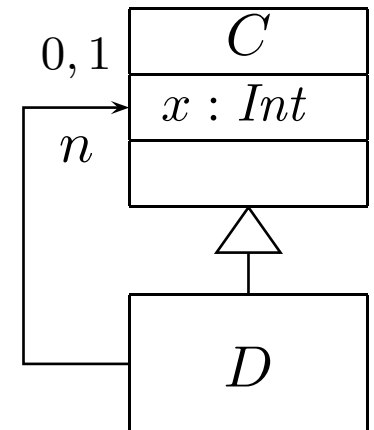
$$\forall \pi = (\sigma_i, \varepsilon_i)_{i \in \mathbb{N}} \in \llbracket \mathcal{M} \rrbracket \quad \forall i \in \mathbb{N} \quad \forall u \in \text{dom}(\sigma_i) \cap \mathcal{D}(C) :$$

$$I[\text{expr}_0](\sigma_i, \{self \mapsto u\}) = 1.$$

- \mathcal{M} is (still) consistent if and only if it satisfies all constraints in $\text{Inv}(\mathcal{M})$.

- Example:** $\sigma: \boxed{v_1: \text{SMS}} \quad \boxed{v_2: \text{TS}}$
 $\in \mathcal{D}(\text{TempSensor}) \not\subseteq \mathcal{D}(\text{Sensor})$

$$\text{dom}(\sigma) \cap \mathcal{D}(\text{Sensor}) = \{v_1, v_2\}$$



Transformers (Domain Inclusion)

- Transformers also remain **the same**, e.g. [VL 12, p. 18]

$$\text{update}(\text{expr}_1, v, \text{expr}_2) : (\sigma, \varepsilon) \mapsto (\sigma', \varepsilon)$$

with

$$\sigma' = \sigma[u \mapsto \sigma(u)[v \mapsto I[\text{expr}_2](\sigma)]]$$

where $u = I[\text{expr}_1](\sigma)$.

Inheritance and State Machines: Triggers

- **Wanted:** triggers shall also be sensitive for inherited events, sub-class shall execute super-class' state-machine (unless overridden).

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon') \text{ if}$$

- $\exists u \in \text{dom}(\sigma) \cap \mathcal{D}(C) \exists u_E \in \mathcal{D}(\mathcal{E}) : u_E \in \text{ready}(\varepsilon, u)$
- u is stable and in state machine state s , i.e. $\sigma(u)(\text{stable}) = 1$ and $\sigma(u)(st) = s$,
- a transition is enabled, i.e.

$$\exists (s, F, \text{expr}, \text{act}, s') \in \rightarrow (\mathcal{SM}_C) : F = E \wedge I[\![\text{expr}]\!](\tilde{\sigma}) = 1$$

where $\tilde{\sigma} = \sigma[u.\text{params}_E \mapsto u_e]$.

and

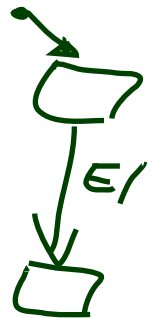
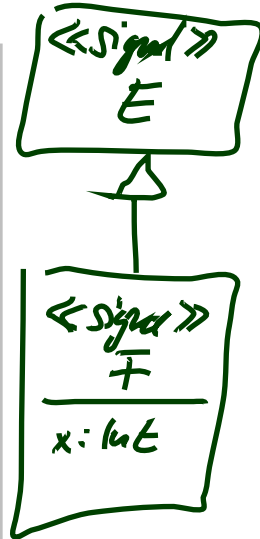
- (σ', ε') results from applying t_{act} to (σ, ε) and removing u_E from the ether, i.e.

$$(\sigma'', \varepsilon') = t_{act}(\tilde{\sigma}, \varepsilon \ominus u_E),$$

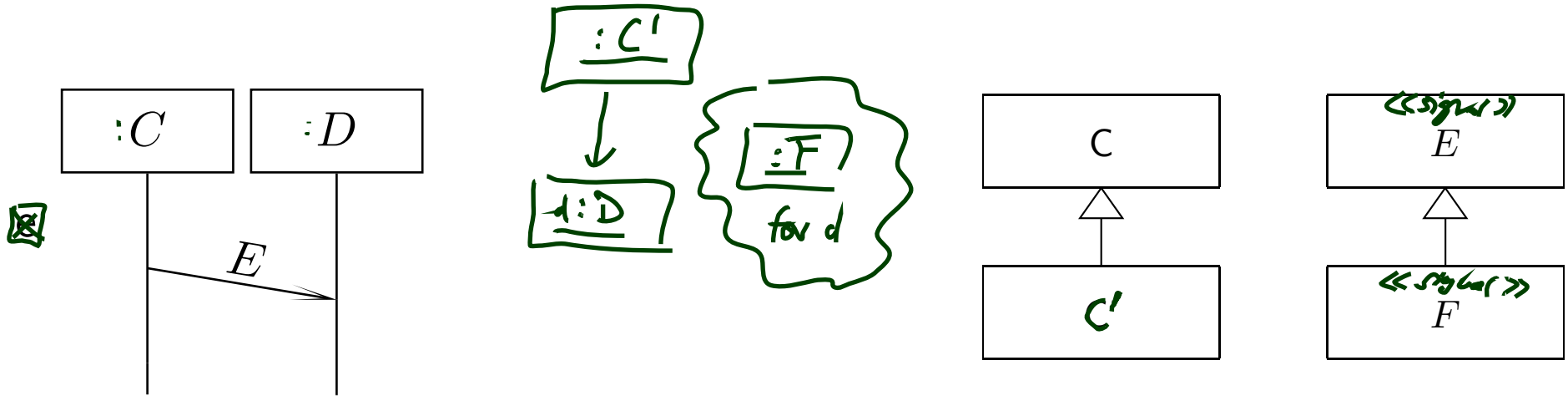
$$\sigma' = (\sigma''[u.st \mapsto s', u.stable \mapsto b, u.\text{params}_E \mapsto \emptyset])|_{\mathcal{D}(\mathcal{E}) \setminus \{u_E\}}$$

where b **depends:**

- If u becomes stable in s' , then $b = 1$. It **does** become stable if and only if there is no transition **without trigger** enabled for u in (σ', ε') .
- Otherwise $b = 0$.



Domain Inclusion and Interactions



- Similar to satisfaction of OCL expressions above:
 - An instance line stands for all instances of C (exact or inheriting).
 - Satisfaction of event observation has to take inheritance into account, too, so we have to **fix**, e.g.

$$\sigma, cons, Snd \models_{\beta} E_{x,y}^!$$

if and only if

$\beta(x)$ sends an F -event to $\beta(y)$ where $E \preceq F$.

- C -instance line also binds to C' -objects.

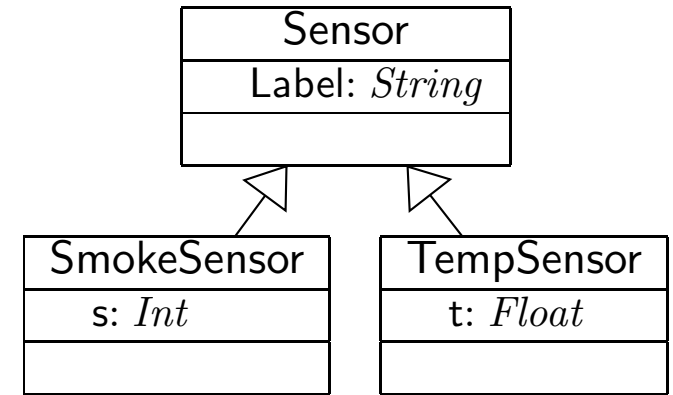
Uplink Semantics

Uplink Semantics: Idea

context $s_1, s_2 : \text{Sensor inv} : v < 0$

$u_1 : \text{SmokeSensor}$
Label = "51.3.1"
s = 3

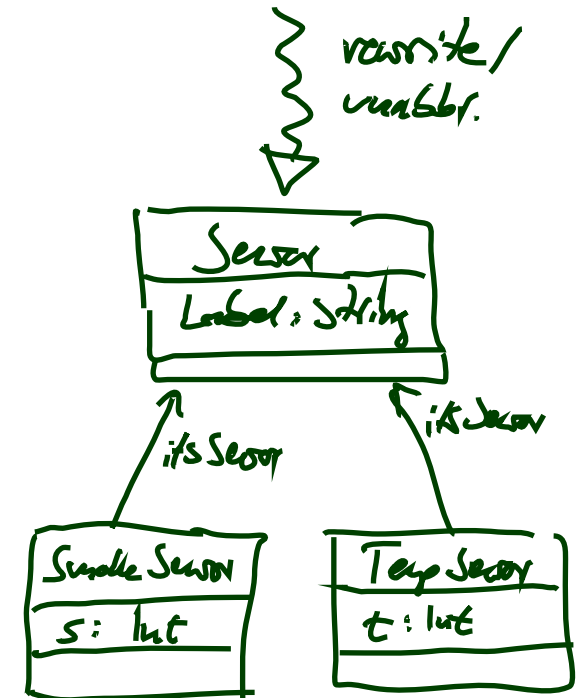
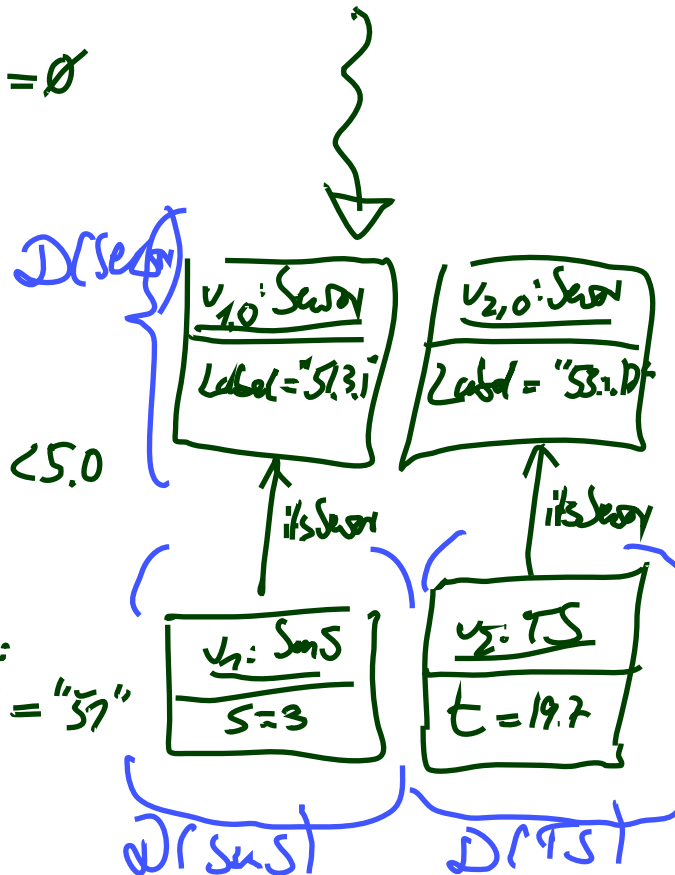
$u_2 : \text{TempSensor}$
Label = "51.3.17"
t = 19.7



$D(\text{Sensor}) \cap D(\text{SmokeSensor}) = \emptyset$

context TempSensor inv:
Label = "51" implies $t < 5.0$

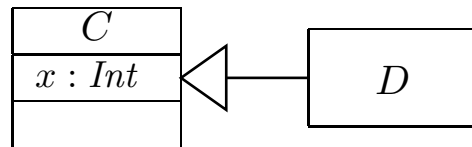
rewrite
context TempSensor inv:
self. itsSensor.Label = "51"
implies $t < 5.0$



Uplink Semantics

- **Idea:**

- Continue with the existing definition of **structure**, i.e. disjoint domains for identities.
- Have an **implicit association** from the child to each parent part (similar to the implicit attribute for stability).



- Apply (a different) pre-processing to make appropriate use of that association, e.g. rewrite (C++)

`x = 0;`

in D to

`uplinkC -> x = 0;`

Pre-Processing for the Uplink Semantics

- For each pair $C \triangleleft D$, extend D by a (fresh) association

$$\text{uplink}_C : C \text{ with } \mu = [1, 1], \xi = +$$

(**Exercise:** public necessary?)

- Given expression v (or f) in the **context** of class D ,
 - let C be the **smallest** class wrt. “ \preceq ” such that
 - $C \preceq D$, and
 - $C::v \in \text{atr}(D)$
 - then there exists (by definition) $C \triangleleft C_1 \triangleleft \dots \triangleleft C_n \triangleleft D$,
 - **normalise** v to (= replace by)

$$\text{uplink}_{C_n} \rightarrow \dots \rightarrow \text{uplink}_{C_1}.C::v$$

- If no (unique) smallest class exists, the model is considered **not well-formed**; the expression is ambiguous.

Uplink Structure, System State, Typing

- Definition of structure remains **unchanged**.
- Definition of system state remains **unchanged**.
- Typing and transformers remain **unchanged** — the preprocessing has put everything in shape.

Satisfying OCL Constraints (Uplink)

- Let $\mathcal{M} = (\mathcal{CD}, \mathcal{OD}, \mathcal{IM}, \mathcal{I})$ be a UML model, and \mathcal{D} a structure.
- We (**continue to**) say

$$\mathcal{M} \models expr$$

for

$$\underbrace{\text{context } C \text{ inv : } expr_0}_{=expr} \in Inv(\mathcal{M})$$

if and only if

$$\forall \pi = (\sigma_i)_{i \in \mathbb{N}} \in \llbracket \mathcal{M} \rrbracket$$

$$\forall i \in \mathbb{N}$$

$$\forall u \in \text{dom}(\sigma_i) \cap \mathcal{D}(C) :$$

$$I \llbracket expr_0 \rrbracket (\sigma_i, \{self \mapsto u\}) = 1.$$

- \mathcal{M} is (still) consistent if and only if it satisfies all constraints in $Inv(\mathcal{M})$.

Transformers (Uplink)

- What **has to change** is the **create** transformer:

$$\text{create}(C, \text{expr}, v)$$

- Assume, C 's inheritance relations are as follows.

$$\begin{aligned} C_{1,1} \triangleleft \dots \triangleleft C_{1,n_1} \triangleleft C, \\ \dots \\ C_{m,1} \triangleleft \dots \triangleleft C_{m,n_m} \triangleleft C. \end{aligned}$$

- Then, we have to
 - create one fresh object for each part, e.g.

$$u_{1,1}, \dots, u_{1,n_1}, \dots, u_{m,1}, \dots, u_{m,n_m},$$

- set up the uplinks recursively, e.g.

$$\sigma(u_{1,2})(\text{uplink}_{C_{1,1}}) = u_{1,1}.$$

- And, if we had constructors, be careful with their order.

Domain Inclusion vs. Uplink Semantics

Cast-Transformers

- C c;
- D d;
- **Identity upcast** (C++):
 - C* cp = &d; *// assign address of 'd' to pointer 'cp'*
- **Identity downcast** (C++):
 - D* dp = (D*)cp; *// assign address of 'd' to pointer 'dp'*
- **Value upcast** (C++):
 - *c = *d; *// copy attribute values of 'd' into 'c', or,
// more precise, the values of the C-part of 'd'*

Casts in Domain Inclusion and Uplink Semantics

	Domain Inclusion	Uplink
$C^* \text{ cp} = \&d;$	easy: immediately compatible (in underlying system state) because $\&d$ yields an identity from $\mathcal{D}(D) \subset \mathcal{D}(C)$.	easy: By pre-processing, $C^* \text{ cp} = d.\text{uplink}_C$;
$D^* \text{ dp} = (D^*)\text{cp};$	easy: the value of cp is in $\mathcal{D}(D) \cap \mathcal{D}(C)$ because the pointed-to object is a D . Otherwise, error condition.	difficult: we need the identity of the D whose C -slice is denoted by cp . (See next slide.)
$c = d;$	bit difficult: set (for all $C \preceq D$) $(C)(\cdot, \cdot) : \tau_D \times \Sigma \rightarrow \Sigma _{\text{atr}(C)}$ $(u, \sigma) \mapsto \sigma(u) _{\text{atr}(C)}$ Note: $\sigma' = \sigma[u_C \mapsto \sigma(u_D)]$ is not type-compatible!	easy: By pre-processing, $c = *(d.\text{uplink}_C)$;

Identity Downcast with Uplink Semantics

- **Recall** (C++): $D\ d; \quad C^* \ cp = \&d; \quad D^* \ dp = (D^*)cp;$
- **Problem**: we need the identity of the D whose C -slice is denoted by cp .
- **One technical solution**:
 - Give up disjointness of domains for **one additional type** comprising all identities, i.e. have

$$\text{all} \in \mathcal{I}, \quad \mathcal{D}(\text{all}) = \bigcup_{C \in \mathcal{C}} \mathcal{D}(C)$$

- In each \preceq -**minimal class** have associations “mostspec” pointing to **most specialised** slices, plus information of which type that slice is.
- Then **downcast** means, depending on the mostspec type (only finitely many possibilities), **going down and then up** as necessary, e.g.

```
switch(mostspec_type){
  case C :
    dp = cp -> mostspec -> uplinkDn -> ... -> uplinkD1 -> uplinkD;
  ...
}
```

Domain Inclusion vs. Uplink: Differences

- **Note:** The uplink semantics views inheritance as an abbreviation:
 - We only need to touch transformers (create) — and if we had constructors, we didn't even need that (we could encode the recursive construction of the upper slices by a transformation of the existing constructors.)
- **So:**
 - Inheritance **doesn't add** expressive power.
 - And it also **doesn't improve** conciseness **soo dramatically**.

As long as we're “**early binding**”, that is...

Domain Inclusion vs. Uplink: Motivations

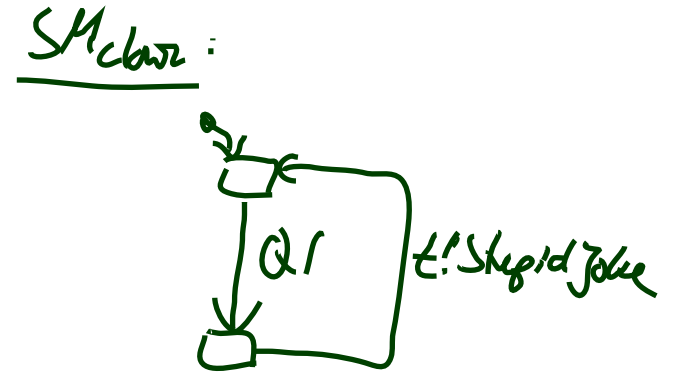
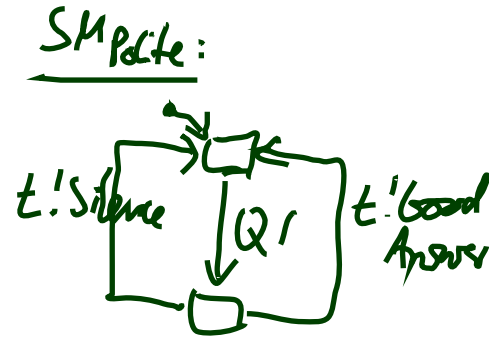
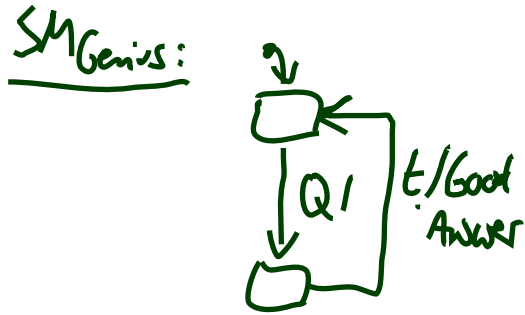
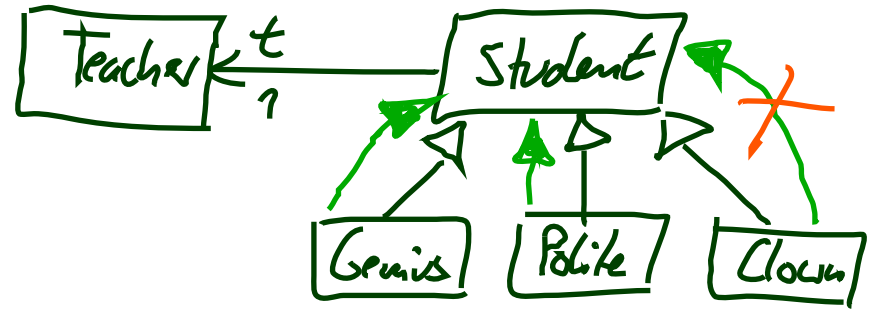
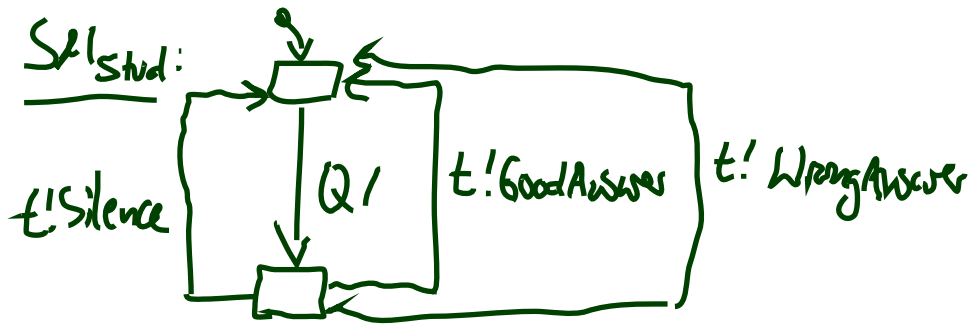
- **Exercise:**

What's the point of

- having the **tedious** adjustments of the **theory** if it can be approached **technically**?
- having the **tedious** technical **pre-processing** if it can be approached **cleanly** in the **theory**?

More Interesting: Behaviour

Example: Behaviour of Kinds of Students



→ inheritance
 ⇒
 → subtype

Desired Semantics of Specialisation: Subtyping

There is a classical description of what one **expects** from **sub-types**, which in the OO domain is closely related to inheritance:

The principle of type substitutability [Liskov, 1988, Liskov and Wing, 1994].
(**Liskov Substitution Principle** (LSP).)

“If for each object o_1 of type S there is an object o_2 of type T such that for all programs P defined in terms of T ,
the behavior of P is unchanged when o_1 is substituted for o_2 then S is a **subtype** of T .”

In other words: [Fischer and Wehrheim, 2000]

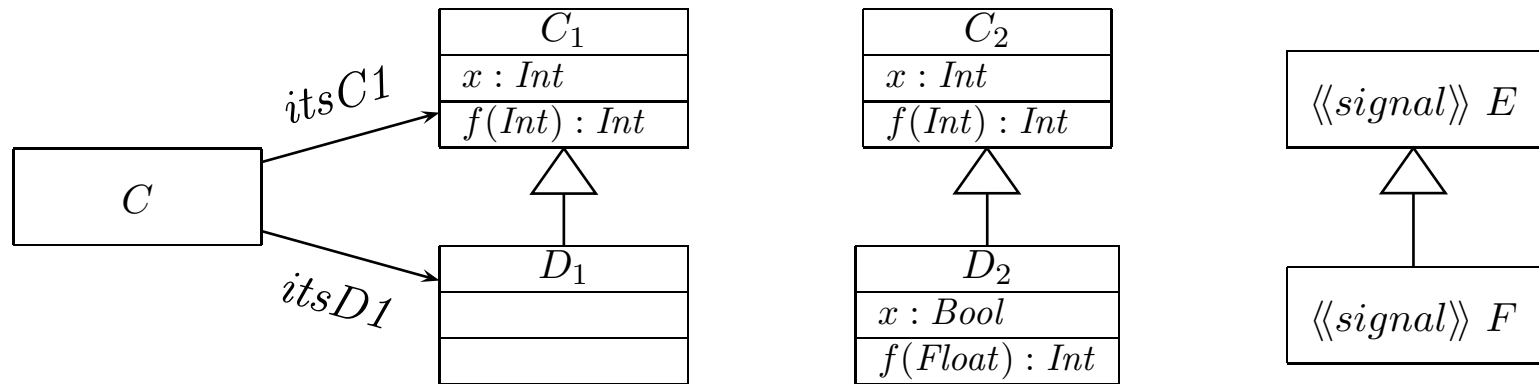
“An instance of the **sub-type** shall be **usable** whenever an instance of the supertype was expected,
without a client being able to tell the difference.”

So, what's “**usable**”? Who's a “**client**”? And what's a “**difference**”?

“...shall be usable...” for UML

Easy: Static Typing for Attributes

Given:



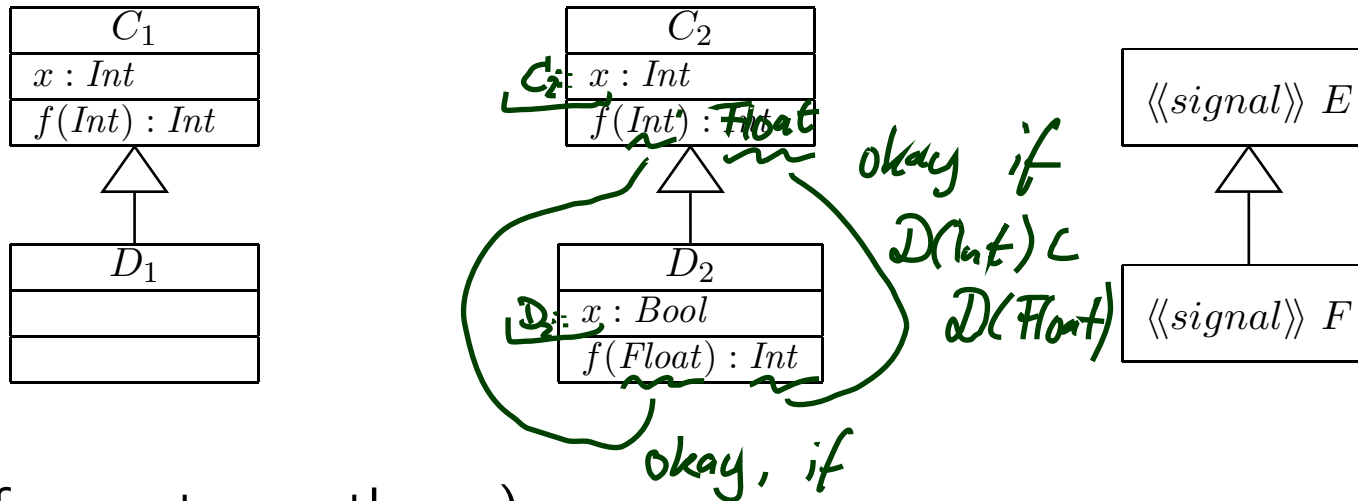
Wanted:

- $x > 0$ also **well-typed** for D_1
- assignment $itsC1 := itsD1$ being **well-typed**
- $itsC1.x = 0$, $itsC1.f(0)$, $itsC1 ! F$ being well-typed (and doing the right thing).

Approach:

- Simply define it as being well-typed, adjust system state definition to do the right thing.

Static Typing for Methods



Notions (from category theory):

- **invariance**,
- **covariance**,
- **contravariance**.

We could call, e.g. a method, **sub-type preserving**, if and only if it

- accepts **more general** types as input (**contravariant**),
- provides a **more specialised** type as output (**covariant**).

This is a notion used by many programming languages — and easily type-checked.

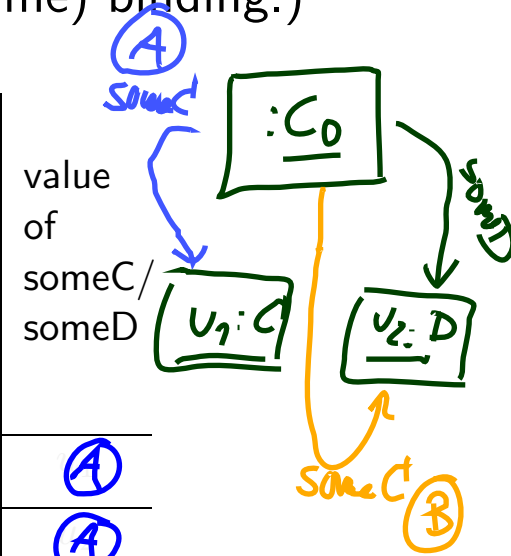
Excursus: Late Binding of Behavioural Features

Late Binding

What transformer applies in what situation? (Early (compile time) binding.)

the type of the link determines which impl. is used at runtime

	f not overridden in D	f overridden in D	
someC -> f()	$C::f()$	$C::f()$	(A)
someD -> f()	$C::f()$	$D::f()$	(A)
<u>someC</u> -> f()	$C::f()$	$C::f()$	(B)



What one could want is something different: (Late binding.)

type of object (at runtime) determines which impl. is used

someC -> f()	$C::f()$	$C::f()$	u_1 (A)
someD -> f()	$D::f()$	$D::f()$	(A)
someC -> f()	$C::f()$	$D::f()$	(B)

Late Binding in the Standard and in Prog. Lang.

- In **the standard**, Section 11.3.10, “CallOperationAction”:

“Semantic Variation Points

The mechanism for determining the method to be invoked as a result of a call operation is unspecified.” [OMG, 2007b, 247]

- In **C++**,
 - methods are by default “(early) compile time binding”,
 - can be declared to be “late binding” by keyword “virtual”,
 - the declaration applies to all inheriting classes.
- In **Java**,
 - methods are “late binding”;
 - there are patterns to imitate the effect of “early binding”

Exercise: What could be the rationale of the designers of C++?

Note: late binding typically applies only to **methods**, **not** to **attributes**.
(But: getter/setter methods have been invented recently.)

Back to the Main Track: “...tell the difference...” for UML

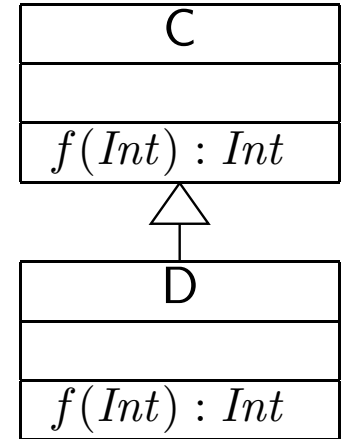
With Only Early Binding...

- ...we're **done** (if we realise it correctly in the framework).
- Then
 - if we're calling method f of an object u ,
 - which is an instance of D with $C \preceq D$
 - via a C -link,
 - then we (by definition) only see and change the C -part.
 - We cannot tell whether u is a C or an D instance.

use the transformer provided by C

So we immediately also have behavioural/dynamic subtyping.

Difficult: Dynamic Subtyping



- $C::f$ and $D::f$ are **type compatible**, but D is **not necessarily** a **sub-type** of C .
- **Examples:** (C++)

```
int C::f(int) {
    return 0;
};
```

vs.

```
int D::f(int) {
    return 1;
};
```

```
int C::f(int) {
    return (rand() %
2);
};
```

vs.

```
int D::f(int x) {
    return (x % 2);
};
```

Sub-Typing Principles Cont'd

- In the standard, Section 7.3.36, “**Operation**”:

“Semantic Variation Points

[...] When operations are redefined in a specialization, rules regarding **invariance**, **covariance**, or **contravariance** of types and preconditions determine whether the specialized classifier is substitutable for its more general parent. Such rules constitute semantic variation points with respect to redefinition of operations.” [OMG, 2007a, 106]

- So, better: call a method **sub-type preserving**, if and only if it
 - (i) accepts **more input values** (**contravariant**),
 - (ii) on the **old values**, has **fewer behaviour** (**covariant**).

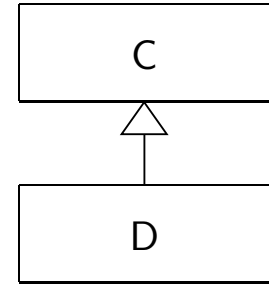
Note: ~~This~~ (ii) is no longer a matter of simple type-checking!

- And not necessarily the end of the story:
 - One could, e.g. want to consider execution time.
 - Or, like [Fischer and Wehrheim, 2000], relax to “fewer observable behaviour”, thus admitting the sub-type to do more work on inputs.

Note: “testing” differences depends on the **granularity** of the semantics.

- **Related:** “has a weaker pre-condition,” (**contravariant**),
“has a stronger post-condition.” (**covariant**).

Ensuring Sub-Typing for State Machines



- In the CASE tool we consider, multiple classes in an inheritance hierarchy can have state machines.
- But the state machine of a sub-class **cannot** be drawn from scratch.
- Instead, the state machine of a sub-class can only be obtained by applying actions from a **restricted** set to a copy of the original one. Roughly (cf. User Guide, p. 760, for details),
 - add things into (hierarchical) states,
 - add more states,
 - attach a transition to a different target (limited).
- They **ensure**, that the sub-class is a **behavioural sub-type** of the super class. (But method implementations can still destroy that property.)
- Technically, the idea is that (by late binding) only the state machine of the most specialised classes are running.

By knowledge of the framework, the (code for) state machines of super-classes is still accessible — but using it is hardly a good idea...

References

- [Fischer and Wehrheim, 2000] Fischer, C. and Wehrheim, H. (2000). Behavioural subtyping relations for object-oriented formalisms. In Rus, T., editor, AMAST, number 1816 in Lecture Notes in Computer Science. Springer-Verlag.
- [Liskov, 1988] Liskov, B. (1988). Data abstraction and hierarchy. SIGPLAN Not., 23(5):17–34.
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- [OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.
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