

# Testing, Abstraction, Theorem Proving: Better Together

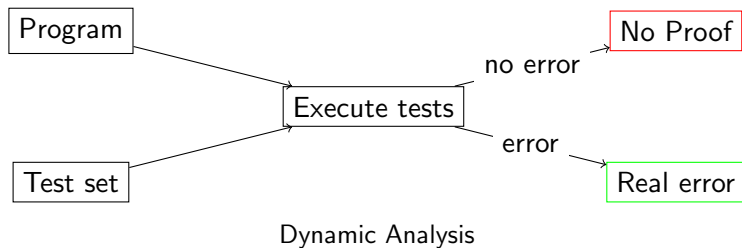
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Thomas Ball  
Mooly Sagiv

Presenter: Michael Rudolph

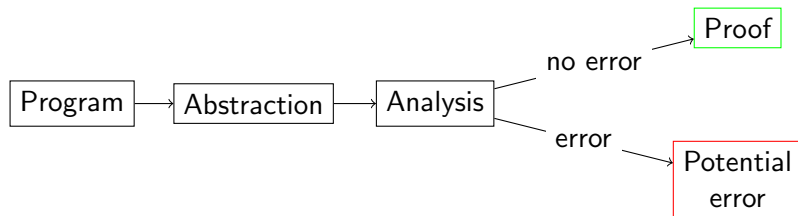
Seminar Program Analysis and Software Testing

University of Freiburg 2016

# Motivation

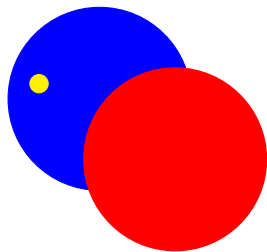





# Motivation





Static Analysis

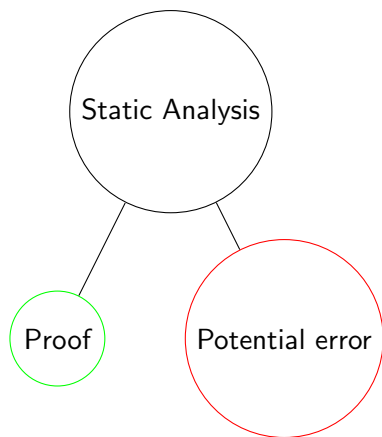
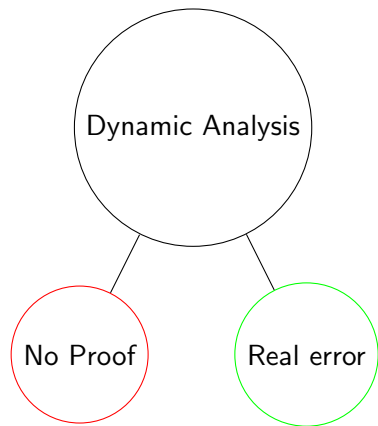
# False error



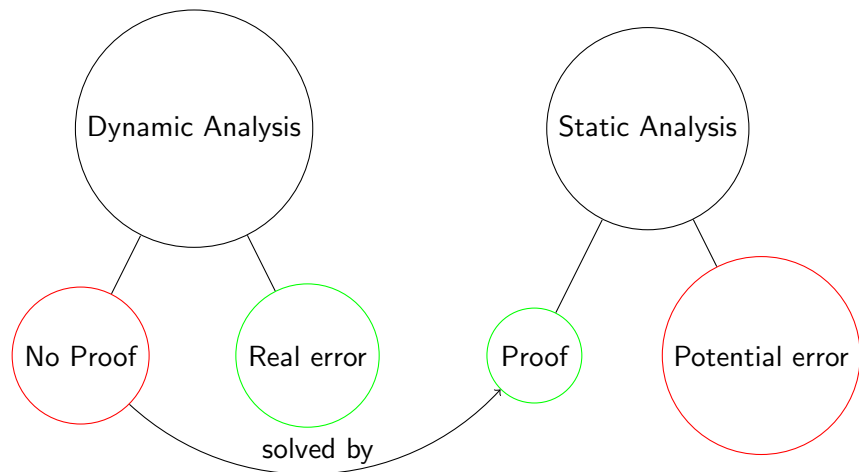
	Unreachable concrete states
	Reachable concrete states
	false error

Abstract state =  + 

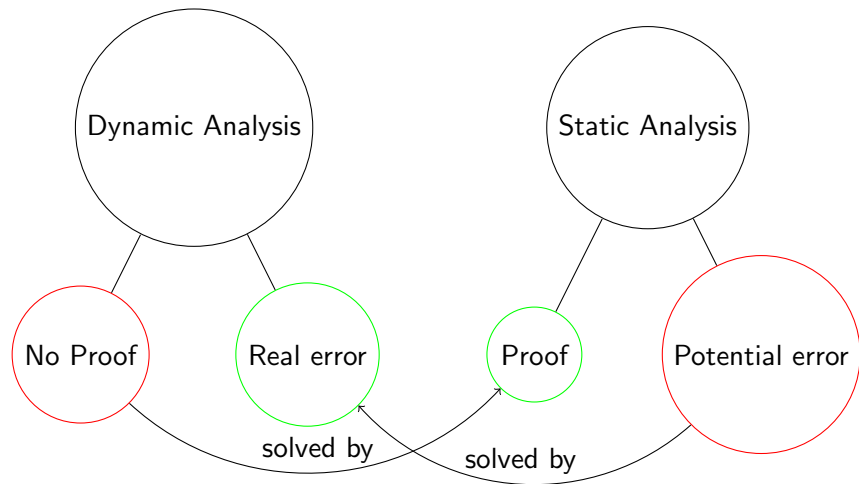
# Motivation



# Motivation



# Motivation





Idea: **Combine** both approaches.



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- 1 Motivation
- 2 Overview method**
- 3 Real error
- 4 Proof
- 5 False error
- 6 Conclusion

# Example

---

**Algorithm:** `foo(int x, int y)`

---

```
I int * px = NULL;
A x = x + 1;
B if x < 4 then
C |   px = &x;
D if px == &y then
E |   x = x + 1;
F if x < 5 then
G |   *px = *px + 1;
```

---

(foo algorithm, G. Yorsh , T. Ball, and M. Sagiv , 2006.)



# Example

---

**Algorithm:** foo(3, 0)

---

I *int \* px = NULL;* // (pc=I, x=3, y=0, px=NULL)  
A *x = x + 1;* // (pc=A, x=3, y=0, px=NULL)

# Example

---

**Algorithm:** foo(3, 0)

---

I *int \* px = NULL;* // (pc=I, x=3, y=0, px=NULL)  
A *x = x + 1;* // (pc=A, x=3, y=0, px=NULL)  
B **if** *x < 4* // (pc=B, x=4, y=0, px=NULL)  
**then**  
C | *px = &x;*

# Example

---

## Algorithm: foo(3, 0)

---

```
I int * px = NULL; // (pc=I, x=3, y=0, px=NULL)
A x = x + 1; // (pc=A, x=3, y=0, px=NULL)
B if x < 4 // (pc=B, x=4, y=0, px=NULL)
  then
C | px = &x;
D if px == &y // (pc=D, x=4, y=0, px=NULL)
  then
E | x = x + 1; // Dead code
```

# Example

---

## Algorithm: foo(3, 0)

---

```
I int * px = NULL;           // (pc=I, x=3, y=0, px=NULL)
A x = x + 1;                 // (pc=A, x=3, y=0, px=NULL)
B if x < 4                   // (pc=B, x=4, y=0, px=NULL)
  then
C   | px = &x;
D if px == &y                // (pc=D, x=4, y=0, px=NULL)
  then
E   | x = x + 1;             // Dead code
F if x < 5                   // (pc=F, x=4, y=0, px=NULL)
  then
G   | *px = *px + 1;        // (pc=G, x=4, y=0, px=NULL)
```

---

# Example

---

**Algorithm:** foo(3, 0)

---

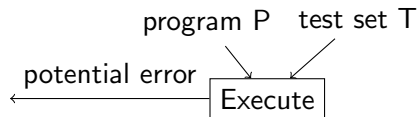
```
I int * px = NULL;           // (pc=I, x=3, y=0, px=NULL)
A x = x + 1;                 // (pc=A, x=3, y=0, px=NULL)
B if x < 4                    // (pc=B, x=4, y=0, px=NULL)
  then
C   | px = &x;
D if px == &y                 // (pc=D, x=4, y=0, px=NULL)
  then
E   | x = x + 1;             // Dead code
F if x < 5                    // (pc=F, x=4, y=0, px=NULL)
  then
G   | *px = *px + 1;         // (pc=G, x=4, y=0, px=NULL)
```

⇒ Null pointer dereference error

---

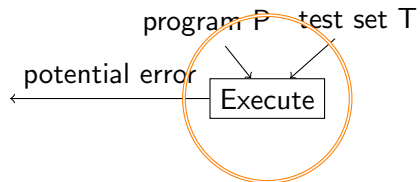


# Execute



(Figure 1, G. Yorsh , T. Ball, and M. Sagiv , 2006.)

# Execute



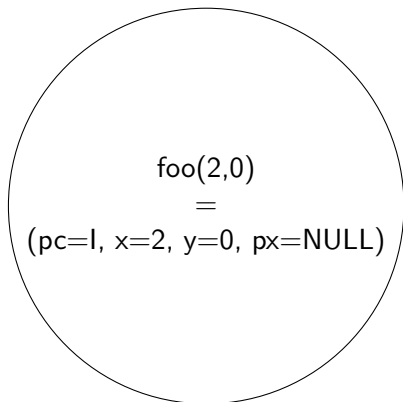
(Figure 1, G. Yorsh , T. Ball, and M. Sagiv , 2006.)

# Execute

Program P



Test set



An example input.

# Execute

---

**Algorithm:** foo(2, 0)

---

| *int \* px = NULL; // (pc=I, x=2, y=0, px=NULL)*

# Execute

---

**Algorithm:** foo(2, 0)

---

I *int \* px = NULL;* // (pc=I, x=2, y=0, px=NULL)  
A *x = x + 1;* // (pc=A, x=2, y=0, px=NULL)

# Execute

---

## Algorithm: foo(2, 0)

---

```
I int * px = NULL;           // (pc=I, x=2, y=0, px=NULL)
A x = x + 1;                 // (pc=A, x=2, y=0, px=NULL)
B if x < 4                   // (pc=B, x=3, y=0, px=NULL)
  then
C | px = &x;                 // (pc=C, x=3, y=0, px=NULL)
```

# Execute

---

## Algorithm: foo(2, 0)

---

```
I  int * px = NULL;           // (pc=I, x=2, y=0, px=NULL)
A  x = x + 1;                 // (pc=A, x=2, y=0, px=NULL)
B  if x < 4                   // (pc=B, x=3, y=0, px=NULL)
    then
C  |   px = &x;               // (pc=C, x=3, y=0, px=NULL)
D  if px == &y                // (pc=D, x=3, y=0, px=¬NULL)
    then
E  |   x = x + 1;
```

---

# Execute

---

## Algorithm: foo(2, 0)

---

```
I  int * px = NULL;           // (pc=I, x=2, y=0, px=NULL)
A  x = x + 1;                 // (pc=A, x=2, y=0, px=NULL)
B  if x < 4                   // (pc=B, x=3, y=0, px=NULL)
    then
C  |   px = &x;               // (pc=C, x=3, y=0, px=NULL)
D  if px == &y               // (pc=D, x=3, y=0, px=¬NULL)
    then
E  |   x = x + 1;
F  if x < 5                   // (pc=F, x=3, y=0, px=¬NULL)
    then
G  |   *px = *px + 1;         // (pc=G, x=3, y=0, px=¬NULL)
```

---



# Execute

Execute test set  $\Rightarrow$  No Error.

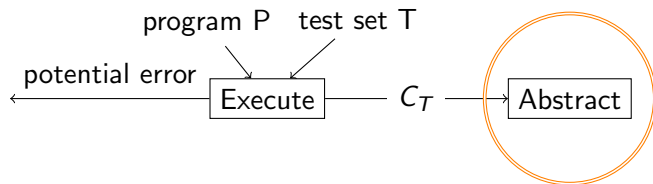
but

Execute `foo(3,0)`  $\Rightarrow$  Error

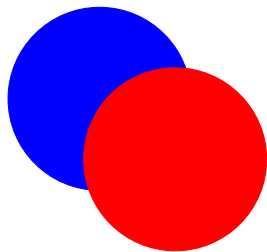


Execute phase gives **no** proof.

# Abstract



# Abstract



■ Unreachable concrete states  
■ Reachable concrete states

Abstract state = ■ + ■

# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

Algorithm: foo(2,0)

| *int* \* *px* = *NULL*;

$C_T$   
pc=1, x=2, y=0, px=NULL

$A_T$   
l, t, t

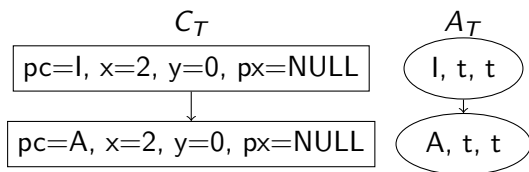
# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) | (pc, x, y, px) \in C\}$$

Algorithm: foo(2,0)

I *int \* px = NULL;*  
A *x = x + 1;*

---



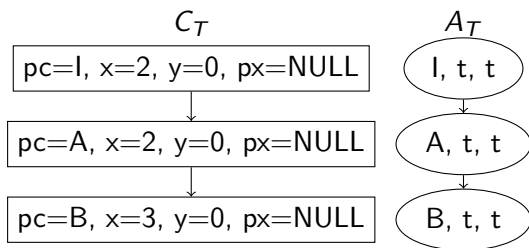
# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

**Algorithm:** `foo(2,0)`

```
I int * px = NULL;  
A x = x + 1;  
B if x < 4 then  
C |   px = &x;
```

---

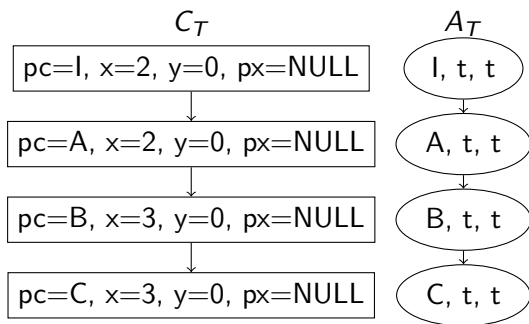


# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

**Algorithm:** `foo(2,0)`

```
I int * px = NULL;  
A x = x + 1;  
B if x < 4 then  
C | px = &x;
```

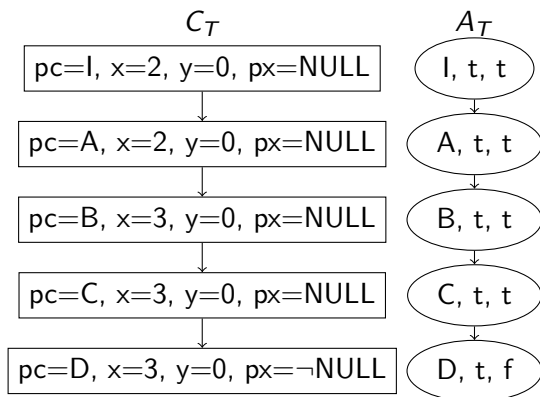


# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

**Algorithm:** `foo(2,0)`

```
I  int * px = NULL;
A  x = x + 1;
B  if x < 4 then
C  |   px = &x;
D  if px == &y then
E  |   x = x + 1;
```



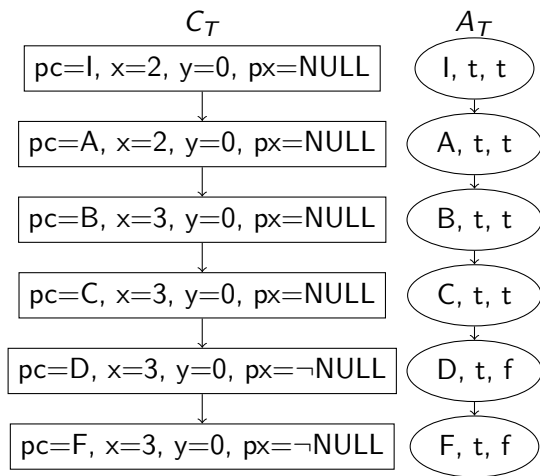


# Abstract

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

**Algorithm:** `foo(2,0)`

```
I  int *px = NULL;
A  x = x + 1;
B  if x < 4 then
C  |   px = &x;
D  if px == &y then
E  |   x = x + 1;
F  if x < 5 then
G  |   *px = *px + 1;
```

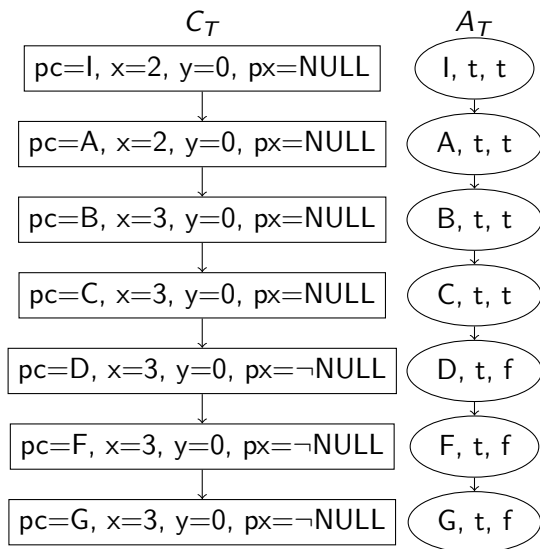


# Abstract

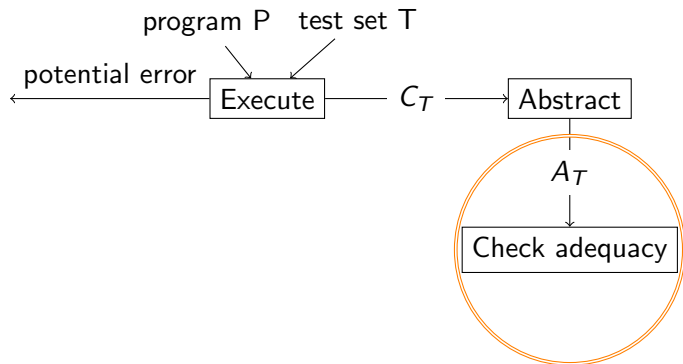
$$\alpha(C) = \{(pc, x < 5, px = \text{NULL}) \mid (pc, x, y, px) \in C\}$$

**Algorithm:** `foo(2,0)`

```
I  int *px = NULL;
A  x = x + 1;
B  if x < 4 then
C  |   px = &x;
D  if px == &y then
E  |   x = x + 1;
F  if x < 5 then
G  |   *px = *px + 1;
```



# Check adequacy



# Check adequacy

What are **successor states** ? Let's look at our example.

---

**Algorithm:** part of `foo(int x, int y)`

---

```
A x = x + 1;
B if x < 4 then
C   | px = &x; // If B is true.
D if px == &y // If B is false.
   then
E   | x = x + 1;
F if x < 5 then
G   | *px = *px + 1;
```

---

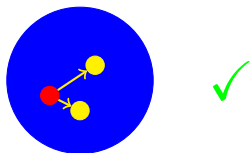
Successor states of B




# Check adequacy

A set of tests  $T$  is adequate under a given abstraction

$\Leftrightarrow$

for all concrete states which are represented by  $A_T$  it holds that their successor states are covered by  $A_T$ , too.



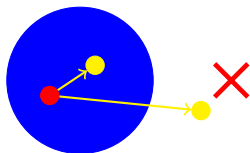
	$A_T$ : abstract states covered by $T$
	concrete state $c$
	successor states of $c$




# Check adequacy

A set of tests  $T$  is adequate under a given abstraction

$\Leftrightarrow$

for all concrete states which are represented by  $A_T$  it holds that their successor states are covered by  $A_T$ , too.



	$A_T$ : abstract states covered by $T$
	concrete state $c$
	successor states of $c$

# Check adequacy

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

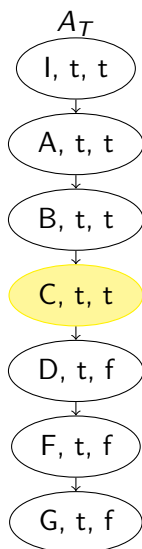
---

**Algorithm:** foo(int x, int y)

---

```
I  int * px = NULL;
A  x = x + 1;
B  if x < 4 then
C  |   px = &x;           // (pc=C, x<4, px=NULL)
D  if px == &y
   then
E  |   x = x + 1;
F  if x < 5 then
G  |   *px = *px + 1;
```

---



# Check adequacy

$$\alpha(C) = \{(pc, x < 5, px = NULL) \mid (pc, x, y, px) \in C\}$$

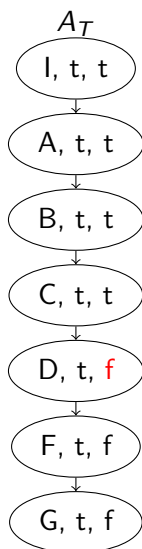
---

**Algorithm:** foo(int x, int y)

---

```
I  int * px = NULL;
A  x = x + 1;
B  if x < 4 then
C  |   px = &x;
D  if px == &y           // (pc=D, x>=4, px=NULL)
   then
E  |   x = x + 1;
F  if x < 5 then
G  |   *px = *px + 1;
```

---





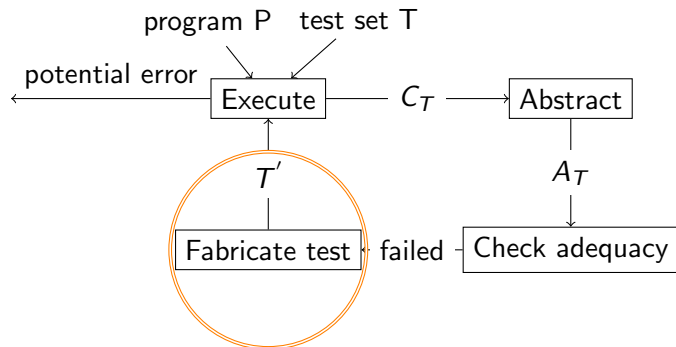
# Check adequacy

The successor state (D,4,0,NULL) is **not** covered by  $A_T$



adequacy check **fails**.

# Fabricate test



# Fabricate test

The successor state (D,4,0,NULL) is **not** covered by  $A_T$



adequacy check **fails**.



model generator fabricates a pair of concrete states, e.g.

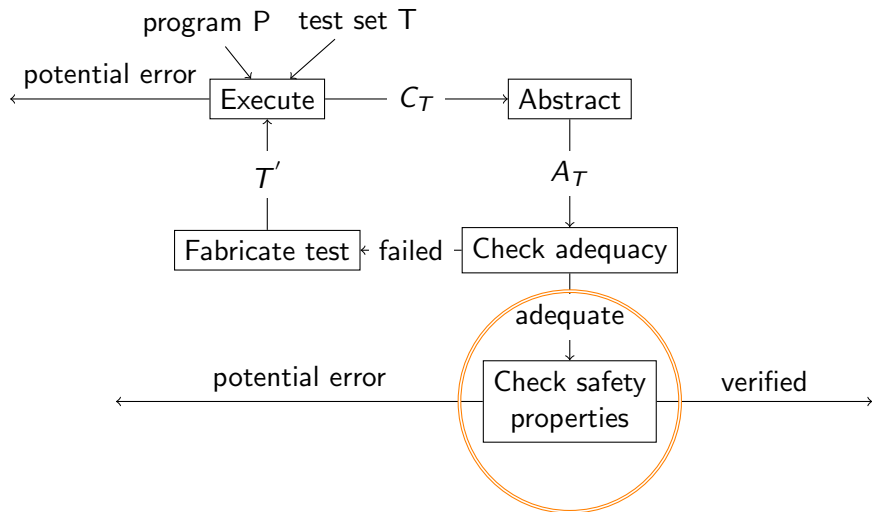
(B,4,0,NULL), (D,4,0,NULL)

**concrete states need not be reachable.**



extend test set T by the generated concrete states.

# Check safety property



# Check safety property

What is a **safety property** ? Let's look at our example.

---

**Algorithm:** part of foo(int x, int y)

---

```
A x = x + 1;
B if x < 4 then
C   | px = &x;
D if px == &y then
E   | x = x + 1;
F if x < 5 then
G   | *px = *px + 1; // error if px = NULL
```

---

⇒ safety property: abstract error state  $(G,t,t) \notin A_T$

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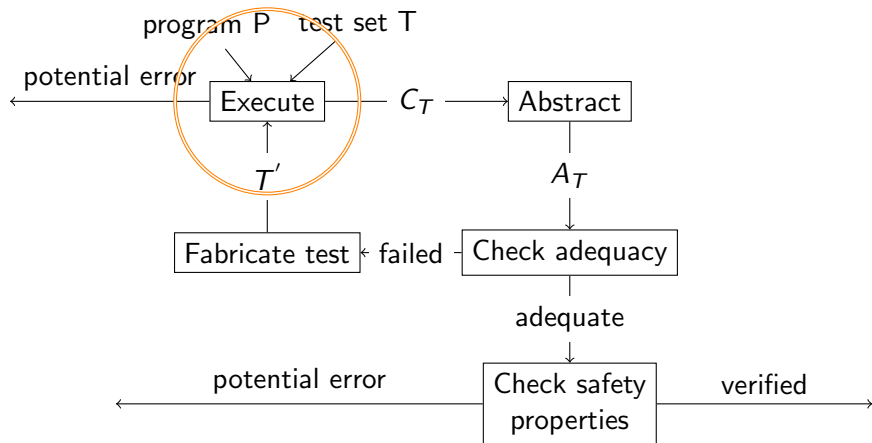
- 1 Motivation
- 2 Overview method
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- 4 Proof
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# Input

- Program P  
foo
- Test set T

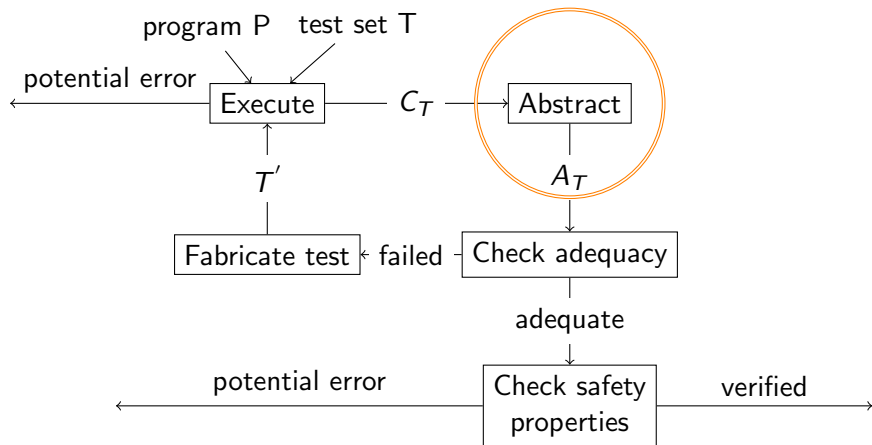
$$\begin{aligned} & \underbrace{\{(pc = 1, x = 2, y = 0, px = NULL)\}}_{fixedFoo(2,0)} \\ & \underbrace{(pc = 1, x = 6, y = 0, px = NULL)}_{fixedFoo(6,0)} \\ & \underbrace{(pc = 1, x = 11, y = 0, px = NULL)}_{fixedFoo(11,0)} \end{aligned}$$

# Run until check adequacy

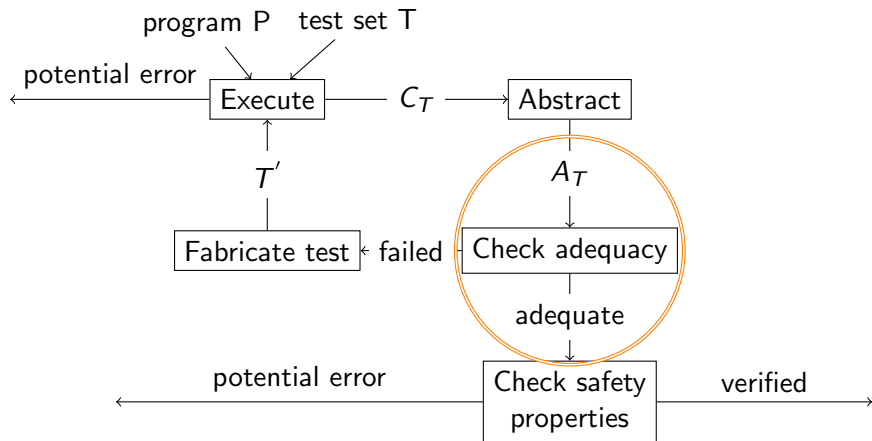




# Run until check adequacy

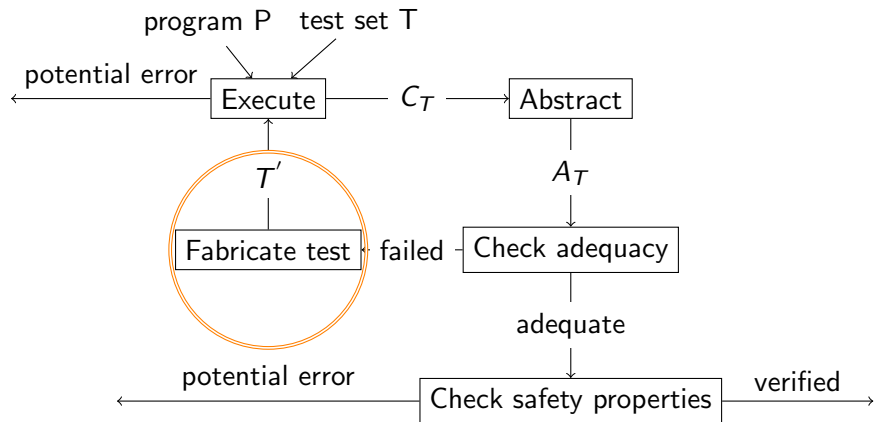


# Run until check adequacy





# Fabricate test



# Fabricate test

---

**Algorithm:** fixedFoo(int x, int y)

---

I *int \* px = NULL;*

A *x = x + 1;*

B **if** *x < 5* **then**

C | *px = &x;*

D **if** *px == &y* **then**

E | *x = x + 1;*

F **if** *x < 5* **then**

G | *\*px = \*px + 1;*

---

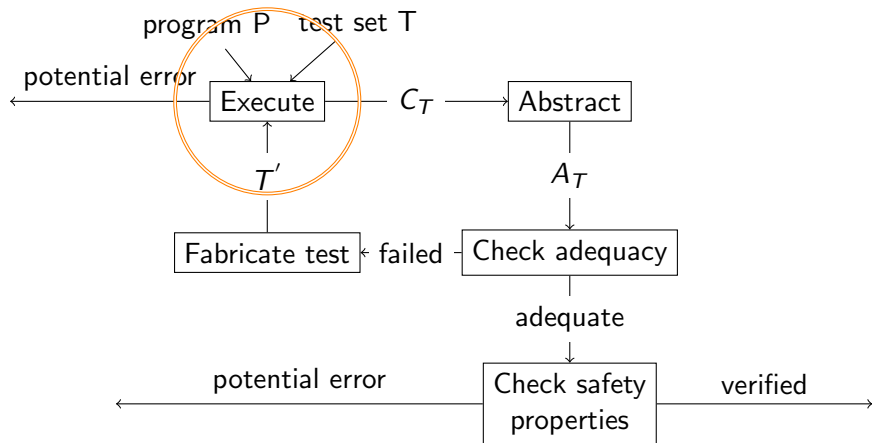
Fabricate tests

(pc=B, x=4, y=0, px=NULL)

(pc=D, x=4, y=0, px=NULL)

Remark: Both tests are  
reachable.

# Execute



# Execute

(pc=B, x=4, y=0, px=NULL)

---

**Algorithm:** foo(int x, int y)

---

```
I  int * px = NULL;
A  x = x + 1;
B  if x < 4 // (pc=B, x=4, y=0, px=NULL)
    then
C  |   px = &x;
D  if px == &y // (pc=D, x=4, y=0, px=NULL)
    then
E  |   x = x + 1;
F  if x < 5 // (pc=F, x=4, y=0, px=NULL)
    then
G  |   *px = *px + 1; // (pc=G, x=4, y=0, px=NULL)
```

# Execute

(pc=B, x=4, y=0, px=NULL)

---

**Algorithm:** foo(int x, int y)

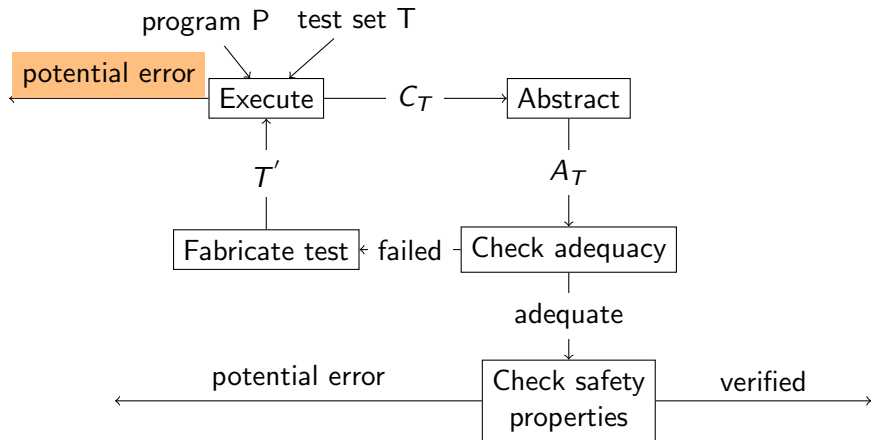
---

```
I int * px = NULL;
A x = x + 1;
B if x < 4 // (pc=B, x=4, y=0, px=NULL)
  then
C   |   px = &x;
D if px == &y // (pc=D, x=4, y=0, px=NULL)
  then
E   |   x = x + 1;
F if x < 5 // (pc=F, x=4, y=0, px=NULL)
  then
G   |   *px = *px + 1; // (pc=G, x=4, y=0, px=NULL)
```

⇒ Null pointer dereference error



# Report potential error



# Table of Contents

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- 6 Conclusion

# Modify example foo

---

**Algorithm:** fixedFoo(int x, int y)

---

```
I int * px = NULL;
A x = x + 1;
B if x < 5 // instead of x < 4
  then
C   |   px = &x;
D if px == &y then
E   |   x = x + 1;
F if x < 5 then
G   |   *px = *px + 1;
```

---

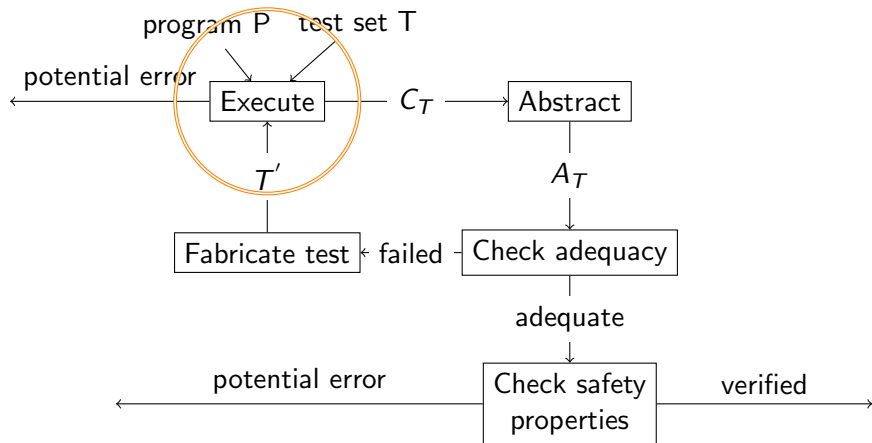
(fixed\_foo algorithm, G. Yorsh , T. Ball, and M. Sagiv , 2006.)

# Input

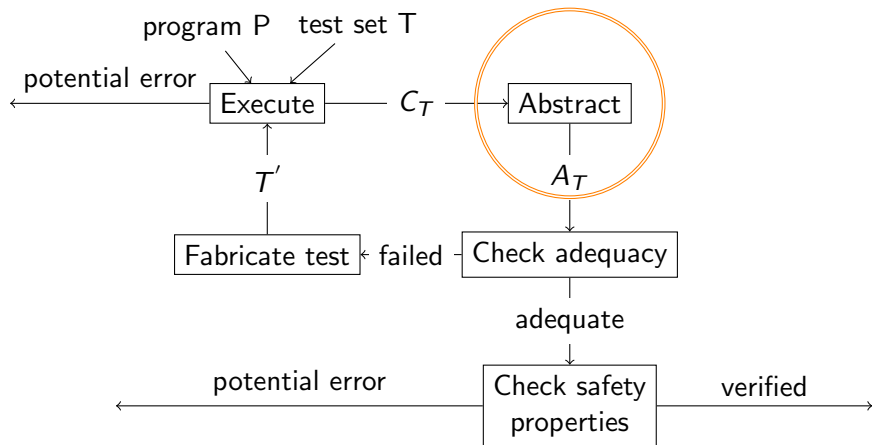
- Program P  
fixedFoo
- Test set T

$$\begin{aligned} & \underbrace{\{(pc = 1, x = 2, y = 0, px = NULL)\}}_{fixedFoo(2,0)} \\ & \underbrace{(pc = 1, x = 6, y = 0, px = NULL)}_{fixedFoo(6,0)} \\ & \underbrace{(pc = 1, x = 11, y = 0, px = NULL)}_{fixedFoo(11,0)} \end{aligned}$$

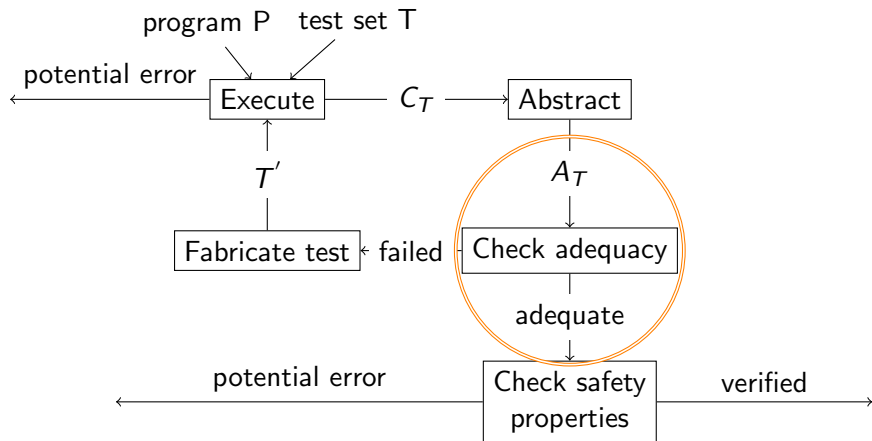
# Run until check adequacy



# Run until check adequacy



# Run until check adequacy



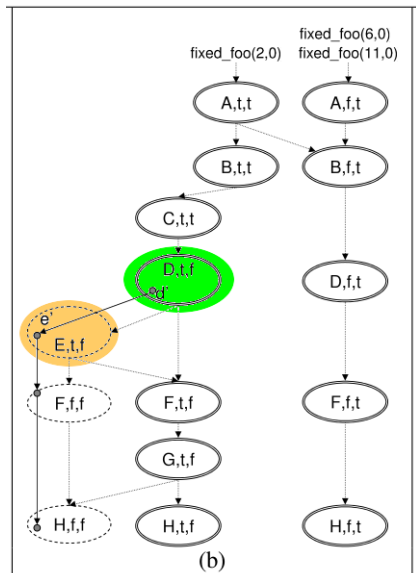
---

**Algorithm:** fixedFoo(int x, int y)

---

```
I int * px = NULL;
A x = x + 1;
B if x < 5 then
C   |   px = &x;
D if px == &y then
E   |   x = x + 1;
F if x < 5 then
G   |   *px = *px + 1;
```

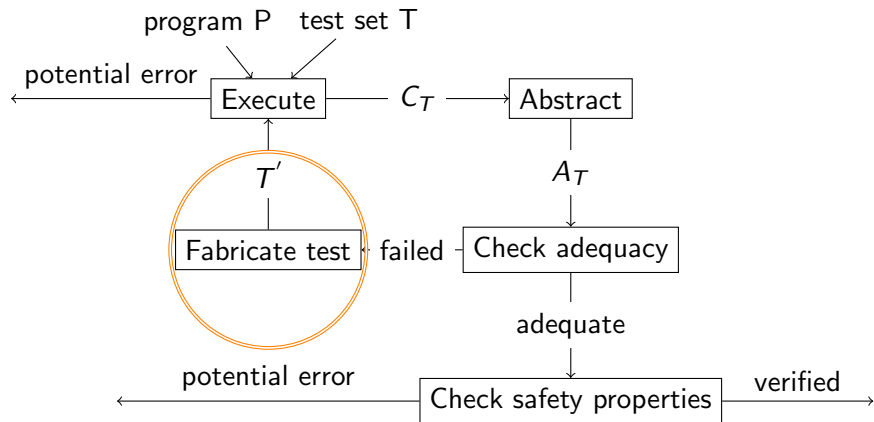
---



(Figure 2(b), G. Yorsh , T. Ball, and M. Sagiv , 2006.)



# Fabricate test



# Fabricate test

---

**Algorithm:** fixedFoo(int x, int y)

---

```
I int * px = NULL;
A x = x + 1;
B if x < 5 then
C |   px = &x;
D if px == &y then
E |   x = x + 1;
F if x < 5 then
G |   *px = *px + 1;
```

---

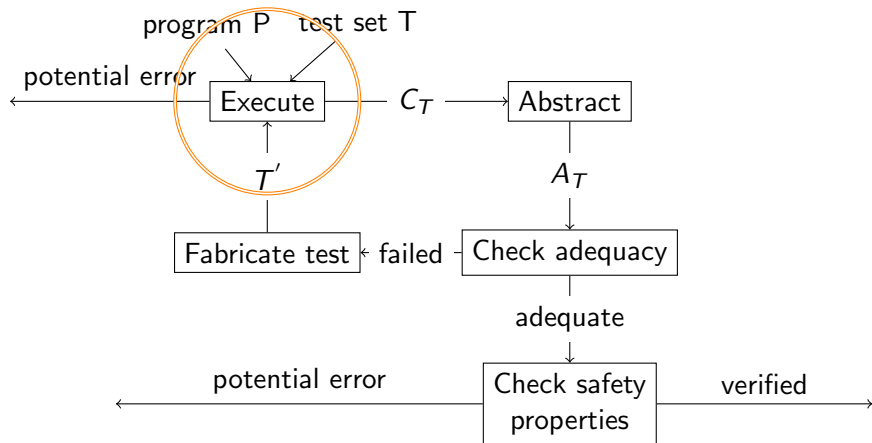
Fabricate tests

(pc=D, x=4, y=0, px=&y)

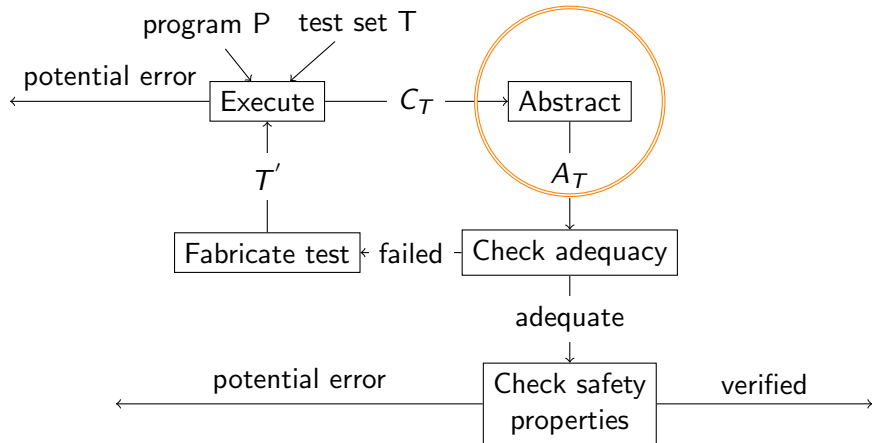
(pc=E, x=4, y=0, px=&y)

Remark: Both tests are **not** reachable, since px never assigned to &y.

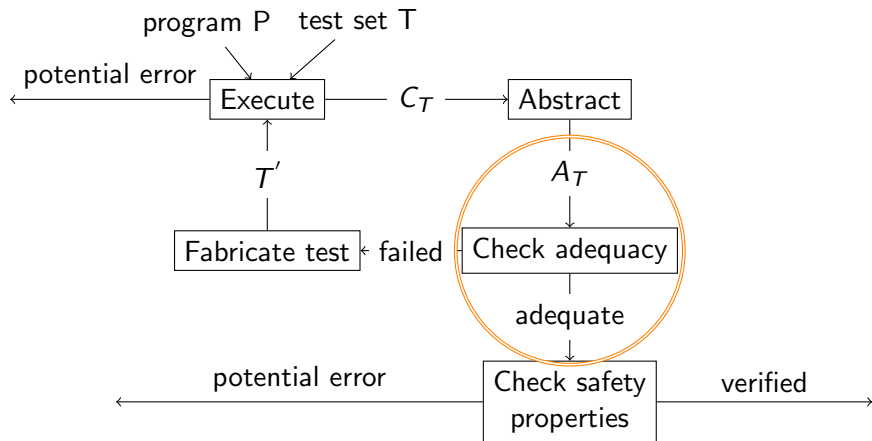
# Run until check adequacy

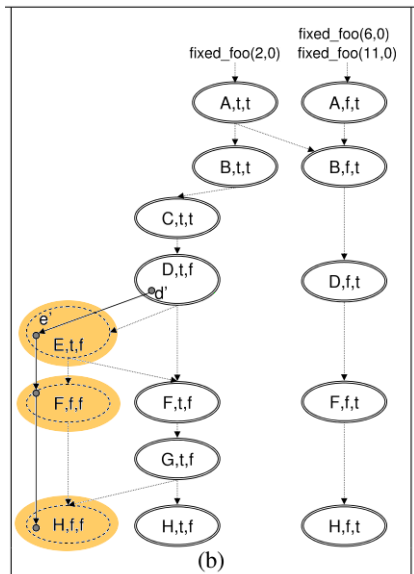


# Run until check adequacy



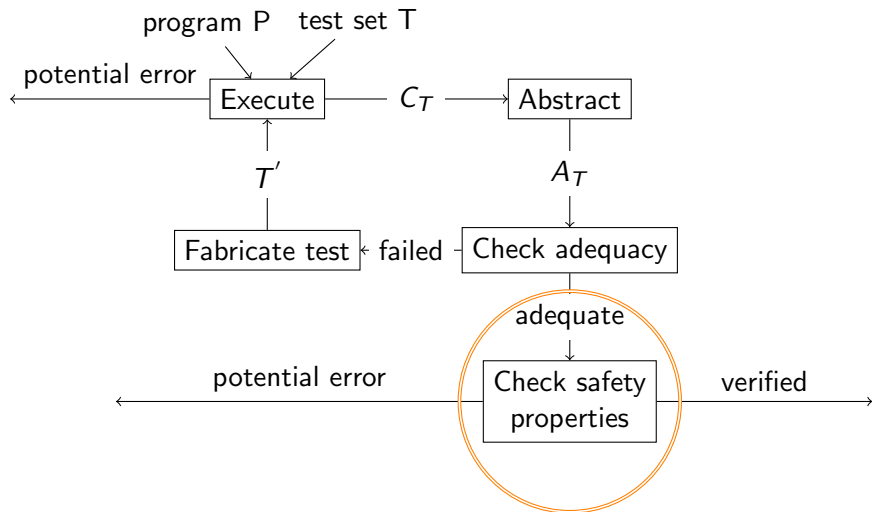
# Run until check adequacy



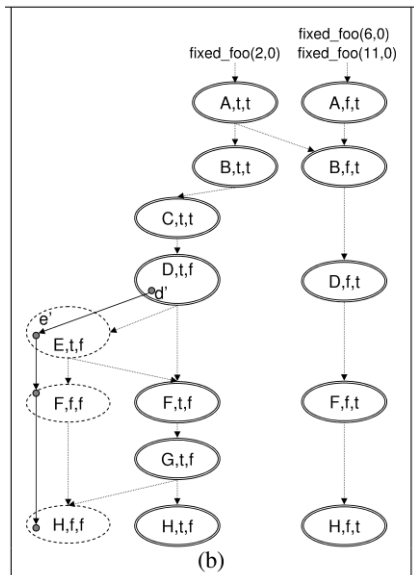


(Figure 2(b), G. Yorsh , T. Ball, and M. Sagiv , 2006.)

# Check safety properties



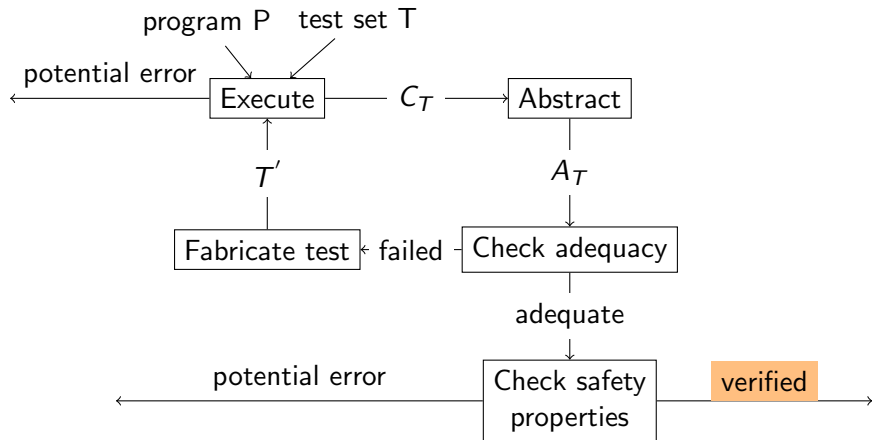
Error state  $(G,t,t) \notin$



(Figure 2(b), G. Yorsh , T. Ball, and M. Sagiv, 2006.)



# Proof



# Table of Contents

- 1 Motivation
- 2 Overview method
- 3 Real error
- 4 Proof
- 5 False error**
- 6 Conclusion

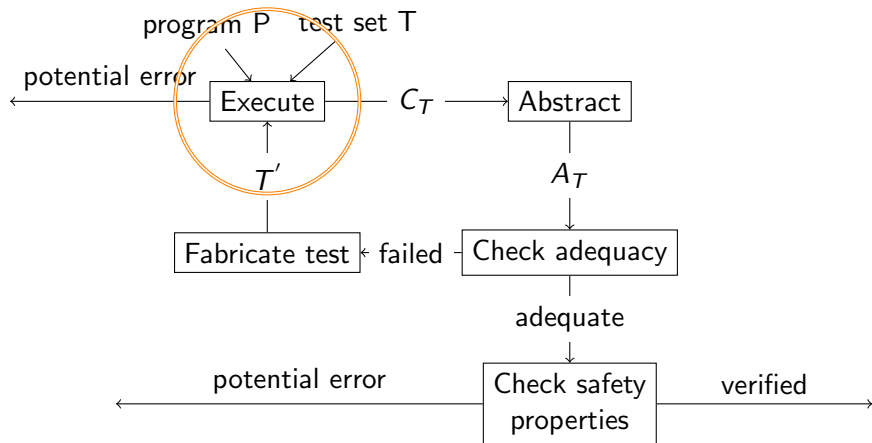
# Input

- Program P  
fixedFoo
- Test set T

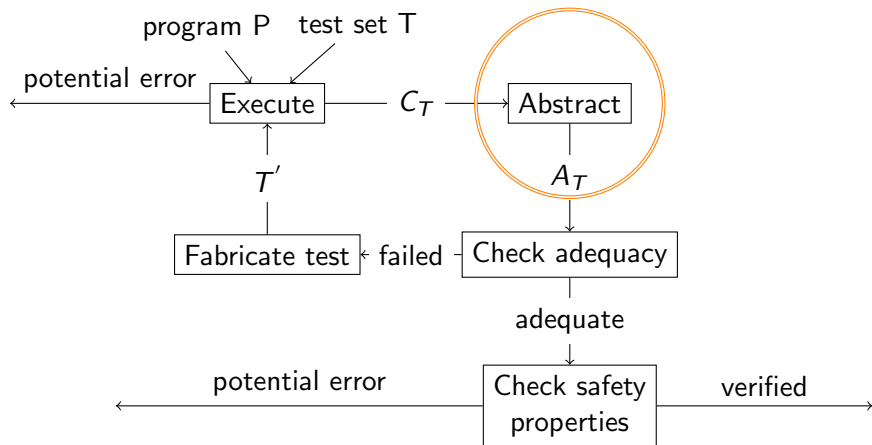
$$\underbrace{\{(pc = l, x = 2, y = 0, px = NULL)\}}_{fixedFoo(2,0)}$$
$$\underbrace{\{(pc = l, x = 6, y = 0, px = NULL)\}}_{fixedFoo(6,0)}$$
$$\underbrace{\{(pc = l, x = 11, y = 0, px = NULL)\}}_{fixedFoo(11,0)}$$

- abstraction function:  
 $\alpha = \{(pc, x < 10, px = NULL) \mid (pc, x, y, px) \in C\}$

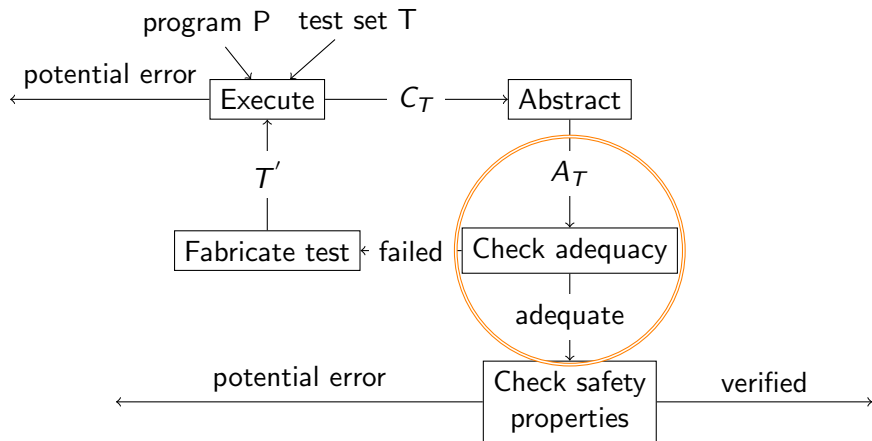
# Run until check adequacy



# Run until check adequacy



# Run until check adequacy



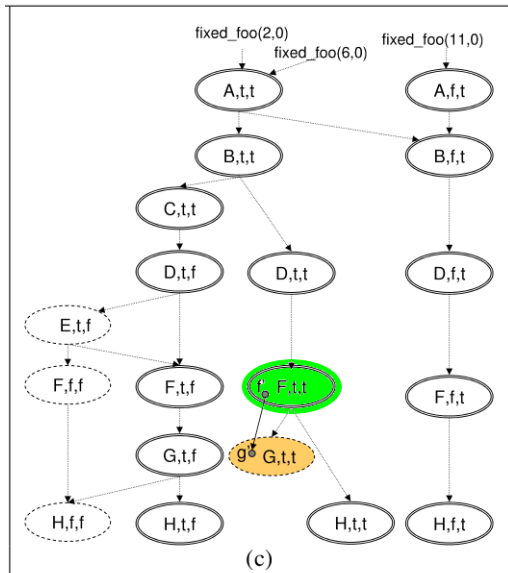
---

**Algorithm:** fixedFoo(int x,  
int y)

---

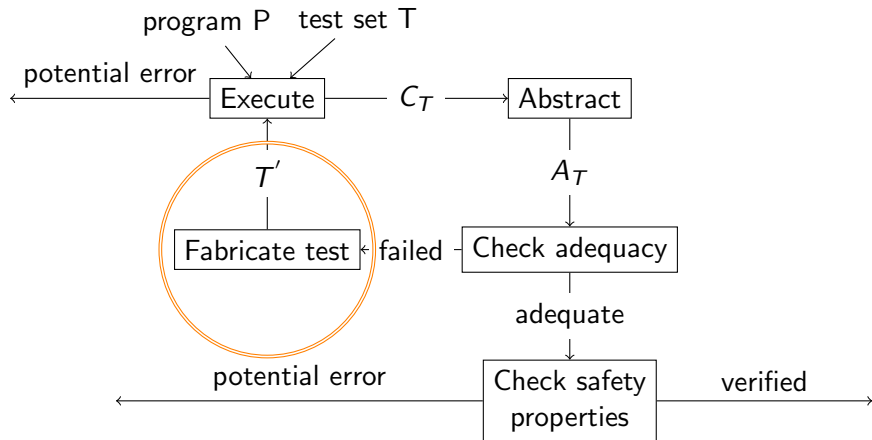
```
I int * px = NULL;  
A x = x + 1;  
B if x < 5 then  
C |   px = &x;  
D if px == &y then  
E |   x = x + 1;  
F if x < 5 then  
G |   *px = *px + 1;
```

---



(Figure 2(c), G. Yorsh , T. Ball, and M. Sagiv , 2006.)

# Fabricate test





# Fabricate test

---

**Algorithm:** fixedFoo(int x, int y)

---

```
I int * px = NULL;
A x = x + 1;
B if x < 5 then
C |   px = &x;
D if px == &y then
E |   x = x + 1;
F if x < 5 then
G |   *px = *px + 1;
```

---

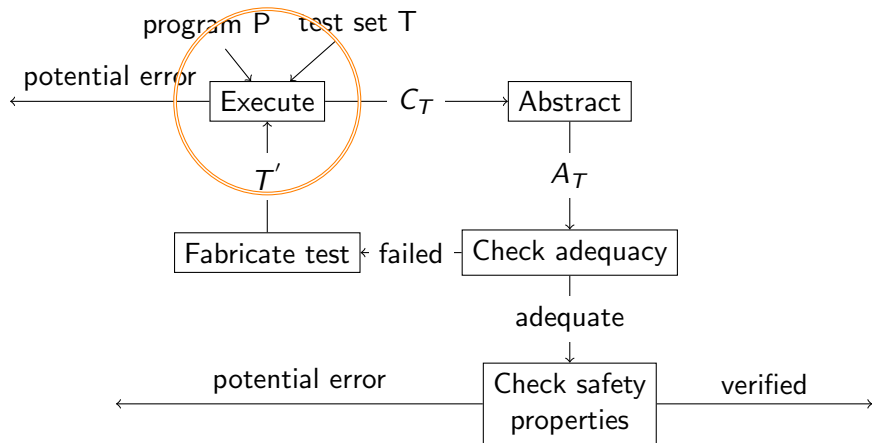
Fabricate tests

(pc=F, x=4, y=0, px=NULL)

(pc=G, x=4, y=0, px=NULL)

Remark: Both tests are **not**  
reachable,  
since  $px = \&x \Leftrightarrow x < 5$ .

# Execute



# Execute

(pc=F, x=4, y=0, px=NULL)

---

**Algorithm:** fixedFoo(int x, int y)

---

```
I  int * px = NULL;
A  x = x + 1;
B  if x < 5 then
C  |   px = &x;
D  if px == &y then
E  |   x = x + 1;
F  if x < 5                // (pc=F, x=4, y=0, px=NULL)
   then
G  |   *px = *px + 1;      // (pc=G, x=4, y=0, px=NULL)
```

---

# Execute

(pc=F, x=4, y=0, px=NULL)

---

**Algorithm:** fixedFoo(int x, int y)

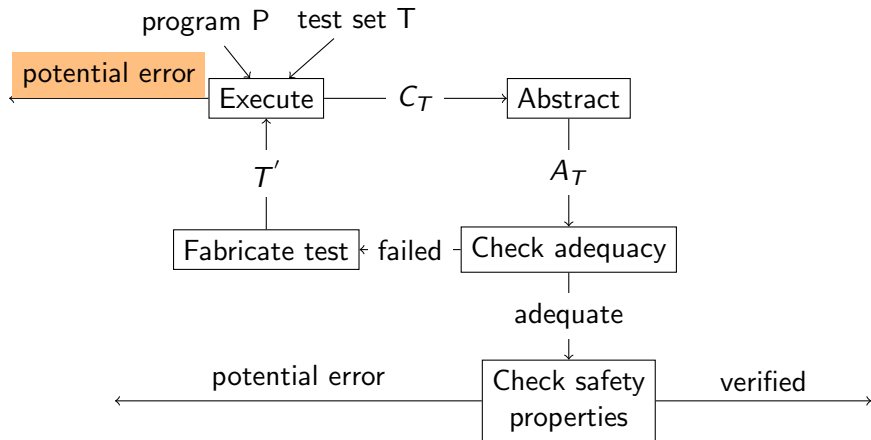
---

```
I int * px = NULL;
A x = x + 1;
B if x < 5 then
C |   px = &x;
D if px == &y then
E |   x = x + 1;
F if x < 5 // (pc=F, x=4, y=0, px=NULL)
  then
G |   *px = *px + 1; // (pc=G, x=4, y=0, px=NULL)
```

⇒ Null pointer dereference error

---

# False error



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# Conclusion

- Given an abstraction function and safety property the procedure can proof the absence of errors in programs.
- Approach cannot distinguish between false and real errors like a dynamic approach.
- Idea of presented approach: Combining the pros of dynamic and static program analysis.

# Conclusion

- Given an abstraction function and safety property the procedure can proof the absence of errors in programs.
- Approach cannot distinguish between false and real errors like a dynamic approach.
- Idea of presented approach: Combining the pros of dynamic and static program analysis.

Is this true ?



# Conclusion question

