

# *Software Design, Modelling and Analysis in UML*

## *Lecture 14: Core State Machines IV*

2016-01-12

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### *Contents & Goals*

#### **Last Lecture:**

- Transitions by Rule (i) to (v).

#### **This Lecture:**

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is a step / run-to-completion step?
  - What is divergence in the context of UML models?
  - How to define what happens at “system / model startup”?
  - What are roles of OCL constraints in behavioural models?
  - Is this UML model consistent with that OCL constraint?
  - What do the actions create / destroy do? What are the options and our choices (why)?
- **Content:**
  - Step / RTC-Step revisited, Divergence
  - Initial states
  - Missing pieces: create / destroy transformer
  - A closer look onto code generation
  - Maybe: hierarchical state machines

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## Step and Run-to-Completion

### Notions of Steps: The Step

**Note:** we call one evolution

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$$

a **step**.

*in case of rules (i) + (ii)*

Thus in our setting, a **step directly corresponds** to

**one object** (namely  $u$ ) taking a **single transition** between regular states.

(We will extend the concept of "single transition" for hierarchical state machines.)

**That is:** We're going for an interleaving semantics without true parallelism.

## Notions of Steps: The Run-to-Completion Step

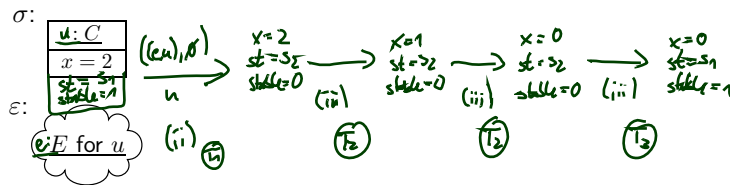
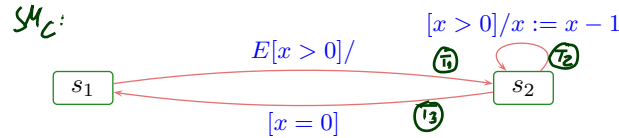
What is a **run-to-completion** step...?

- **Intuition:** a **maximal** sequence of steps of one object, where the first step is a **dispatch** step, all later steps are **continue** steps, and the last step establishes stability (or object disappears).

**Note:** while one step corresponds to one transition in the state machine, a run-to-completion step is in general **not syntactically definable**:

one transition may be taken multiple times during an RTC-step.

**Example:**



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## Notions of Steps: The Run-to-Completion Step Cont'd

**Proposal:** Let

$$(\sigma_0, \epsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} \dots \xrightarrow[u_{n-1}]{(cons_{n-1}, Snd_{n-1})} (\sigma_n, \epsilon_n), \quad n > 0,$$

be a finite (!), non-empty, **maximal**, consecutive sequence **such that**

- $(cons_0, Snd_0)$  indicates dispatching to  $u := u_0$  (by Rule (ii)  $\alpha(i)$ )  
i.e.  $cons = \{u_E\}$ ,  $u_E \in \text{dom}(\sigma_0) \cap \mathcal{D}(\mathcal{E})$ ,
- if  $u$  becomes stable or disappears, then in the last step, i.e.

$$\forall i > 0 \bullet (\sigma_i(u)(stable) = 1 \vee u \notin \text{dom}(\sigma_i)) \implies i = n$$

Let  $0 = k_1 < k_2 < \dots < k_N < n$  be the maximal sequence of indices such that  $u_{k_i} = u$  for  $1 \leq i \leq N$ . Then we call the sequence

$$(\sigma_0(u) =) \sigma_{k_1}(u), \sigma_{k_2}(u), \dots, \sigma_{k_N}(u), \sigma_n(u)$$

a (!) **run-to-completion step** of  $u$  (from (local) configuration  $\sigma_0(u)$  to  $\sigma_n(u)$ ).

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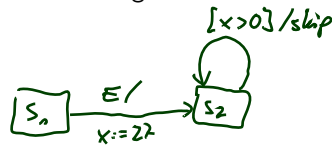
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## Divergence

We say, object  $u$  **can diverge** on reception  $cons_0$  from (local) configuration  $\sigma_0(u)$  if and only if there is an **infinite**, consecutive sequence

$$(\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \xrightarrow[u_1]{(cons_1, Snd_1)} \dots$$

where  $u_i = u$  for infinitely many  $i \in \mathbb{N}_0$  and  $\sigma_i(u)(stable) = 0$ ,  $i > 0$ , i.e.  $u$  does not become stable again.



## Run-to-Completion Step: Discussion.

Our definition of RTC-step takes a **global** and **non-compositional** view, that is:

- In the projection onto a single object we still **see** the effect of interaction with other objects.
- Adding classes (or even objects) may change the divergence behaviour of existing ones.
- Compositional would be: the behaviour of a set of objects is determined by the behaviour of each object "in isolation".

Our semantics and notion of RTC-step doesn't have this (often desired) property.

Can we give (syntactical) criteria such that any (global) run-to-completion step is an interleaving of local ones?

**Maybe: Strict interfaces.**

(Proof left as exercise...)

- **(A)**: Refer to private features only via "self".  
(Recall that other objects of the same class can modify private attributes.)
- **(B)**: Let objects only communicate by events, i.e. don't let them modify each other's local state via links **at all**.

## Putting It All Together

### Initial States

**Recall:** a labelled transition system is  $(S, A, \rightarrow, S_0)$ . We **have**

- $S$ : system configurations  $(\sigma, \varepsilon)$
- $\rightarrow$ : labelled transition relation  $(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$ .

**Wanted:** initial states  $S_0$ .

**Proposal:**

Require a (finite) set of **object diagrams**  $\mathcal{OD}$  as part of a UML model

$$(\mathcal{CD}, \mathcal{LM}, \mathcal{OD}).$$

And set

$$S_0 = \{(\sigma, \varepsilon) \mid \sigma \in G^{-1}(\mathcal{OD}), \quad \mathcal{OD} \in \mathcal{OD}, \quad \varepsilon \text{ empty}\}.$$

**Other Approach:** (used by Rhapsody tool) multiplicity of classes (plus initialisation code).  
We can read that as an abbreviation for an object diagram.

## Semantics of UML Model (So Far)

The **semantics** of the **UML model**

$$\mathcal{M} = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$$

where

- some classes in  $\mathcal{CD}$  are stereotyped as 'signal' (standard), some signals and attributes are stereotyped as 'external' (non-standard),
- there is a 1-to-1 relation between classes and state machines,
- $\mathcal{OD}$  is a set of object diagrams over  $\mathcal{CD}$ ,

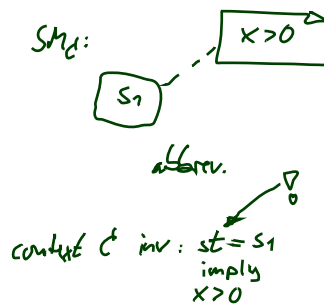
is the **transition system**  $(S, A, \rightarrow, S_0)$  constructed on the previous slide(s).

The **computations of  $\mathcal{M}$**  are the computations of  $(S, A, \rightarrow, S_0)$ .

## OCL Constraints and Behaviour

- Let  $\mathcal{M} = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$  be a UML model.
- We call  $\mathcal{M}$  **consistent** iff, for each OCL constraint  $expr \in Inv(\mathcal{CD}) \cup Inv(\mathcal{SM})$   
 $\sigma \models expr$  for each “reasonable point”  $(\sigma, \varepsilon)$  of computations of  $\mathcal{M}$ .  
(Cf. exercises and tutorial for discussion of “reasonable point”.)

**Note:** we could define  $Inv(\mathcal{SM})$  similar to  $Inv(\mathcal{CD})$ .



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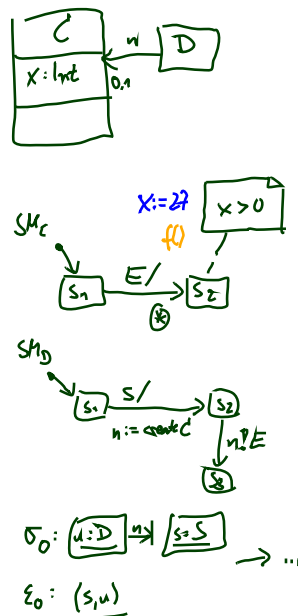
### Pragmatics:

- In **UML-as-blueprint mode**, if  $\mathcal{SM}$  doesn't exist yet, then  $\mathcal{M} = (\mathcal{CD}, \emptyset, \mathcal{OD})$  is typically asking the developer to provide  $\mathcal{SM}$  such that  $\mathcal{M}' = (\mathcal{CD}, \mathcal{SM}, \mathcal{OD})$  is consistent.

If the developer makes a mistake, then  $\mathcal{M}'$  is inconsistent.

*(and not completely uncommon)*

- Not common:** if  $\mathcal{SM}$  is given, then constraints are also considered when choosing transitions in the RTC-algorithm. In other words: even in presence of mistakes, the  $\mathcal{SM}$  never move to inconsistent configurations.



① model is "broken"

② model behaviour considers constraints,  
 (\*) would not be taken if  $x \leq 0$  after creation

## Last Missing Piece: Create and Destroy Transformer

### Transformer: Create

|   |                                  |
|---|----------------------------------|
| <b>abstract syntax</b>  | <b>concrete syntax</b>           |
| $\text{create}(C, \text{expr}, v)$  | $\text{expr}.v := \text{new } C$ |
| <b>intuitive semantics</b>  |                                  |
| Create an object of class $C$ and assign it to attribute $v$ of the object denoted by expression $\text{expr}$ .                  |                                  |
| <b>well-typedness</b>   |                                  |
| $\text{expr} : T_D, v \in \text{atr}(D), v = C_{0,1},$<br>$\text{atr}(C) = \{(v_1 : T_1, \text{expr}_i^0) \mid 1 \leq i \leq n\}$ |                                  |
| <b>semantics</b>  |                                  |
| ...   |                                  |
| <b>observables</b>  |                                  |
| ...   |                                  |
| <b>(error) conditions</b>   |                                  |
| $I[\![\text{expr}]\!](\sigma, \beta)$ not defined.  |                                  |

instead  
 $x := (\text{new } C).y + (\text{new } D).z;$   
 write  
 $\text{temp}_1 := \text{new } C;$   
 $\text{temp}_2 := \text{new } D;$   
 $x := \text{temp}_1.y + \text{temp}_2.z;$



## Transformer: Create

| abstract syntax   | concrete syntax                                    |
|---|--|
| $\text{create}(C, \text{expr}, v)$  |  |
| <b>intuitive semantics</b>  |  |
| Create an object of class $C$ and assign it to attribute $v$ of the object denoted by expression $\text{expr}$ .  |  |
| <b>well-typedness</b>   |  |
| $\text{expr} : T_D, v \in \text{atr}(D),$ $\text{atr}(C) = \{(v_1 : T_1, \text{expr}_i^0) \mid 1 \leq i \leq n\}$ |  |
| <b>semantics</b>  | ...  |
| <b>observables</b>  | ...  |
| <b>(error) conditions</b>   | $I[\![\text{expr}]\!](\sigma, \beta)$ not defined. |

- We use an “and assign”-action for simplicity — it doesn’t add or remove expressive power, but moving creation to the expression language raises all kinds of other problems since then expressions would need to modify the system state.
- Also for simplicity: no parameters to construction ( $\sim$  parameters of constructor). Adding them is straightforward (but somewhat tedious).

## How To Choose New Identities?

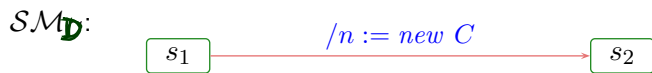
- **Re-use**: choose any identity that is not alive **now**, i.e. not in  $\text{dom}(\sigma)$ .
  - Doesn’t depend on history.
  - May “undangle” dangling references – may happen on some platforms.
- **Fresh**: choose any identity that has not been alive **ever**, i.e. not in  $\text{dom}(\sigma)$  and any predecessor in current run.
  - Depends on history.
  - Dangling references remain dangling – could mask “dirty” effects of platform.

# Transformer: Create

|  |                        |
|--|------------------------|
| <b>abstract syntax</b>   | <b>concrete syntax</b> |
| $\text{create}(C, \text{expr}, v)$   |                        |
| <b>intuitive semantics</b>   |                        |
| Create an object of class $C$ and assign it to attribute $v$ of the object denoted by expression $\text{expr}$ .   |                        |
| <b>well-typedness</b>  |                        |
| $\text{expr} : T_D, v \in \text{atr}(D),$<br>$\text{atr}(C) = \{\langle v_1 : T_1, \text{expr}_i^0 \rangle \mid 1 \leq i \leq n\}$   |                        |
| <b>semantics</b>   |                        |
| $((\sigma, \varepsilon), (\sigma', \varepsilon')) \in t_{\text{create}(C, \text{expr}, v)[u_x]$  |                        |
| iff  |                        |
| $\sigma' = \sigma[u_0 \mapsto \sigma(u_0)[v \mapsto \{u\}] \cup \{u \mapsto \{v_i \mapsto d_i \mid 1 \leq i \leq n\}\},$   |                        |
| $\varepsilon' = [u](\varepsilon); u \in \mathcal{D}(C)$ fresh, i.e. $u \notin \text{dom}(\sigma);$   |                        |
| $u_0 = I[\text{expr}](\sigma, u_x); d_i = I[\text{expr}_i^0](\sigma, \emptyset)$ if $\text{expr}_i^0 \neq \downarrow$ and arbitrary value from $\mathcal{D}(T_i)$ otherwise. |                        |
| <b>observables</b>   |                        |
| $\text{Obs}_{\text{create}[u_x]} = \{(*, u)\}$   |                        |
| <b>(error) conditions</b>  |                        |
| $I[\text{expr}](\sigma, u_x)$ not defined.   |                        |

similar to update  
similar to send  
clean exits  
(do it on slide 20!)

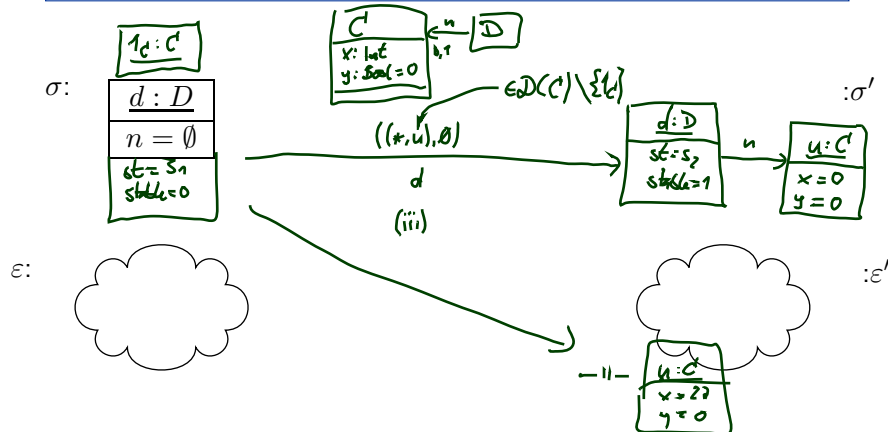
# Create Transformer Example



$\text{create}(C, \text{expr}, v)$

$t_{\text{create}(C, \text{expr}, v)[u_x](\sigma, \varepsilon) = \dots$

$\mathcal{D}(\text{int}) = \mathbb{Z}$



## Transformer: Destroy

|                            |   |
|----------------------------|---|
| <b>abstract syntax</b>     | <b>concrete syntax</b>  |
| $\text{destroy}(expr)$     |   |
| <b>intuitive semantics</b> | Destroy the object denoted by expression $expr$ .                 |
| <b>well-typedness</b>      | $expr : T_C, C \in \mathcal{C}$                                   |
| <b>semantics</b>           | ...   |
| <b>observables</b>         | $Obs_{\text{destroy}}[u_x] = \{(u_x, \perp, (+, \emptyset), u)\}$ |
| <b>(error) conditions</b>  | $I[[expr]](\sigma, \beta)$ not defined.                           |

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## What to Do With the Remaining Objects?

Assume object  $u_0$  is destroyed...

- object  $u_1$  may still refer to it via association  $r$ :
  - allow dangling references?
  - or remove  $u_0$  from  $\sigma(u_1)(r)$ ?
- object  $u_0$  may have been the last one linking to object  $u_2$ :
  - leave  $u_2$  alone?
  - or remove  $u_2$  also? (garbage collection)
- Plus: (temporal extensions of) OCL may have dangling references.

**Our choice:** Dangling references and no garbage collection!

This is in line with “expect the worst”, because there are target platforms which don’t provide garbage collection — and models shall (in general) be correct without assumptions on target platform.

**But:** the more “dirty” effects we see in the model, the more expensive it often is to analyse. Valid proposal for simple analysis: monotone frame semantics, no destruction at all.

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## Transformer: Destroy

| abstract syntax   | concrete syntax |
|---|-----------------|
| $\text{destroy}(expr)$  |                 |
| <b>intuitive semantics</b>  |                 |
| Destroy the object denoted by expression $expr$ .   |                 |
| <b>well-typedness</b>   |                 |
| $expr : TC, C \in \mathcal{C}$  |                 |
| <b>semantics</b>  |                 |
| $t_{\text{destroy}(expr)}[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon')\}, \quad \varepsilon' = [u](\varepsilon)$<br>where $\sigma' = \sigma _{\text{dom}(\sigma) \setminus \{u\}}$ with $u = I[expr](\sigma, u_x)$ . |                 |
| <b>observables</b>  |                 |
| $Obs_{\text{destroy}(expr)}[u_x] = \{(+, u)\}$  |                 |
| <b>(error) conditions</b>   |                 |
| $I[expr](\sigma, u_x)$ not defined.   |                 |

## Hierarchical State-Machines

# The Full Story

UML distinguishes the following **kinds of states**:

|                        | example |                         | example |
|------------------------|---------|-------------------------|---------|
| <b>simple state</b>    |         | <b>pseudo-state</b>     |         |
| <b>final state</b>     |         |                         |         |
| <b>composite state</b> |         |                         |         |
| OR                     |         |                         |         |
| AND                    |         |                         |         |
|                        |         | <b>submachine state</b> |         |

## References

## *References*

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OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.