A Fixpoint Antichain Algorithm

A faster algorithm to check universality of NFA

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Seminar on Automata Theory at the chair of Software Engineering.
Winter semester 2016/2017

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Universality of NFA

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- An NFA $\mathscr{A} = (Loc, Init, Fin, \delta, \Sigma)$ is universal $\Leftrightarrow L(\mathscr{A}) = \Sigma^*$
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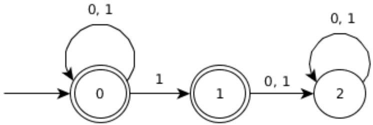

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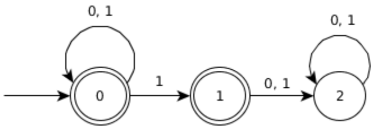

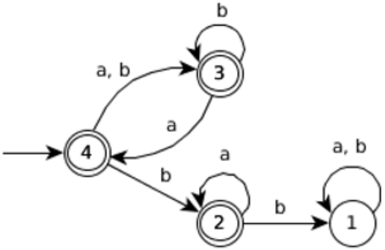

Content

Classical subset construction algorithm

- Consider NFA $\mathscr{A}$ with n states.
- Build corresponding DFA $\mathscr{A}'$ with 2^n states.
- Traverse the DFA $\mathscr{A}'$ starting in $\{Init\}$.
- If a non accepting state is found, $\mathscr{A}'$ hence $\mathscr{A}$ is **not** universal.
- **Problem:** Exponential blow-up of the set of states.

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$cpre^{\mathscr{A}}_{\sigma}(s)$ exclusive predecessors of a state set

Definition

Consider NFA $\mathscr{A} = (Loc, Init, Fin, \delta, \Sigma)$
For $s \subseteq Loc$ we define:

$$cpre^{\mathscr{A}}_{\sigma}(s) = \{l \in Loc \,|\, \forall l' \in Loc : \delta(l, \sigma, l') \Rightarrow l' \in s\}$$

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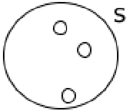

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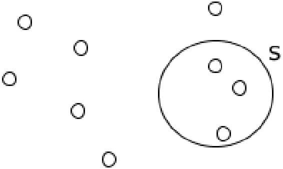

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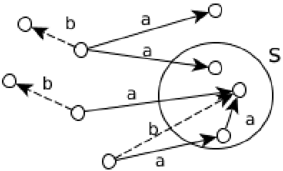

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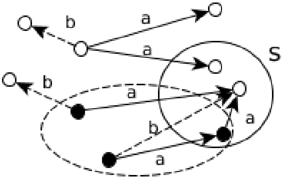

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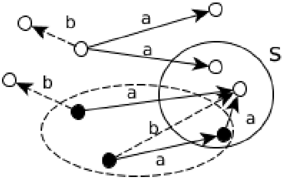
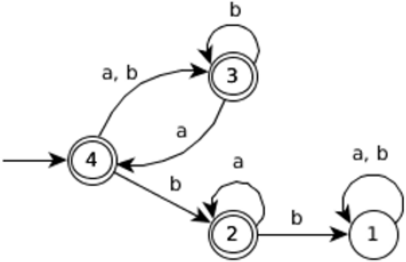


Thus $cpre^{\mathscr{A}}_{a}(s)$ contains all states that with letter a have a transition to some state in s and nowhere else.

$cpre_{\sigma}^{\mathscr{A}}(s)$ and $post_{\sigma}^{\mathscr{A}}(s)$



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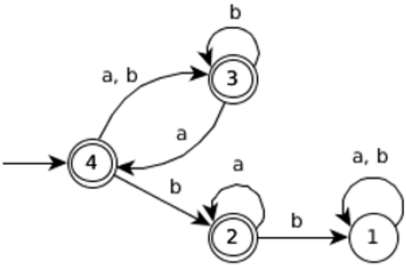

Example $cpre_\sigma^{\mathscr{A}}(s)$:

- $cpre_a^{\mathscr{A}}(\{1\}) = \{1\}$

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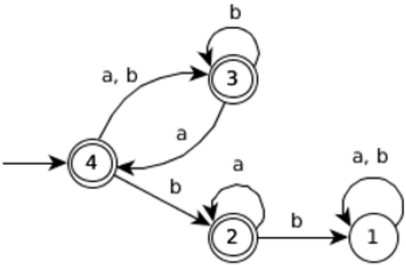

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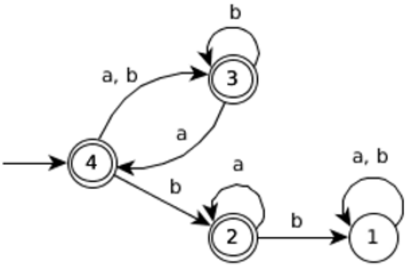

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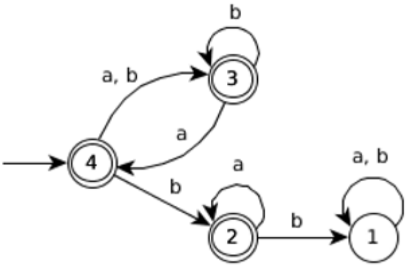

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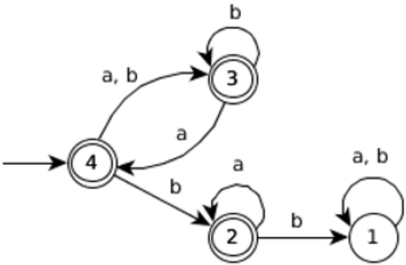

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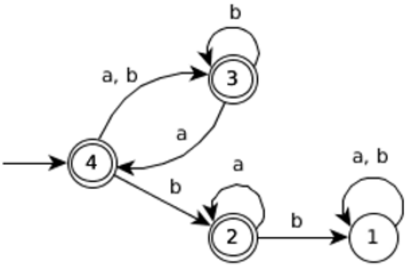

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Content

A partial order $\sqsubseteq$ on Antichains

Definition

Let L denote the set of all antichains over 2^{Loc}

$$\forall q, q' \in L : q \sqsubseteq q' \Leftrightarrow \forall s \in q \, \exists s' \in q' : s \subseteq s'$$

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- $\{\{1\},\{2\},\{3\}\} \sqsubseteq \{\{1,2\},\{2,3\}\}$

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For two antichains $q, q' \in L$ the least upper bound (lub) is:

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- Lattice property is needed later on for correctness of the algorithm

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Monotone function on antichains $CPre^{\mathscr{A}}(q)$

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The concept of predecessors is extended to antichains by:

$$CPre^{\mathscr{A}} : L \rightarrow L$$

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- follows from subset inclusion order and Def. of $cpre_{\sigma}^{\mathscr{A}}(s)$

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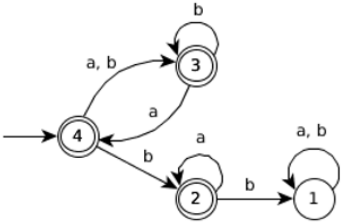

Example: $CPre^{\mathscr{A}}(\{\{1\}\})$

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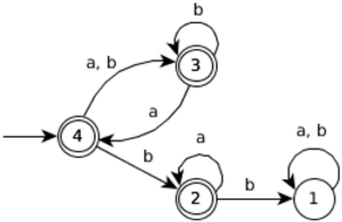

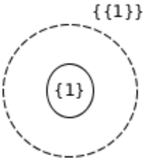

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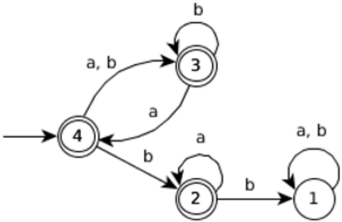

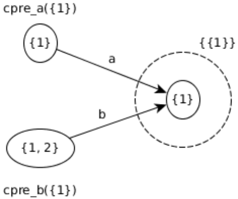

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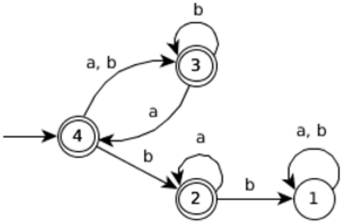

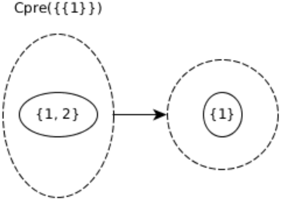

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- we start with the antichain $\{\{1\}\}$
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- $CPre^{\mathscr{A}}(\{\{1\}\}) = Max(\{\{1,2\},\{1\}\}) = \{\{1,2\}\}$

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The general idea

- Start with antichain $F = \{\overline{Fin}\}$ and set $Frontier = F$
- Repeatedly compute $F = F \sqcup CPre^{\mathscr{A}}(Frontier)$ in a loop
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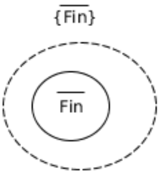
- Thus after some iteration n, F stops growing, i.e. $F_n = F_{n-1}$
- Iff $\{Init\} \sqsubseteq F$ $\mathscr{A}$ is not universal.

Algorithm 0



Initialization

- We start with the antichain of the set of non accepting states

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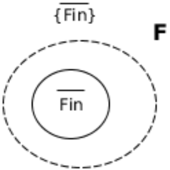

Initialization

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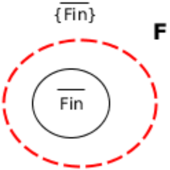

Initialization

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- *Frontier* $\leftarrow F$

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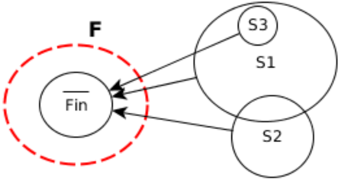

First Iteration

- s_1, s_2, s_3 are $cpre_\sigma(s)$ for all σ and all $s \in Frontier$

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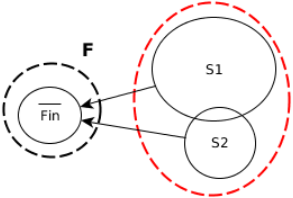

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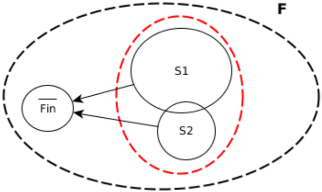

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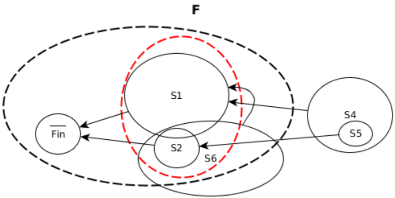
- $F \leftarrow F \sqcup Frontier$

Algorithm 2



Second Iteration

- s_4, s_5, s_6 are $cpre(s)$ for all σ and all $s \in Frontier$

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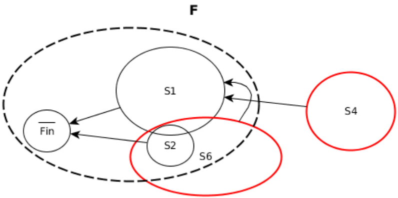

Second Iteration

- s_4, s_5, s_6 are $cpre(s)$ for all σ and all $s \in Frontier$
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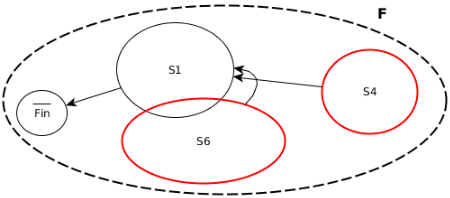
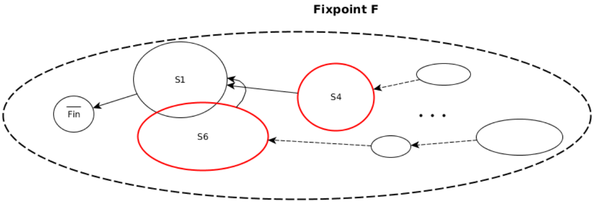


Second Iteration

- s_4, s_5, s_6 are $cpre(s)$ for all σ and all $s \in Frontier$
- $Frontier = CPre^{\mathscr{A}}(Frontier) = \{s_4, s_6\}$
- $F \leftarrow F \sqcup Frontier$

Algorithm Termination



Termination

- The Algorithm computes a series of antichains $q_0 \sqsubseteq q_1 \sqsubseteq \cdots \sqsubseteq q_n = \mathscr{F}$ where $q_i = CPre^{\mathscr{A}}(q_{i-1}) \sqcup \{\overline{Fin}\}$

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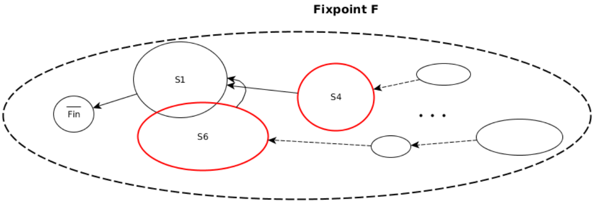

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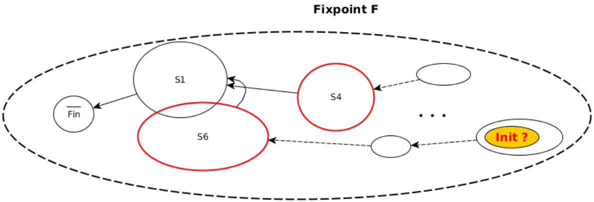

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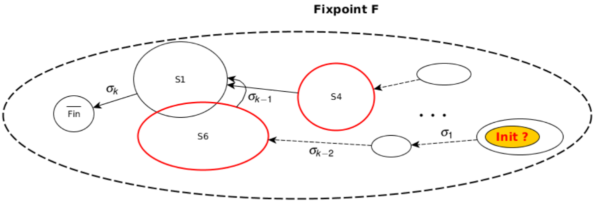

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Content

Comparison of Classical and Antichain Algorithm

Theorem

For the familiy of $\mathscr{A}_k$, $k \geq 2$ with $k+1$ states, the Backward Antichain Algorithm is polynomial in k, whereas the classical subset construction algorithm is exponential in k

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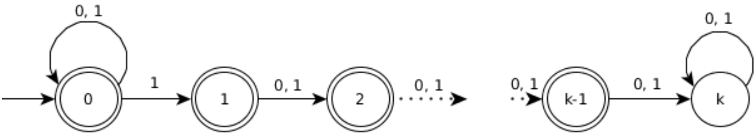
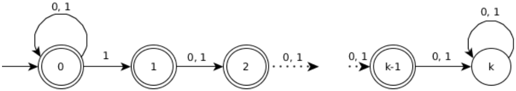


Classical



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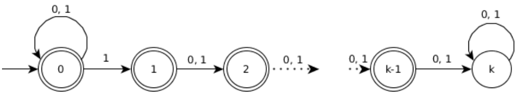

Classical

- DFA of 2^{k+1} states

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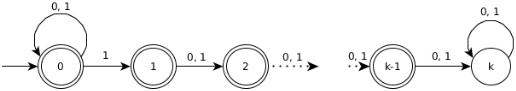

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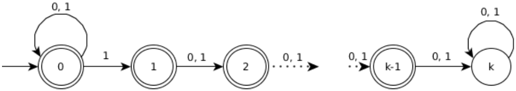

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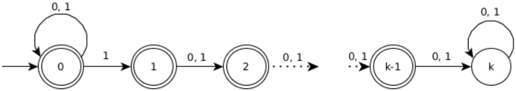

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Comparison of Classical and Antichain Algorithm

Antichain Algorithm

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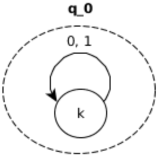

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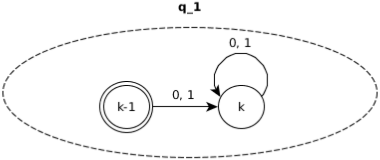

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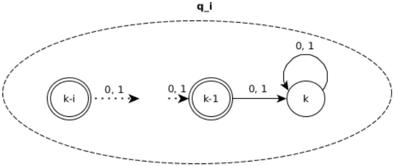

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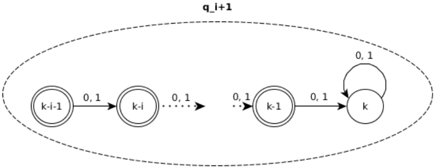

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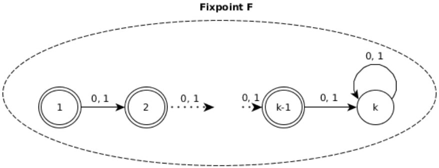

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- The computation of the $CPre()$ in each iteration takes linear time

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- Empirical comparisons of antichain and classical algorithm on randomly generated NFA show, that antichain is up to 200 times faster.
- The higher the density of accepting states the more advantageous is the antichain approach.
- Antichain algorithms are also applied to other problems like language inclusion

References

- M. De Wulf, L. Doyen, T. A. Henzinger, and J.-F. Raskin. (2006). Antichains: A new algorithm for checking universality of finite automata. In Proc. of CAV: Computer Aided Verification, LNCS 4144, pages 17-30. Springer, 2006.
- L. Doyen and J.-F. Raskin. (2010). Antichain Algorithms for Finite Automata. In Proc. of TACAS: Tools and Algorithms for the Construction and Analysis of Systems, LNCS 6015, pages 2-22. Springer, 2010.