

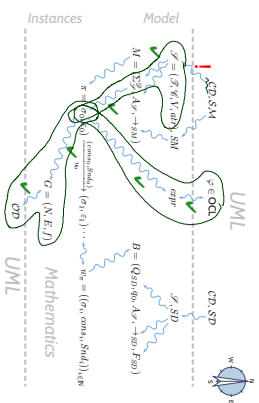
# Software Design, Modelling and Analysis in UML

## Lecture 6: Class Diagrams I

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Prof. Dr. Andreas Poddick, Dr. Bernd Westphal  
Albert-Ludwigs-Universität Freiburg, Germany

### Course Map



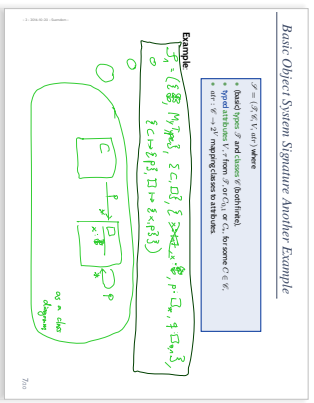
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### Content

- Stocktaking
- Extended Signatures
- Structures for Extended Signatures
- Semantically Relevant
- Mapping Class Diagrams to Extended Signatures
- What if things are missing?
- (Temporary) Abbreviations
- Stereotypes

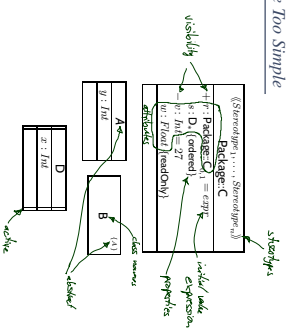
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### Recall: Signature vs. Class Diagram



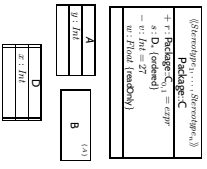
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### That'd Be Too Simple



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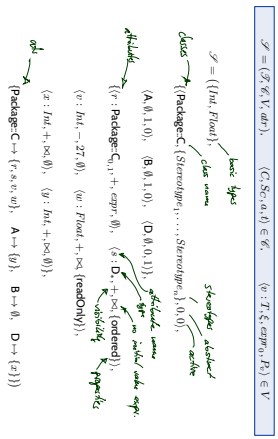
- A class**
- has a set of stereotypes;
  - has a name.
  - (belongs to a package)
  - can be abstract
  - can be active
  - has a set of attributes.
  - has a set of operations. ( $\leftarrow$ ,  $\leftarrow$ ,  $\leftarrow$ )
- Each attribute has
- a visibility,
  - a name, a type,  $\leftarrow$ ,  $\leftarrow$
  - a multiplicity, an order,
  - an initial value, and
  - a set of properties, such as **readOnly**, **ordered**, etc.



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**Warning:** places in the signature to represent the information from the picture.

Extended Signature Example



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Extended Signature

- We write  $(C: S_C, a, b)$  if we want to refer to all aspects of class C.
- If the new aspects are irrelevant (for a given context), we simply write  $C$  (i.e. old definitions written in terms of  $C$ ) are still valid
- Similarly, we write  $(c: T, \xi, expr_0, P_1)$  if we want to refer to all aspects of attribute  $c$ .
- While only  $c: T$  or  $a$  if details are irrelevant.

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Conventions

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**Definition:** An (Extended) Object System Signature is a quadruple  $\mathcal{S} = (\mathcal{C}, \mathcal{A}, \mathcal{V}, \text{attr})$  where

- $\mathcal{C}$  is a finite set of classes  $(C: S_C, a, b)$  where
- $\mathcal{V}$  is a finite set of basic types,
- $S_C$  is a finite (possibly empty) set of stereotypes,
- $a \in \mathcal{A}$  is a boolean flag indicating whether  $C$  is abstract. ( $a = 1$  if  $C$  is abstract)
- $b \in \mathcal{B}$  is a boolean flag indicating whether  $C$  is active,
- $V$  is a finite set of attributes  $(c: T, \xi, expr_0, P_1)$  where
- $T$  is a type from  $\mathcal{V}$  or  $C_0, 1, C_1$  for some  $C \in \mathcal{C}$ ,
- $\xi \in \{\text{public, private, protected, package}\}$  is the visibility,
- an initial value expression  $expr_0$  given as a word from a language for initial value expressions, e.g. OCL, or C++ in the Rhapsody tool,
- $P_1$  is a finite (possibly empty) set of properties  $P_i$ ,
- $\text{attr}: \mathcal{C} \rightarrow 2^{\mathcal{V}}$  maps each class to its set of attributes.

We use  $S_C$  to denote the set  $\bigcup_{C \in \mathcal{C}} S_C$  of stereotypes in  $\mathcal{S}$ .

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Structures of Extended Signatures

**Recall:**

**Definition:** A Basic Object System Structure of a Basic Object System Signature  $\mathcal{S} = (\mathcal{C}, \mathcal{A}, \mathcal{V}, \text{attr})$  is a domain function  $\mathcal{D}$  which assigns to each type a domain, i.e.

- $T \in \mathcal{V}$  is mapped to  $\mathcal{D}(T)$ ,
- $C \in \mathcal{C}$  is mapped to an infinite set  $\mathcal{D}(C)$  of object identities.

Note: Object identities only have the "=" operation

- Sets of object identities for different classes are disjoint, i.e.

$$\forall C, D \in \mathcal{C}: C \neq D \rightarrow \mathcal{D}(C) \cap \mathcal{D}(D) = \emptyset$$

- $C_0$  and  $C_{0,1}$  for  $C \in \mathcal{C}$  are mapped to  $2^{\mathcal{D}(C)}$ .

We use  $\mathcal{D}(c)$  to denote  $\bigcup_{C \in \mathcal{C}} \mathcal{D}(C)$ , analogously  $\mathcal{D}(K_1)$ .

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New:

**Definition.** An (Object System) Structure of an (Extended Object System) Signature  $\mathcal{S} = (\mathcal{C}, \mathcal{V}, \text{act})$  is a domain function  $\mathcal{D}$  which assigns to each type a domain, i.e.

- $\tau \in \mathcal{D}$  is mapped to  $\mathcal{D}(\tau)$ .
- $C \in \mathcal{C}$  is mapped to an infinite set  $\mathcal{D}(C)$  of (object) identities.
- Note: Object identities only have the "=" operation.
- Sets of object identities for different classes are disjoint, i.e.  $\forall C, D \in \mathcal{C} : C \neq D \Rightarrow \mathcal{D}(C) \cap \mathcal{D}(D) = \emptyset$ .
- $C_1$  and  $C_{n1}$  for  $C \in \mathcal{C}$  are mapped to  $2^{\mathcal{D}(C)}$ .
- We use  $\mathcal{D}(\mathcal{C})$  to denote  $\bigcup_{C \in \mathcal{C}} \mathcal{D}(C)$ ; analogously  $\mathcal{D}(\mathcal{V})$ .

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Recall:

**Definition.** Let  $\mathcal{D}$  be a **basic structure of basic signature**  $\mathcal{S} = (\mathcal{C}, \mathcal{V}, \text{act})$ . A **system state of  $\mathcal{S}$**  wrt.  $\mathcal{D}$  is a **type-consistent mapping**

$$\sigma : \mathcal{D}(\mathcal{V}) \rightarrow (\mathcal{V} \rightarrow (\mathcal{D}(\mathcal{C}) \cup \mathcal{D}(\mathcal{V})))$$

That is, for each  $u \in \mathcal{D}(C)$ ,  $C \in \mathcal{C}$ , if  $u \in \text{dom}(\sigma)$

- $\text{dom}(\sigma(u)) = \text{act}(C)$
- $\sigma(u)(v) \in \mathcal{D}(\tau)$  if  $v : \tau \in \mathcal{C}$
- $\sigma(u)(v) \in \mathcal{D}(D)$  if  $v : D_{n1}$  or  $v : D$ , with  $D \in \mathcal{C}$ .

We call  $u \in \mathcal{D}(\mathcal{C})$  **alive in  $\sigma$**  and **only if**  $u \in \text{dom}(\sigma)$ .

We use  $\Sigma_{\mathcal{D}}^{\mathcal{S}}$  to denote the set of all system states of  $\mathcal{S}$  wrt.  $\mathcal{D}$ .

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New:

**Definition.** Let  $\mathcal{D}$  be a **structure of extended signature**  $\mathcal{S} = (\mathcal{C}, \mathcal{V}, \text{act})$ . A **system state of  $\mathcal{S}$**  wrt.  $\mathcal{D}$  is **type-consistent mapping**

$$\sigma : \mathcal{D}(\mathcal{V}) \rightarrow (\mathcal{V} \rightarrow (\mathcal{D}(\mathcal{C}) \cup \mathcal{D}(\mathcal{V})))$$

That is, for each  $u \in \mathcal{D}(C)$ ,  $C \in \mathcal{C}$ , if  $u \in \text{dom}(\sigma)$

- $\text{dom}(\sigma(u)) = \text{act}(C)$
- $\sigma(u)(v) \in \mathcal{D}(\tau)$  if  $v : \tau \in \mathcal{C}$
- $\sigma(u)(v) \in \mathcal{D}(D)$  if  $v : D_{n1}$  or  $v : D$ , with  $D \in \mathcal{C}$
- $\forall (C, S; i, l) \in \mathcal{C} : \text{dom}(\sigma) \cap \mathcal{D}(C) = \emptyset$

We call  $u \in \mathcal{D}(\mathcal{C})$  **alive in  $\sigma$**  and **only if**  $u \in \text{dom}(\sigma)$ .

We use  $\Sigma_{\mathcal{D}}^{\mathcal{S}}$  to denote the set of all system states of  $\mathcal{S}$  wrt.  $\mathcal{D}$ .

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Semantical Relevance

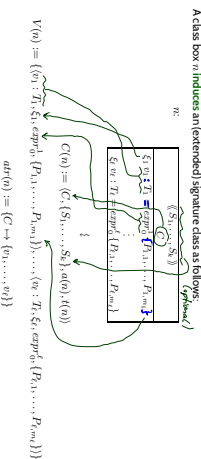
- The semantics for meaning of an extended object system signature  $\mathcal{S}$  wrt. a structure  $\mathcal{D}$  is the set of system states  $\Sigma_{\mathcal{D}}^{\mathcal{S}}$ .
- The semantics for meaning of an extended object system signature  $\mathcal{S}$  is the set of sets of system states wrt. some structure of  $\mathcal{S}$ , i.e. the set  $\{\Sigma_{\mathcal{D}}^{\mathcal{S}} \mid \mathcal{D} \text{ is structure of } \mathcal{S}\}$ .

Which of the following aspects is semantically relevant, i.e. **does contribute** to the constitution of system states?

- Class
- has a set of **ternary** ops. **X**
  - has a **ternary** **X**
  - has **ternary** **operations** **X**
  - can be **active** **X**
  - has a set of **attributes** **X**
  - has a set of **operations** (like)
- Each attribute has
- a **variability** **X**
  - a **arity** type **X**
  - a **multiplicity** **X**
  - a **multiplicity** **X**
  - a set of **properties** **X**
  - such as **readOnly**, **ordered**, etc

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Mapping UML Class Diagrams to Extended Signatures



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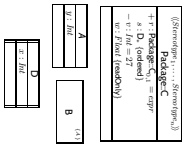
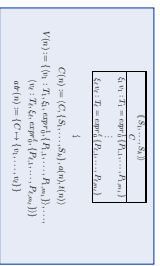
From Class Boxes to Extended Signatures

Where

- 'abstract' is determined by the box:  $\text{act}(n_i) = \{ \text{true} \mid \text{if } n_i \in \square \text{ or } n_i = \square \}$
- 'active' is determined by the frame:  $\text{act}(n_i) = \{ \text{true} \mid \text{if } n_i \in \square \text{ or } n_i = \square \}$

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### Example



$\mathcal{V} = \{ \text{int, float} \}$   
 $\{ \langle \text{Package C}, \text{int} \rangle, \langle \text{Package C}, \text{float} \rangle \}$   
 $\{ \langle \text{Package D}, \text{int} \rangle, \langle \text{Package D}, \text{float} \rangle \}$   
 $\{ \langle \text{Package E}, \text{int} \rangle, \langle \text{Package E}, \text{float} \rangle \}$   
 $\{ \langle \text{Package F}, \text{int} \rangle, \langle \text{Package F}, \text{float} \rangle \}$   
 $\{ \langle \text{Package G}, \text{int} \rangle, \langle \text{Package G}, \text{float} \rangle \}$   
 $\{ \langle \text{Package H}, \text{int} \rangle, \langle \text{Package H}, \text{float} \rangle \}$   
 $\{ \langle \text{Package I}, \text{int} \rangle, \langle \text{Package I}, \text{float} \rangle \}$   
 $\{ \langle \text{Package J}, \text{int} \rangle, \langle \text{Package J}, \text{float} \rangle \}$   
 $\{ \langle \text{Package K}, \text{int} \rangle, \langle \text{Package K}, \text{float} \rangle \}$   
 $\{ \langle \text{Package L}, \text{int} \rangle, \langle \text{Package L}, \text{float} \rangle \}$   
 $\{ \langle \text{Package M}, \text{int} \rangle, \langle \text{Package M}, \text{float} \rangle \}$   
 $\{ \langle \text{Package N}, \text{int} \rangle, \langle \text{Package N}, \text{float} \rangle \}$   
 $\{ \langle \text{Package O}, \text{int} \rangle, \langle \text{Package O}, \text{float} \rangle \}$   
 $\{ \langle \text{Package P}, \text{int} \rangle, \langle \text{Package P}, \text{float} \rangle \}$   
 $\{ \langle \text{Package Q}, \text{int} \rangle, \langle \text{Package Q}, \text{float} \rangle \}$   
 $\{ \langle \text{Package R}, \text{int} \rangle, \langle \text{Package R}, \text{float} \rangle \}$   
 $\{ \langle \text{Package S}, \text{int} \rangle, \langle \text{Package S}, \text{float} \rangle \}$   
 $\{ \langle \text{Package T}, \text{int} \rangle, \langle \text{Package T}, \text{float} \rangle \}$   
 $\{ \langle \text{Package U}, \text{int} \rangle, \langle \text{Package U}, \text{float} \rangle \}$   
 $\{ \langle \text{Package V}, \text{int} \rangle, \langle \text{Package V}, \text{float} \rangle \}$   
 $\{ \langle \text{Package W}, \text{int} \rangle, \langle \text{Package W}, \text{float} \rangle \}$   
 $\{ \langle \text{Package X}, \text{int} \rangle, \langle \text{Package X}, \text{float} \rangle \}$   
 $\{ \langle \text{Package Y}, \text{int} \rangle, \langle \text{Package Y}, \text{float} \rangle \}$   
 $\{ \langle \text{Package Z}, \text{int} \rangle, \langle \text{Package Z}, \text{float} \rangle \}$

### What If Things Are Missing?

It depends.

- What does the standard say? (OMG, 2011a, 120)

**Presentation Options.**

The type visibility default, *multiplicity, property, string* may be suppressed from being displayed, even if there are values in the model.

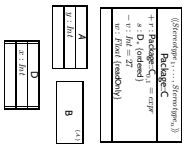
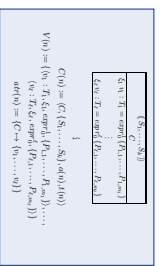
- **Visibility:** There is no "no visibility" – an attribute has a visibility in the extended signature. Some (and we'll assume public) as default, but conventions may vary.

Some (and we'll assume) given by domain (such as "nearest value", but what is "nearest" of Z?)

Some (and we'll understand) non-deterministic initialization if not given.

- **Properties:** probably safe to assume 0 if not given at all.

### Example Cont'd



$\mathcal{V} = \{ \text{int, float} \}$   
 $(\text{Package C}, \{ \text{ShortInteger}, \dots, \text{ShortInteger} \}, 0, 0)$   
 $(A, 0, 1, 0), (B, 0, 1, 0), (D, 0, 0, 1)$   
 $(\langle \text{Package C}, \text{int} \rangle, \langle \text{Package C}, \text{float} \rangle)$   
 $(\langle \text{Package A}, \text{int} \rangle, \langle \text{Package A}, \text{float} \rangle)$   
 $(\langle \text{Package B}, \text{int} \rangle, \langle \text{Package B}, \text{float} \rangle)$   
 $(\langle \text{Package D}, \text{int} \rangle, \langle \text{Package D}, \text{float} \rangle)$   
 $(\langle \text{Package E}, \text{int} \rangle, \langle \text{Package E}, \text{float} \rangle)$   
 $(\langle \text{Package F}, \text{int} \rangle, \langle \text{Package F}, \text{float} \rangle)$   
 $(\langle \text{Package G}, \text{int} \rangle, \langle \text{Package G}, \text{float} \rangle)$   
 $(\langle \text{Package H}, \text{int} \rangle, \langle \text{Package H}, \text{float} \rangle)$   
 $(\langle \text{Package I}, \text{int} \rangle, \langle \text{Package I}, \text{float} \rangle)$   
 $(\langle \text{Package J}, \text{int} \rangle, \langle \text{Package J}, \text{float} \rangle)$   
 $(\langle \text{Package K}, \text{int} \rangle, \langle \text{Package K}, \text{float} \rangle)$   
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 $(\langle \text{Package Y}, \text{int} \rangle, \langle \text{Package Y}, \text{float} \rangle)$   
 $(\langle \text{Package Z}, \text{int} \rangle, \langle \text{Package Z}, \text{float} \rangle)$

### From Class Diagrams to Extended Signatures

- We view a class diagram (CD) as a graph with nodes  $\{n_1, \dots, n_N\}$  (each class rectangle is a node).
- $\mathcal{V}(CD) := \{ \langle n_i, n_j \rangle \mid 1 \leq i \leq N \}$
- $V(CD) := \bigcup_{i=1}^N V(n_i)$
- $dir(CD) := \bigcup_{i=1}^N dir(n_i)$

In a UML model, we can have finitely many class diagrams.

$$\mathcal{V}(\mathcal{D}) = \{ CD_1, \dots, CD_N \}$$

which induce the following signature:

$$\mathcal{V}(\mathcal{D}) = \left( \mathcal{V}(\mathcal{V}(CD_1)), \dots, \mathcal{V}(\mathcal{V}(CD_N)), \bigcup_{i=1}^N dir(CD_i) \right)$$

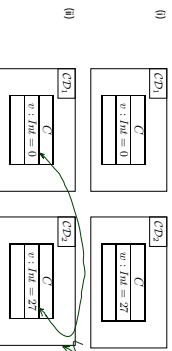
(Assuming  $\mathcal{D}$  given in "reality" (ie. in full UML), we can introduce types in class diagrams; the class diagram then contributes to  $\mathcal{V}$ . Example: enumeration types)

### Is the Mapping a Function?

Question: Is  $\mathcal{V}(\mathcal{V}(\mathcal{D}))$  well-defined?

There are two possible sources for problems:

- (1) A class  $C$  may appear in multiple class diagrams:



Simply forbid the case (1) – easy syntactical check on diagram.

### Is the Mapping a Function?

- (2) An attribute  $v$  may appear in multiple classes with different type:



Two approaches:

- Require unique attribute names. This requirement can easily be established (implicitly, behind the scenes) by viewing  $v$  as an abbreviation for  $\langle C, v \rangle$  or  $\langle D, v \rangle$  ... depending on the context ( $C := \text{float}$  and  $D := \text{int}$  are then unique)
- Slightly formalist's approach: observe that

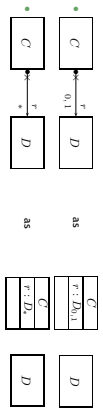
$$\langle v, \text{float} \rangle \quad \text{and} \quad \langle v, \text{int} \rangle$$

$$\langle \langle C, v \rangle, \text{float} \rangle \quad \text{and} \quad \langle \langle D, v \rangle, \text{int} \rangle$$

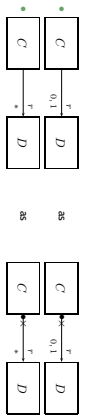
are different things in  $V$ .

## Abbreviations

Since we have not yet discussed **associations**, for now we read

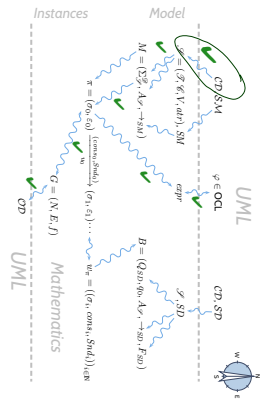


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## Course Map



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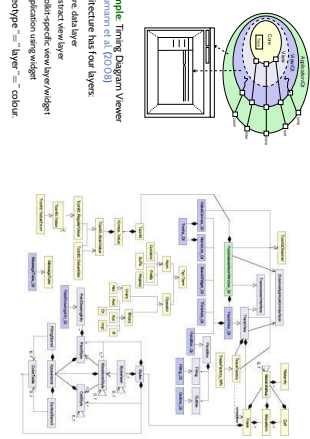
## Stereotypes

### Stereotypes as Labels or Tags

- What are Stereotypes?
- Not represented in system states.
- Not contributing to typing rules / well-formedness.
- Osterweik (2004)
- View stereotypes as (additional) "labeling" ("tags") or as "grouping".
- Useful for documentation and model-driven development, e.g. code-generation.
- Documentation e.g. layers of an architecture.
- Sometimes, packages (cf. OMG (2014b)) are sufficient and "right".
- Model Driven Architecture (MDA) later.

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### Example: Stereotypes for Documentation



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### Other Examples

- Use stereotypes "Team", "Team", "Team", and assign stereotypes Team, to class C. Team is responsible for class C.
- Use stereotypes to label classes with licensing information (e.g. LGPL vs. proprietary).
- Use stereotypes "Server", "Server" to indicate where objects should be stored.
- Use stereotypes to label classes with states in the development process like "under development", "submitted" or "testing", "accepted".
- etc. etc.
- Necessary: a common idea of what each stereotype stands for. (to be defined / agreed on by the team, not the job of the UML consumer.)

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## Tell Them What You've Told Them...

- **Extended Signatures** allow us to represent aspects like
  - abstract, active, visibility, final, value expression, ...
- Not all of these aspects are **semantically relevant**
- The only change on system states is that abstract classes cannot have instances.
- **Class Diagrams map to Extended Signatures:**
  - ie the meaning of a class diagram is the extended signature which it **encapsulates**.
- Thus a Class Diagram (nominatively) denotes a set of system states (given a structure).
- **Stereotypes** are just labels.

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## References

- References**
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