

Software Design, Modelling and Analysis in UML

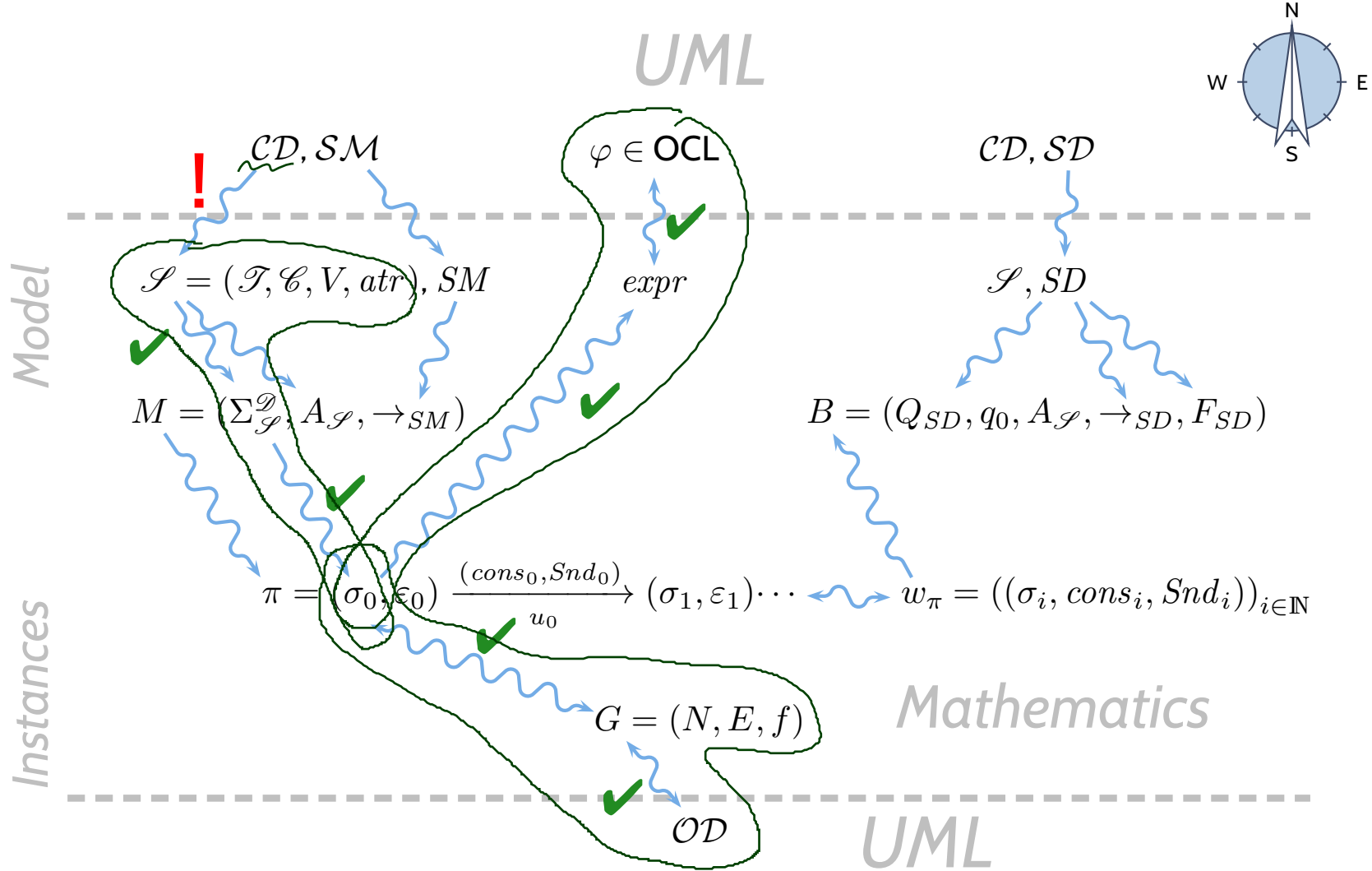
Lecture 6: Class Diagrams I

2016-11-15

Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Course Map



- **Stocktaking**
- **Extended Signatures**
- **Structures** for Extended Signatures
- **Semantically Relevant**
- Mapping **Class Diagrams** to Extended Signatures
- What if things **are missing**?
- (Temporary) Abbreviations
- **Stereotypes**

UML Class Diagrams: Stocktaking

Recall: Signature vs. Class Diagram

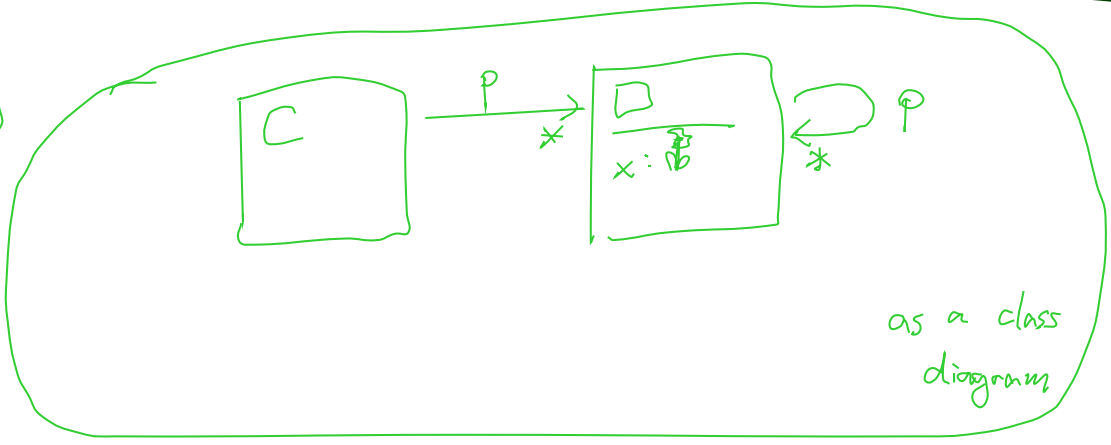
Basic Object System Signature Another Example

$\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ where

- (basic) types \mathcal{T} and classes \mathcal{C} (both finite),
- typed attributes V, τ from \mathcal{T} , or $C_{0,1}$ or C_* , for some $C \in \mathcal{C}$,
- $atr : \mathcal{C} \rightarrow 2^V$ mapping classes to attributes.

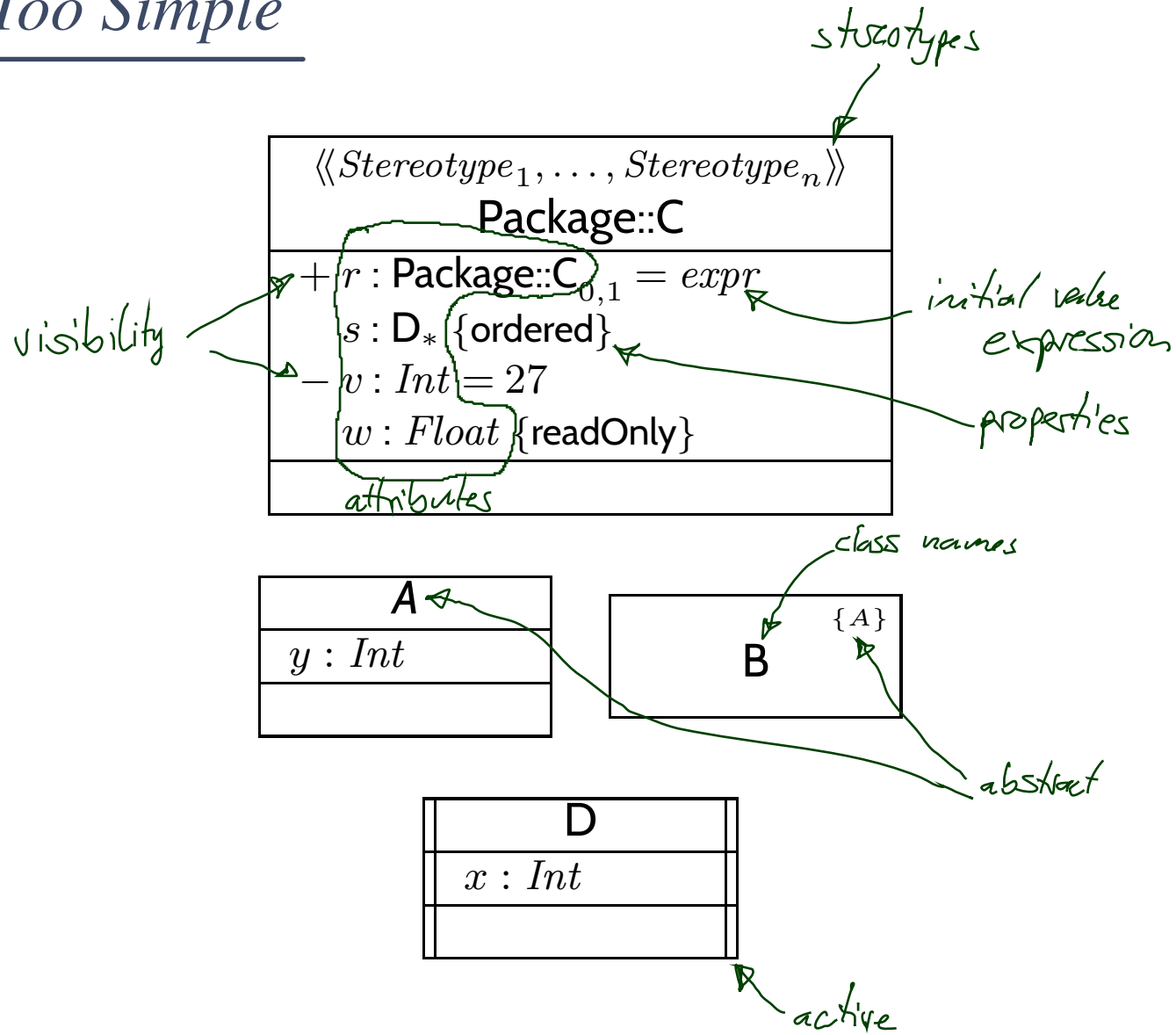
Example:

$\mathcal{S}_1 = (\{\mathbb{B}\}, \{MyType\}, \{C, D\}, \{x: \mathbb{B}, p: D_*, q: D_{0,1}\},$
 $\{C \mapsto \{p\}, D \mapsto \{x, p\}\})$



as a class diagram

That'd Be Too Simple



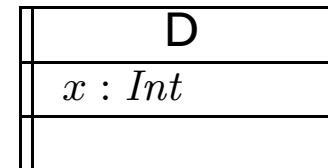
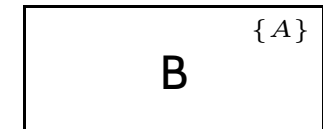
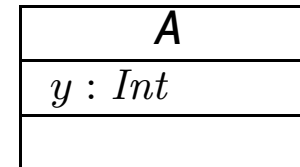
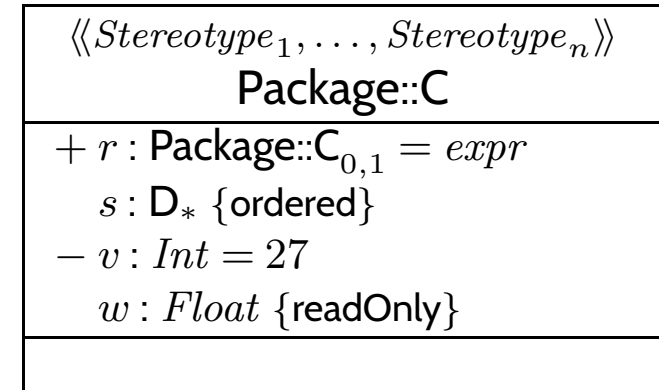
What Do We Want / Have to Cover?

A class

- has a set of **stereotypes**,
- has a **name**,
- (belongs to a **package**,)
- can be **abstract**,
- can be **active**,
- has a set of **attributes**,
- has a set of **operations**. (\rightarrow later)

Each **attribute** has

- a **visibility**,
- a **name**, a **type**, \nearrow later
- a **multiplicity**, an **order**,
- an **initial value**, and
- a set of **properties**, such as **readOnly**, **ordered**, etc.



Wanted: places in the signature to represent the information from the picture.

Extended Signature

Extended Signature

Definition. An (Extended) Object System **Signature** is a quadruple

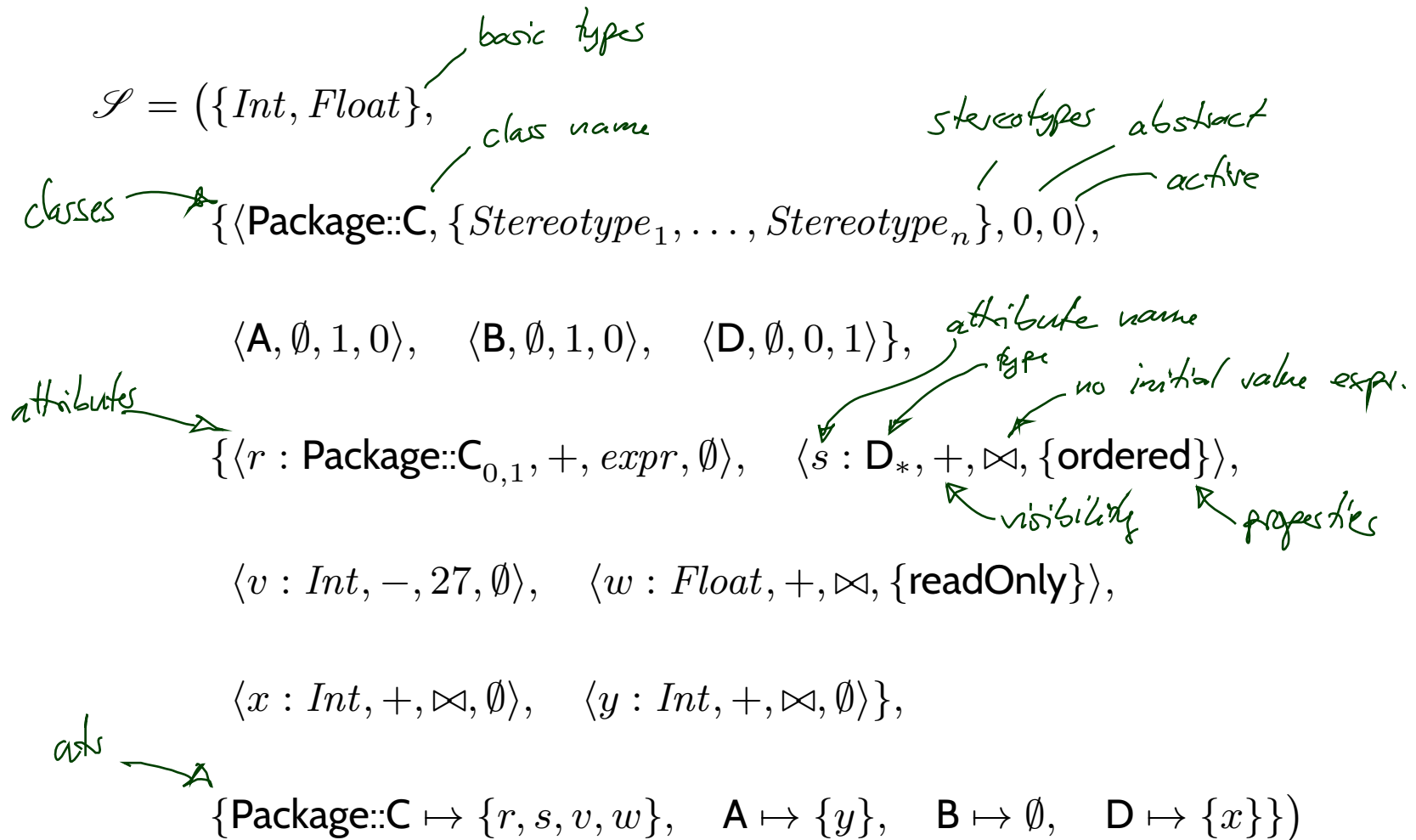
$\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ where

- \mathcal{T} is a set of (basic) **types**,
- \mathcal{C} is a finite set of **classes** $\langle C, S_C, a, t \rangle$ where
 - S_C is a finite (possibly empty) set of **stereotypes**,
 - $a \in \mathbb{B}$ is a boolean flag indicating whether C is **abstract**, ($a=1$ iff C is abstract)
 - $t \in \mathbb{B}$ is a boolean flag indicating whether C is **active**,
- V is a finite set of **attributes** $\langle v : T, \xi, expr_0, P_v \rangle$ where
 - T is a type from \mathcal{T} , or $C_{0,1}, C_*$ for some $C \in \mathcal{C}$,
 - $\xi \in \{\underbrace{\text{public}}_{:=+}, \underbrace{\text{private}}_{:= -}, \underbrace{\text{protected}}_{:=\#}, \underbrace{\text{package}}_{:=\sim}\}$ is the **visibility**,
 - an **initial value expression** $expr_0$ given as a word from a language for initial value expressions, e.g. OCL, or C++ in the Rhapsody tool; write '⊠' to explicitly **not** give an initial value expression.
 - a finite (possibly empty) set of **properties** P_v .
- $atr : \mathcal{C} \rightarrow 2^V$ maps each class to its set of attributes.

We use $S_{\mathcal{C}}$ to denote the set $\bigcup_{C \in \mathcal{C}} S_C$ of stereotypes in \mathcal{S} .

Extended Signature Example

$$\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr), \quad \langle C, S_C, a, t \rangle \in \mathcal{C}, \quad \langle v : T, \xi, expr_0, P_v \rangle \in V$$



Conventions

- We write $\langle C, S_C, a, t \rangle$ if we want to refer to **all aspects** of class C .
- If the new aspects are irrelevant (for a given context), we simply write C i.e. **old definitions** (written in terms of C) are still valid.

- Similarly, we write $\langle v : T, \xi, expr_0, P_v \rangle$ if we want to refer **to all aspects** of attribute v .
- Write only $v : T$ or v **if details are irrelevant**.

Structures of Extended Signatures

Recall:

Definition. A **Basic Object System Structure** of a **Basic Object System Signature** $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ is a domain function \mathcal{D} which assigns to each type a domain, i.e.

- $\tau \in \mathcal{T}$ is mapped to $\mathcal{D}(\tau)$,
- $C \in \mathcal{C}$ is mapped to an infinite set $\mathcal{D}(C)$ of **(object) identities**.

Note: Object identities only have the "=" operation.

- Sets of object identities for different classes are disjoint, i.e.

$$\forall C, D \in \mathcal{C} : C \neq D \rightarrow \mathcal{D}(C) \cap \mathcal{D}(D) = \emptyset.$$

- C_* and $C_{0,1}$ for $C \in \mathcal{C}$ are mapped to $2^{\mathcal{D}(C)}$.

We use $\mathcal{D}(\mathcal{C})$ to denote $\bigcup_{C \in \mathcal{C}} \mathcal{D}(C)$; analogously $\mathcal{D}(\mathcal{C}_*)$.

Structures of Extended Signatures

New:

Definition. An **(Object System) Structure** of an **(Extended Object System) Signature** $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ is a domain function \mathcal{D} which assigns to each type a domain, i.e.

- $\tau \in \mathcal{T}$ is mapped to $\mathcal{D}(\tau)$,
- $C \in \mathcal{C}$ is mapped to an infinite set $\mathcal{D}(C)$ of **(object) identities**.

Note: Object identities only have the "=" operation.

- Sets of object identities for different classes are disjoint, i.e.

$$\forall C, D \in \mathcal{C} : C \neq D \rightarrow \mathcal{D}(C) \cap \mathcal{D}(D) = \emptyset.$$

- C_* and $C_{0,1}$ for $C \in \mathcal{C}$ are mapped to $2^{\mathcal{D}(C)}$.

We use $\mathcal{D}(\mathcal{C})$ to denote $\bigcup_{C \in \mathcal{C}} \mathcal{D}(C)$; analogously $\mathcal{D}(\mathcal{C}_*)$.

System States of Extended Signatures

Recall:

Definition. Let \mathcal{D} be a **basic structure** of **basic signature** $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$.

A **system state** of \mathcal{S} wrt. \mathcal{D} is a **type-consistent** mapping

$$\sigma : \mathcal{D}(\mathcal{C}) \rightarrow (V \rightarrow (\mathcal{D}(\mathcal{T}) \cup \mathcal{D}(\mathcal{C}_*))).$$

That is, for each $u \in \mathcal{D}(C)$, $C \in \mathcal{C}$, if $u \in \text{dom}(\sigma)$

- $\text{dom}(\sigma(u)) = atr(C)$
- $\sigma(u)(v) \in \mathcal{D}(\tau)$ if $v : \tau, \tau \in \mathcal{T}$
- $\sigma(u)(v) \in \mathcal{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathcal{C}$

We call $u \in \mathcal{D}(\mathcal{C})$ **alive** in σ if and only if $u \in \text{dom}(\sigma)$.

We use $\Sigma_{\mathcal{D}}^{\mathcal{S}}$ to denote the set of all system states of \mathcal{S} wrt. \mathcal{D} .

System States of Extended Signatures

New:

Definition. Let \mathcal{D} be a **structure of extended signature** $\mathcal{S} = (\mathcal{I}, \mathcal{C}, V, atr)$.

A **system state** of \mathcal{S} wrt. \mathcal{D} is a **type-consistent** mapping

$$\sigma : \mathcal{D}(\mathcal{C}) \rightarrow (V \rightarrow (\mathcal{D}(\mathcal{I}) \cup \mathcal{D}(\mathcal{C}_*))).$$

That is, for each $u \in \mathcal{D}(C)$, $C \in \mathcal{C}$, if $u \in \text{dom}(\sigma)$

- $\text{dom}(\sigma(u)) = atr(C)$
- $\sigma(u)(v) \in \mathcal{D}(\tau)$ if $v : \tau, \tau \in \mathcal{I}$
- $\sigma(u)(v) \in \mathcal{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathcal{C}$
- $\forall \langle C, S_C, \mathbb{1}, t \rangle \in \mathcal{C} \bullet \text{dom}(\sigma) \cap \mathcal{D}(C) = \emptyset$.

We call $u \in \mathcal{D}(\mathcal{C})$ **alive** in σ if and only if $u \in \text{dom}(\sigma)$.

We use $\Sigma_{\mathcal{D}}$ to denote the set of all system states of \mathcal{S} wrt. \mathcal{D} .

Semantical Relevance

- The **semantics** (or meaning) of an extended object system signature \mathcal{S} wrt. a structure \mathcal{D} is **the set of system states** $\Sigma_{\mathcal{S}}^{\mathcal{D}}$.
- The **semantics** (or meaning) of an extended object system signature \mathcal{S} is **the set of sets of system states** wrt. some structure of \mathcal{S} , i.e. the set

$$\{\Sigma_{\mathcal{S}}^{\mathcal{D}} \mid \mathcal{D} \text{ is structure of } \mathcal{S}\}.$$

$$\mathcal{S}_1: \langle C, \emptyset, 0, 0 \rangle$$

$$\rightsquigarrow \Sigma_{\mathcal{S}_1}^{\mathcal{D}} = \Sigma_{\mathcal{S}_2}^{\mathcal{D}}$$

$$\mathcal{S}_2: \langle C, \emptyset, 0, 1 \rangle \rightsquigarrow \Sigma_{\mathcal{S}_2}^{\mathcal{D}}$$

(only difference in $\mathcal{S}_1, \mathcal{S}_2$ is activeness of C)

Which of the following aspects is **semantically relevant**, i.e. **does contribute** to the constitution of system states?

A class

- has a set of **stereotypes**, ✗
- has a **name**, ✓
- belongs to a **package**,
- can be **abstract**, ✓
- can be **active**, ✗
- has a set of **attributes**, ✓
- has a set of **operations** (later).

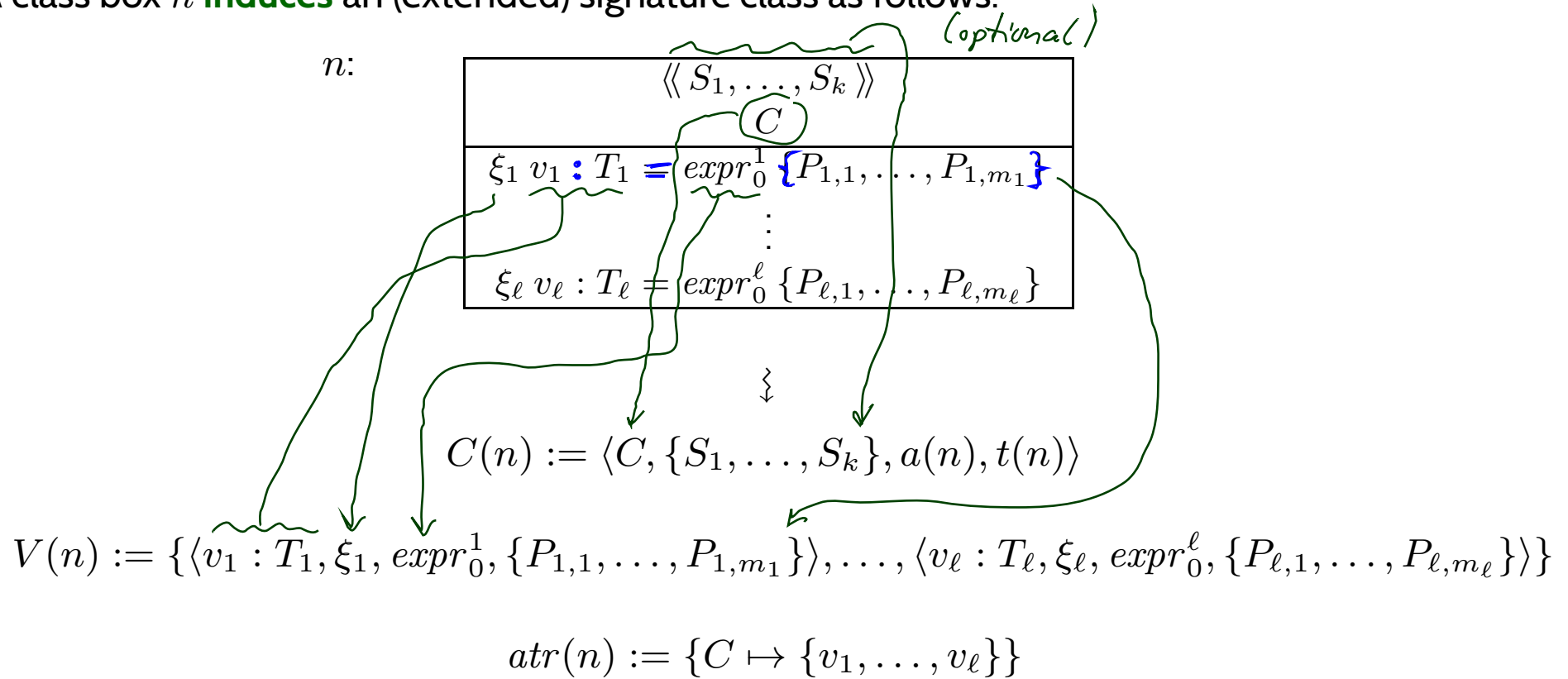
Each attribute has

- a **visibility**, ✗
- a **name**, a **type**, ✓
- a **multiplicity**, an **order**, ✗
- an **initial value**, and ✗
- a set of **properties**, (✗) such as **readOnly**, **ordered**, etc.

Mapping UML Class Diagrams to Extended Signatures

From Class Boxes to Extended Signatures

A class box n **induces** an (extended) signature class as follows:



where

- “abstract” is determined by the font:

$$a(n) = \begin{cases} \text{true} & , \text{ if } n = \boxed{C} \text{ or } n = \boxed{C \{A\}} \\ \text{false} & , \text{ otherwise} \end{cases}$$

- “active” is determined by the frame:

$$t(n) = \begin{cases} \text{true} & , \text{ if } n = \boxed{\boxed{C}} \text{ or } n = \boxed{\boxed{C}} \\ \text{false} & , \text{ otherwise} \end{cases}$$

Example

$\langle\langle S_1, \dots, S_k \rangle\rangle$ C
$\xi_1 v_1 : T_1 = \text{expr}_0^1 \{P_{1,1}, \dots, P_{1,m_1}\}$
\vdots
$\xi_\ell v_\ell : T_\ell = \text{expr}_0^\ell \{P_{\ell,1}, \dots, P_{\ell,m_\ell}\}$

\Downarrow

$$C(n) := \langle C, \{S_1, \dots, S_k\}, a(n), t(n) \rangle$$

$$V(n) := \{ \langle v_1 : T_1, \xi_1, \text{expr}_0^1, \{P_{1,1}, \dots, P_{1,m_1}\} \rangle, \dots, \langle v_\ell : T_\ell, \xi_\ell, \text{expr}_0^\ell, \{P_{\ell,1}, \dots, P_{\ell,m_\ell}\} \rangle \}$$

$$\text{atr}(n) := \{ C \mapsto \{v_1, \dots, v_\ell\} \}$$

$\langle\langle \text{Stereotype}_1, \dots, \text{Stereotype}_n \rangle\rangle$ Package::C
$+ r : \text{Package::C}_{0,1} = \text{expr}$
$s : D_* \{ \text{ordered} \}$
$- v : \text{Int} = 27$
$w : \text{Float} \{ \text{readOnly} \}$

A
$y : \text{Int}$

B {A}

D
$x : \text{Int}$

$$\begin{aligned}
 \mathcal{Y} = & \left(\{ \text{Int}, \text{Float} \}, \right. \\
 & \{ \langle \text{Package::C}, \{ \text{Stereotype}_1, \dots, \text{Stereotype}_n \}, 0, 0 \rangle, \langle A, \emptyset, 1, 0 \rangle, \langle B, \emptyset, 1, 0 \rangle, \langle D, \emptyset, 0, 1 \rangle \}, \\
 & \{ \langle r : \text{Package::C}_{0,1}, +, \text{expr}, \emptyset \rangle, \langle s : D_*, \text{?}, \text{?}, \{ \text{ordered} \} \rangle, \\
 & \langle v : \text{Int}, -, 27, \emptyset \rangle, \langle w : \text{Float}, \text{?}, \text{?}, \{ \text{readOnly} \} \rangle, \langle y : \text{Int}, \text{?}, \text{?}, \emptyset \rangle, \\
 & \langle x : \text{Int}, \text{?}, \text{?}, \emptyset \rangle \}, \\
 & \{ \text{Package::C} \mapsto \{ r, s, v, w \}, A \mapsto \{ y \}, B \mapsto \emptyset, D \mapsto \{ x \} \} \left. \right)
 \end{aligned}$$

What If Things Are Missing?

It depends.

- What does the standard say? (OMG, 2011a, 121)

“Presentation Options.

The type, visibility, default, multiplicity, property string may be suppressed from being displayed, even if there are values in the model.”

- **Visibility**: There is no “no visibility” – an attribute **has** a visibility in the (extended) signature. Some (and we) assume **public** as default, but conventions may vary.
- **Initial value**: some assume it **given by domain** (such as “leftmost value”, but what is “leftmost” of \mathbb{Z} ?). Some (and we) understand **non-deterministic initialisation** if not given.
- **Properties**: probably safe to assume \emptyset if not given at all.

Example Cont'd

$\langle\langle S_1, \dots, S_k \rangle\rangle$ C
$\xi_1 v_1 : T_1 = \text{expr}_0^1 \{P_{1,1}, \dots, P_{1,m_1}\}$
\vdots
$\xi_\ell v_\ell : T_\ell = \text{expr}_0^\ell \{P_{\ell,1}, \dots, P_{\ell,m_\ell}\}$

\Downarrow

$$C(n) := \langle C, \{S_1, \dots, S_k\}, a(n), t(n) \rangle$$

$$V(n) := \{ \langle v_1 : T_1, \xi_1, \text{expr}_0^1, \{P_{1,1}, \dots, P_{1,m_1}\} \rangle, \dots, \langle v_\ell : T_\ell, \xi_\ell, \text{expr}_0^\ell, \{P_{\ell,1}, \dots, P_{\ell,m_\ell}\} \rangle \}$$

$$\text{atr}(n) := \{ C \mapsto \{v_1, \dots, v_\ell\} \}$$

$\langle\langle \text{Stereotype}_1, \dots, \text{Stereotype}_n \rangle\rangle$ Package::C
$+ r : \text{Package::C}_{0,1} = \text{expr}$
$s : \mathbf{D}_* \{ \text{ordered} \}$
$- v : \text{Int} = 27$
$w : \text{Float} \{ \text{readOnly} \}$

A
$y : \text{Int}$

$\{A\}$
B

D
$x : \text{Int}$

$$\mathcal{S} = (\{ \text{Int}, \text{Float} \},$$

$$\{ \langle \text{Package::C}, \{ \text{Stereotype}_1, \dots, \text{Stereotype}_n \}, 0, 0 \rangle,$$

$$\langle \text{A}, \emptyset, 1, 0 \rangle, \quad \langle \text{B}, \emptyset, 1, 0 \rangle, \quad \langle \text{D}, \emptyset, 0, 1 \rangle \},$$

$$\{ \langle r : \text{Package::C}_{0,1}, +, \text{expr}, \emptyset \rangle, \quad \langle s : \mathbf{D}_*, +, \boxtimes, \{ \text{ordered} \} \rangle,$$

$$\langle v : \text{Int}, -, 27, \emptyset \rangle, \quad \langle w : \text{Float}, +, \boxtimes, \{ \text{readOnly} \} \rangle,$$

$$\langle x : \text{Int}, +, \boxtimes, \emptyset \rangle, \quad \langle y : \text{Int}, +, \boxtimes, \emptyset \rangle \},$$

$$\{ \text{Package::C} \mapsto \{ r, s, v, w \}, \quad \text{A} \mapsto \{ y \}, \quad \text{B} \mapsto \emptyset, \quad \text{D} \mapsto \{ x \} \}$$

From Class Diagrams to Extended Signatures

- We view a **class diagram** \mathcal{CD} as a graph with nodes $\{n_1, \dots, n_N\}$ (each “class rectangle” is a node).
 - $\mathcal{C}(\mathcal{CD}) := \{C(n_i) \mid 1 \leq i \leq N\}$
 - $V(\mathcal{CD}) := \bigcup_{i=1}^N V(n_i)$
 - $atr(\mathcal{CD}) := \bigcup_{i=1}^N atr(n_i)$
- In a **UML model**, we can have **finitely many** class diagrams,

$$\mathcal{CD} = \{\mathcal{CD}_1, \dots, \mathcal{CD}_k\},$$

which **induce** the following signature:

$$\mathcal{S}(\mathcal{CD}) = \left(\mathcal{I}, \bigcup_{i=1}^k \mathcal{C}(\mathcal{CD}_i), \bigcup_{i=1}^k V(\mathcal{CD}_i), \bigcup_{i=1}^k atr(\mathcal{CD}_i) \right).$$

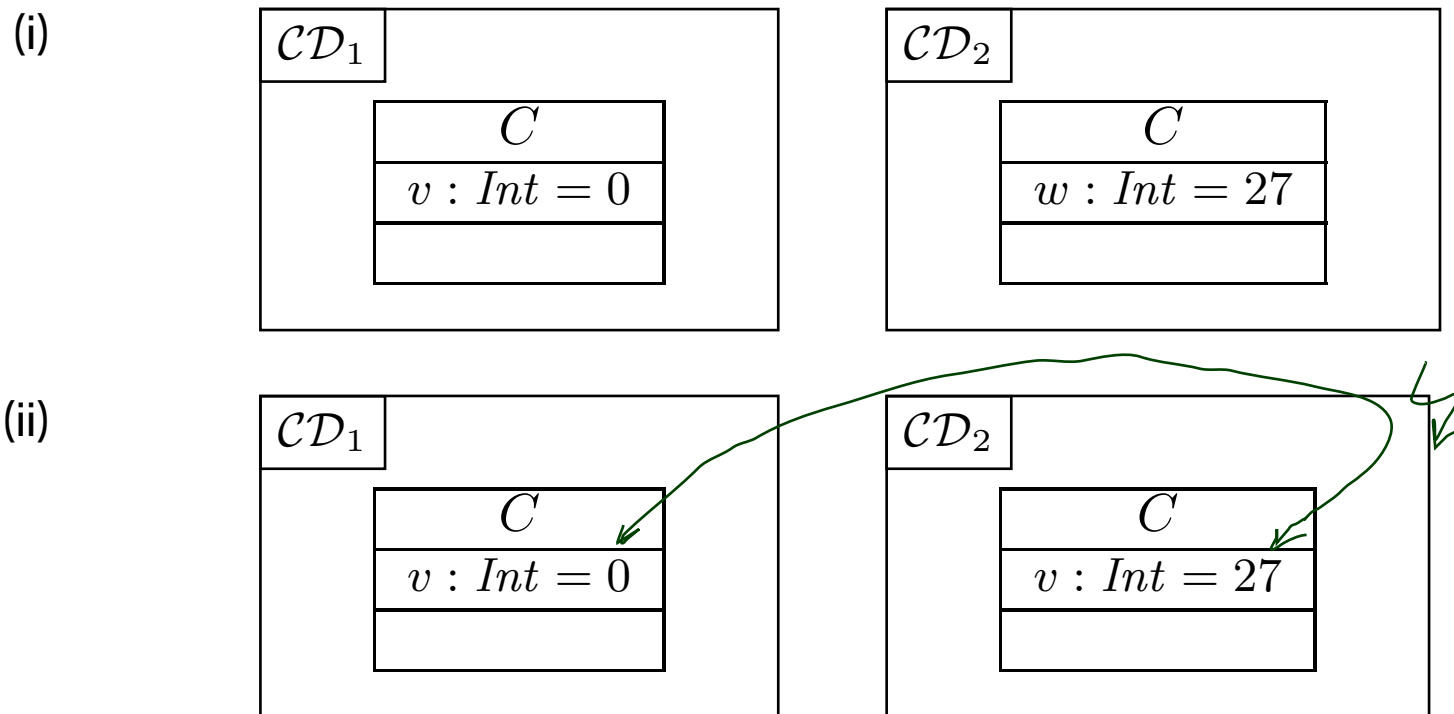
(Assuming \mathcal{I} given. In “reality” (i.e. in full UML), we can introduce types in class diagrams, the class diagram then contributes to \mathcal{I} . Example: enumeration types.)

Is the Mapping a Function?

Question: Is $\mathcal{S}(\mathcal{CD})$ **well-defined**?

There are two possible **sources for problems**:

(1) A **class** C may appear in **multiple** class **diagrams**:



Simply **forbid** the case (ii) – easy syntactical check on diagram.

Is the Mapping a Function?

(2) An **attribute** v may appear in **multiple classes** with different type:

C
$v : Bool$

D
$v : Int$

Two approaches:

- Require **unique** attribute names.

This requirement can easily be established (implicitly, behind the scenes) by viewing v as an abbreviation for

$$\underbrace{C::v} \quad \text{or} \quad \underbrace{D::v} \quad \mathcal{D}(C::v) \dots \quad \mathcal{D}(D::v) \dots$$

depending on the context. ($C::v : Bool$ and $D::v : Int$ are then unique.)

- Subtle, formalist's approach: observe that

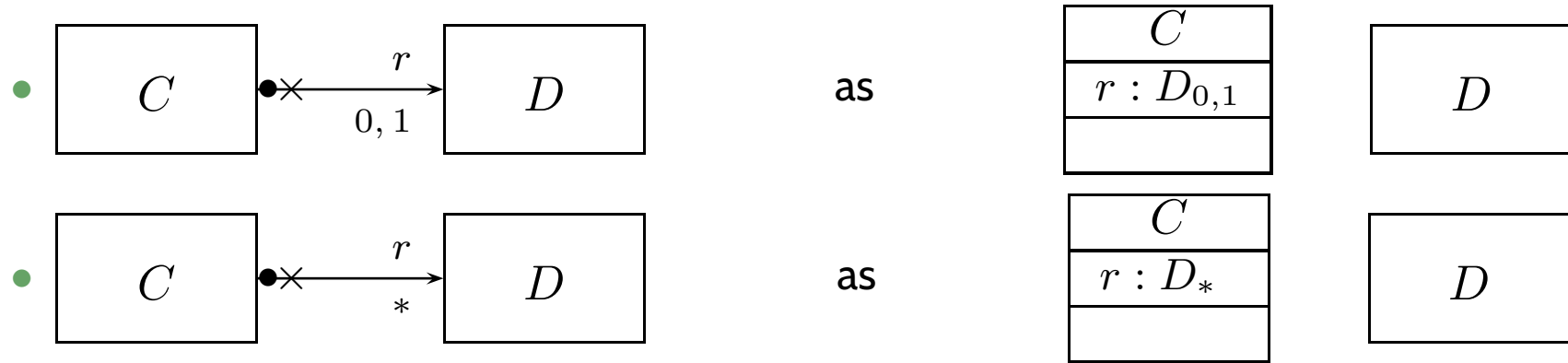
$$\langle v : Bool, \dots \rangle \quad \text{and} \quad \langle v : Int, \dots \rangle$$

are **different things** in V .

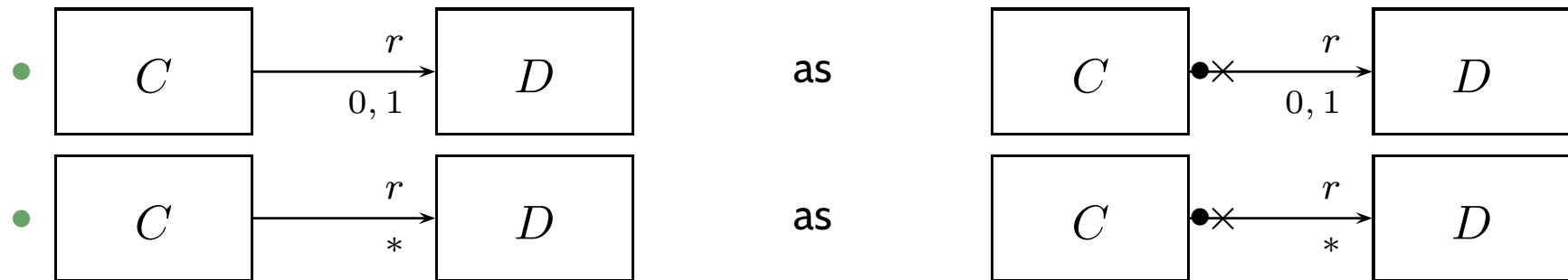
$$\mathcal{D}(\langle v : Bool, \dots \rangle) \\ \mathcal{D}(\langle v : Int, \dots \rangle)$$

Abbreviations

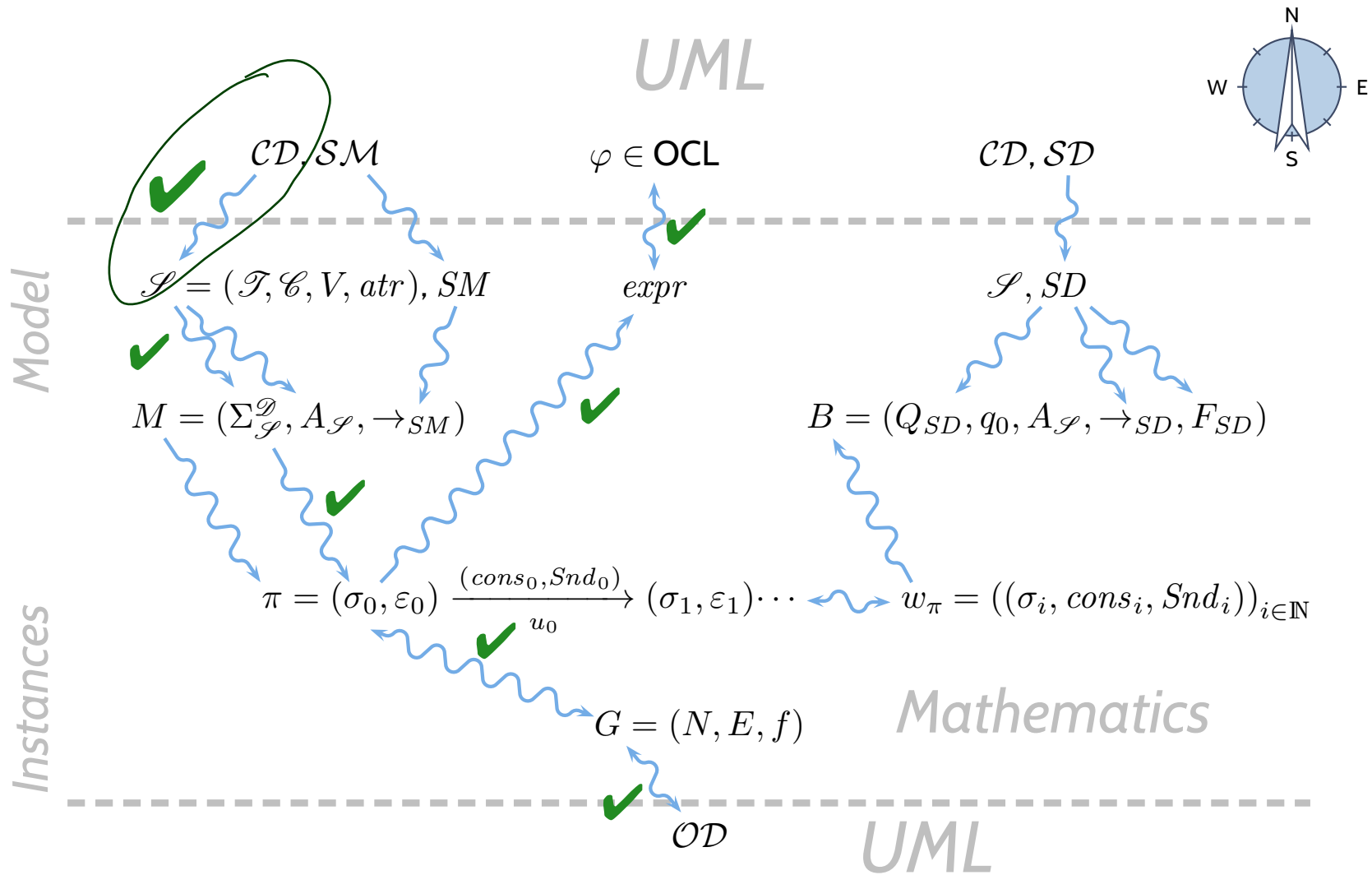
Since we have not yet discussed **associations**, for now we read



and



Course Map



Stereotypes

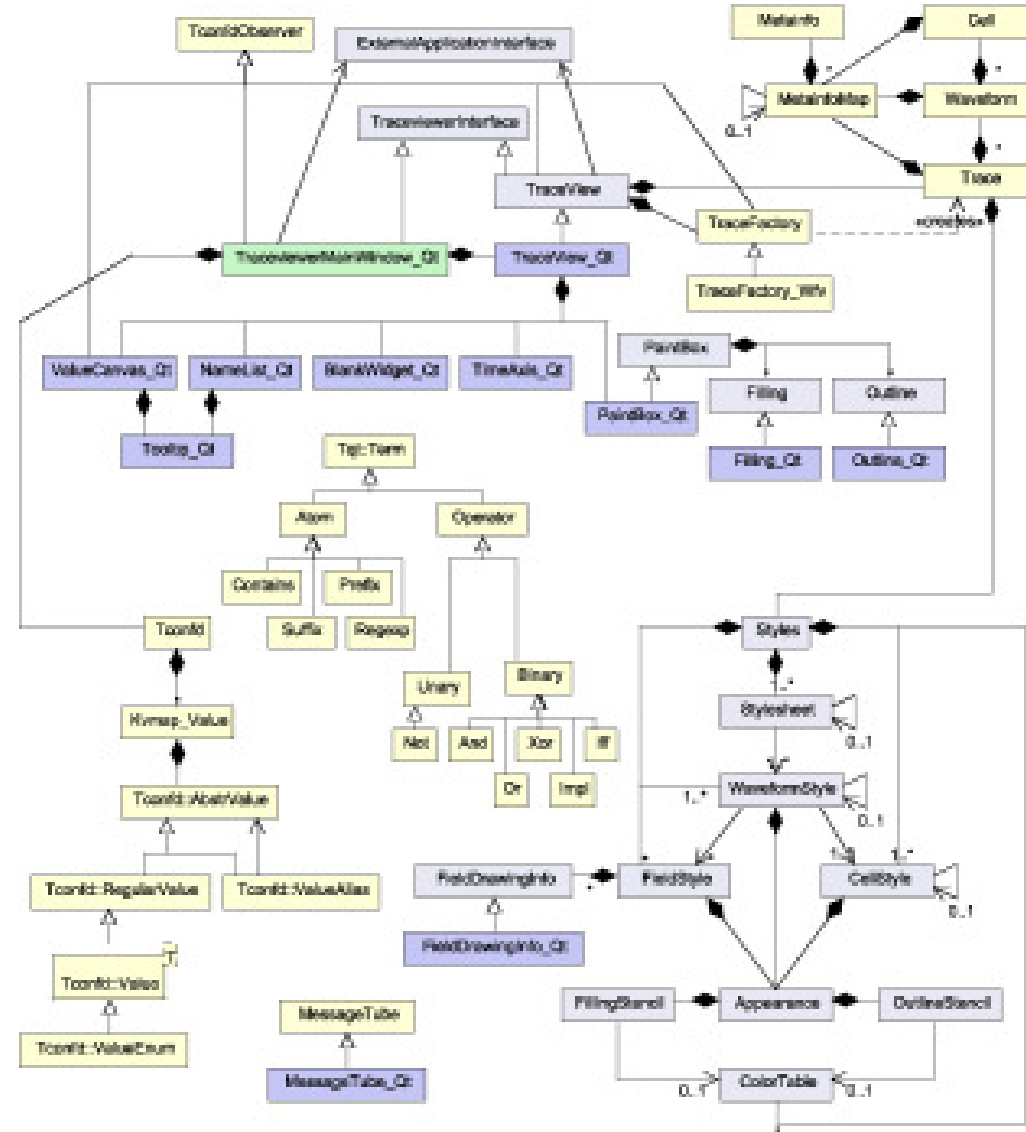
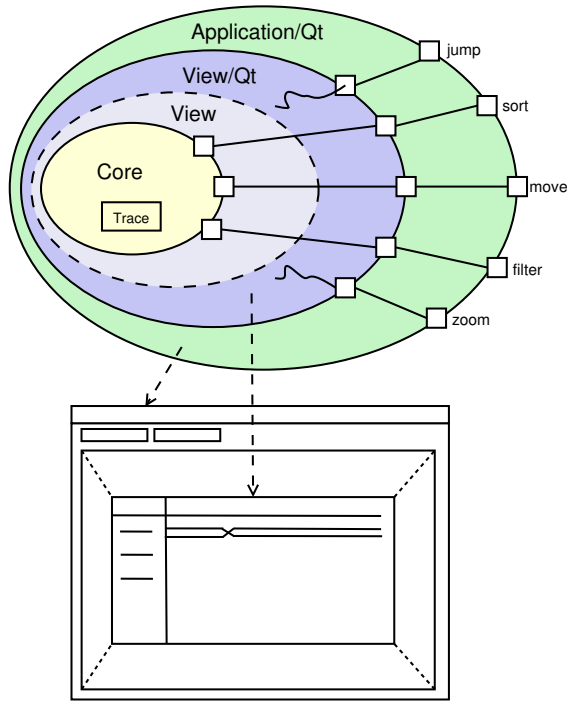
Stereotypes as Labels or Tags

- What are Stereotypes?
 - **Not** represented in system states.
 - **Not** contributing to typing rules / well-formedness.
- Oestereich (2006):

View stereotypes as (additional) “**labelling**” (“tags”) or as “**grouping**”.
- Useful for documentation and model-driven development, e.g. code-generation:
 - **Documentation**: e.g. layers of an architecture.

Sometimes, packages (cf. [OMG \(2011a,b\)](#)) are sufficient and “right”.
 - **Model Driven Architecture (MDA)**: **later**.

Example: Stereotypes for Documentation



- **Example:** Timing Diagram Viewer Schumann et al. (2008)
- Architecture has four layers:
 - core, data layer
 - abstract view layer
 - toolkit-specific view layer/widget
 - application using widget

Stereotype “=” layer “=” colour.

Other Examples

- Use stereotypes 'Team₁', 'Team₂', 'Team₃' and assign stereotype Team_{*i*} to class *C* if Team_{*i*} is responsible for class *C*.
- Use stereotypes to label classes with licensing information (e.g., LGPL vs. proprietary).
- Use stereotypes 'Server_{*A*}', 'Server_{*B*}' to indicate where objects should be stored.
- Use stereotypes to label classes with states in the development process like "under development", "submitted for testing", "accepted".
- etc. etc.

Necessary: a **common idea** of what each stereotype stands for.

(To be defined / agreed on by the team, not the job of the UML consortium.)



Tell Them What You've Told Them. . .

- **Extended Signatures** allow us to represent aspects like
 - abstract, active, visibility, initial value expression, ...
- Not all of these aspects are **semantically relevant**.
- The only change on **system states** is that abstract classes **cannot have instances**.
- **Class Diagrams** map to **Extended Signatures**, i.e. the meaning of a class diagram is the extended signature which it **uniquely** denotes.
- Thus a **Class Diagram** (transitively) denotes a set of system states (given a structure).
- **Stereotypes** are just labels.

References

References

Oestereich, B. (2006). *Analyse und Design mit UML 2.1, 8. Auflage*. Oldenbourg, 8. edition.

OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.

Schumann, M., Steinke, J., Deck, A., and Westphal, B. (2008). Traceviewer technical documentation, version 1.0. Technical report, Carl von Ossietzky Universität Oldenburg und OFFIS.