Software Design, Modelling and Analysis in UML

Lecture 8: Class Diagrams III

2016-11-24

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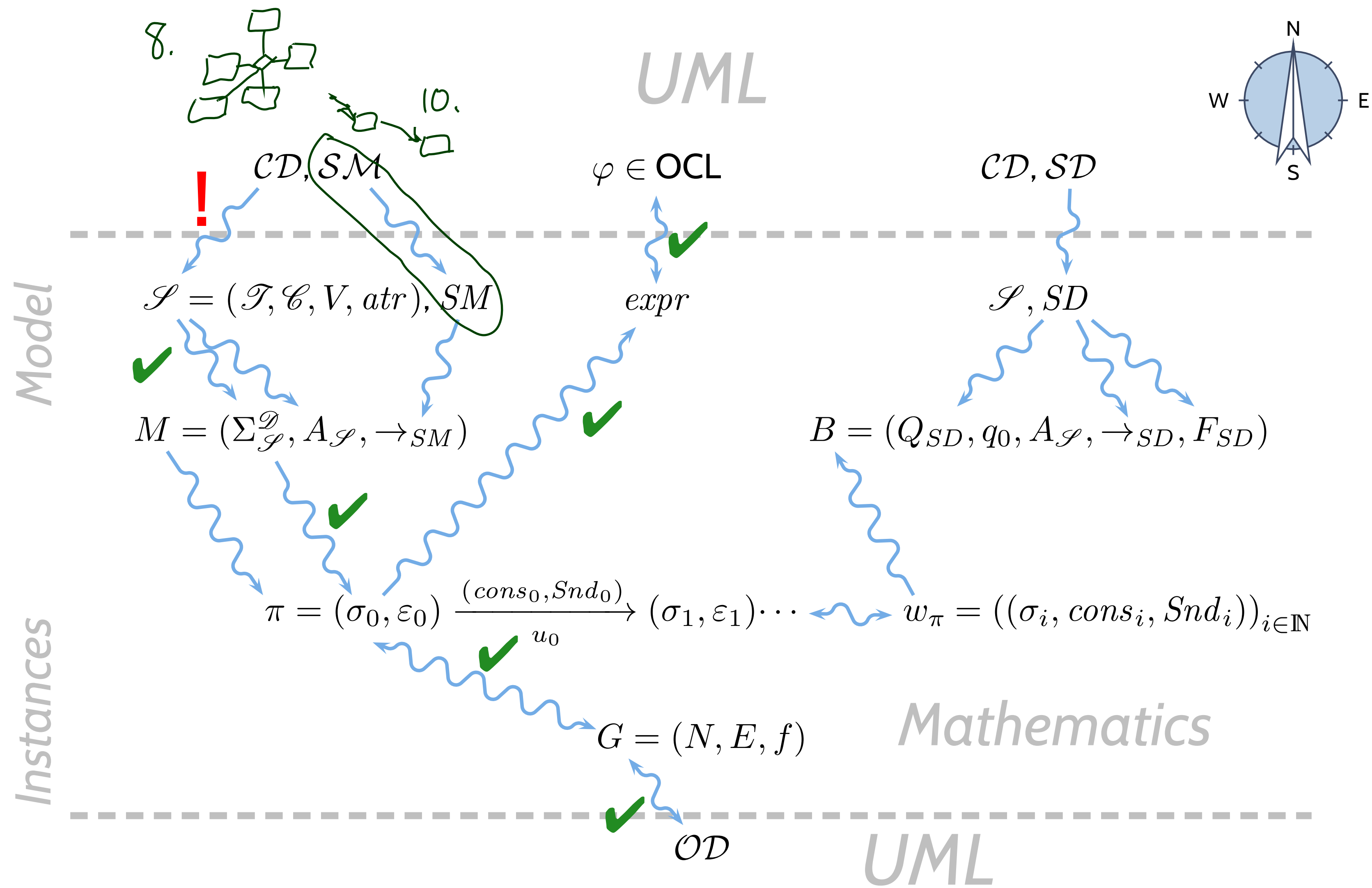
Course Map



8.
10.
UML
N
W
E
S
$\mathcal{CD}, \mathcal{SM}$
!
$\varphi \in$ OCL
$\mathcal{CD}, \mathcal{SD}$
Model
$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr), SM$
$expr$
$\mathscr{S}, SD$
$M = (\Sigma_{\mathscr{S}}^{\mathscr{D}}, A_{\mathscr{S}}, \to_{SM})$
$B = (Q_{SD}, q_0, A_{\mathscr{S}}, \to_{SD}, F_{SD})$
Instances
$\pi = (\sigma_0, \varepsilon_0) \xrightarrow[u_0]{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \cdots$
$w_\pi = ((\sigma_i, cons_i, Snd_i))_{i \in \mathbb{N}}$
$G = (N, E, f)$
Mathematics
$\mathcal{OD}$
UML

Content

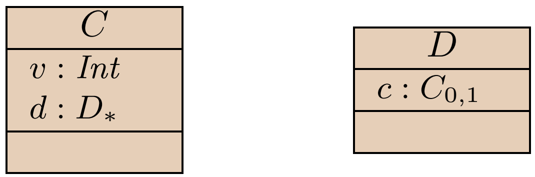
- **Recall**: **Associations**
  - Overview & Plan
  - (Temporarily) **Extend Signature**
- From **Class Diagrams** to **Signatures**
  - What if Things are Missing?
- **Association Semantics**
  - **Links** in System States
  - Associations and **OCL**
- **The Rest**
  - **Visibility**, **Navigability**
  - **Multiplicity**, **Properties**,
  - **Ownership**, "**Diamonds**"

- **Back to the Main Track**

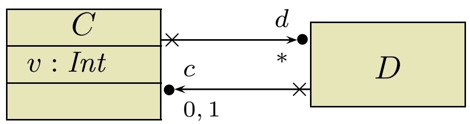
Recall: Plan & Extended Signature

Overview

- **Class diagram**:



Alternative presentation:



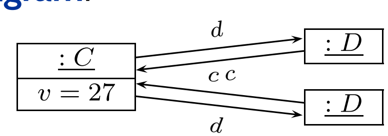
- **Signature**:

$$\mathscr{S} = (\{Int\}, \{C, D\}, \{v : Int, d : D_*, c : C_{0,1}\},$$
$$\{C \mapsto \{v, d\}, D \mapsto \{c\}\})$$

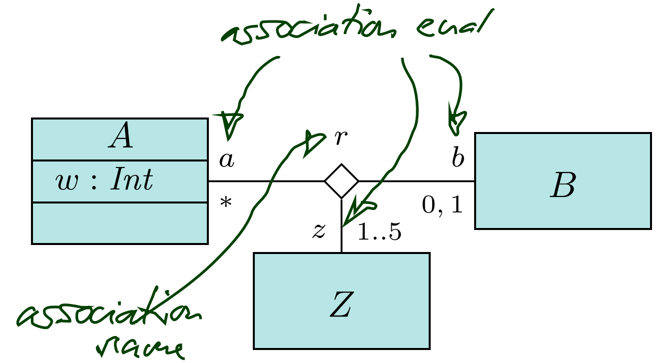
- **Example system state**:

$$\sigma = \{1_C \mapsto \{v \mapsto 27, d \mapsto \{5_D, 7_D\}\},$$
$$5_D \mapsto \{c \mapsto \{1_C\}\}, 7_D \mapsto \{c \mapsto \{1_C\}\}\}$$

- **Object diagram**:



- **Class diagram** (with ternary association):



- **Signature**: extend again to represent
  - **association** r with
    - **association ends** a, b, and z
      (each with multiplicity, visibility, etc.)

- **Example system state**: (σ, λ)

$$\sigma = \{1_A \mapsto \{w \mapsto 13\}, 1_B \mapsto \emptyset, 1_Z \mapsto \emptyset\}$$
$$\lambda = \{\, r \mapsto \{(1_A, 1_B, 1_Z), (1_A, 1_B, 2_Z)\}\,\}$$

		z
1_A	1_B	1_z
1_A	1_B	2_z

- **Object diagram**: No...

So, What Do We (Have to) Cover?

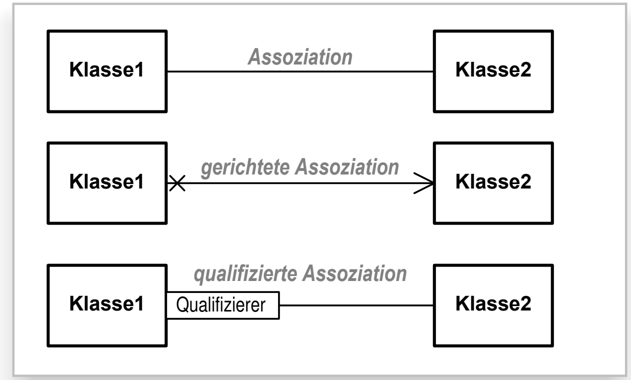
An **association** has

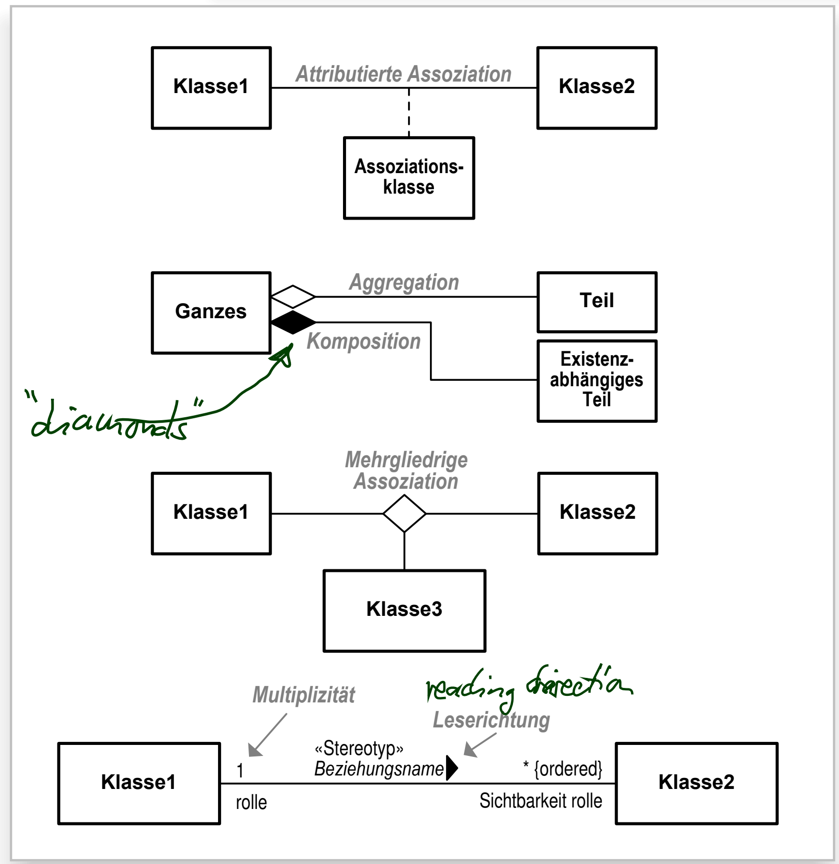
- a **name**,
- a **reading direction**, and
- at least two **ends**.

Each **end** has

- a **role name**,
- a **multiplicity**,
- a set of **properties**, such as **unique**, **ordered**, etc.
- a **qualifier**, (not in lect.)
- a **visibility**,
- a **navigability**,
- an **ownership**,
- and possibly a **diamond**.

Wanted: places in the signature to represent the information from the picture.

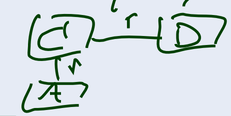




Temporarily (Lecture 7 – 9) Extended Signature

Definition. An (Extended) Object System Signature (with Associations) is a quadruple $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

Rhapsody:
assoc. names must be unique, so NOT



- …
- each element of V is
  - **either** a **basic type attribute** $\langle v : T, \xi, expr_0, P_v\rangle$ with $T \in \mathscr{T}$
  - **or** an **association** of the form

association end

$$\langle r : \quad \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1\rangle,$$
$$\vdots$$
$$\langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n\rangle\rangle$$

association name

the class where this end is located

  (ends with multiplicity μ_i, properties P_i, visibility ξ_i, navigability ν_i, ownership o_i, $1 \le i \le n$)
- …
- $atr : \mathscr{C} \to 2^{\{v \in V \mid v:T,\ T \in \mathscr{T}\}}$ maps classes to **basic type** (!) attributes.

$\mu ::= N..M \mid N..* \mid \mu,\mu$

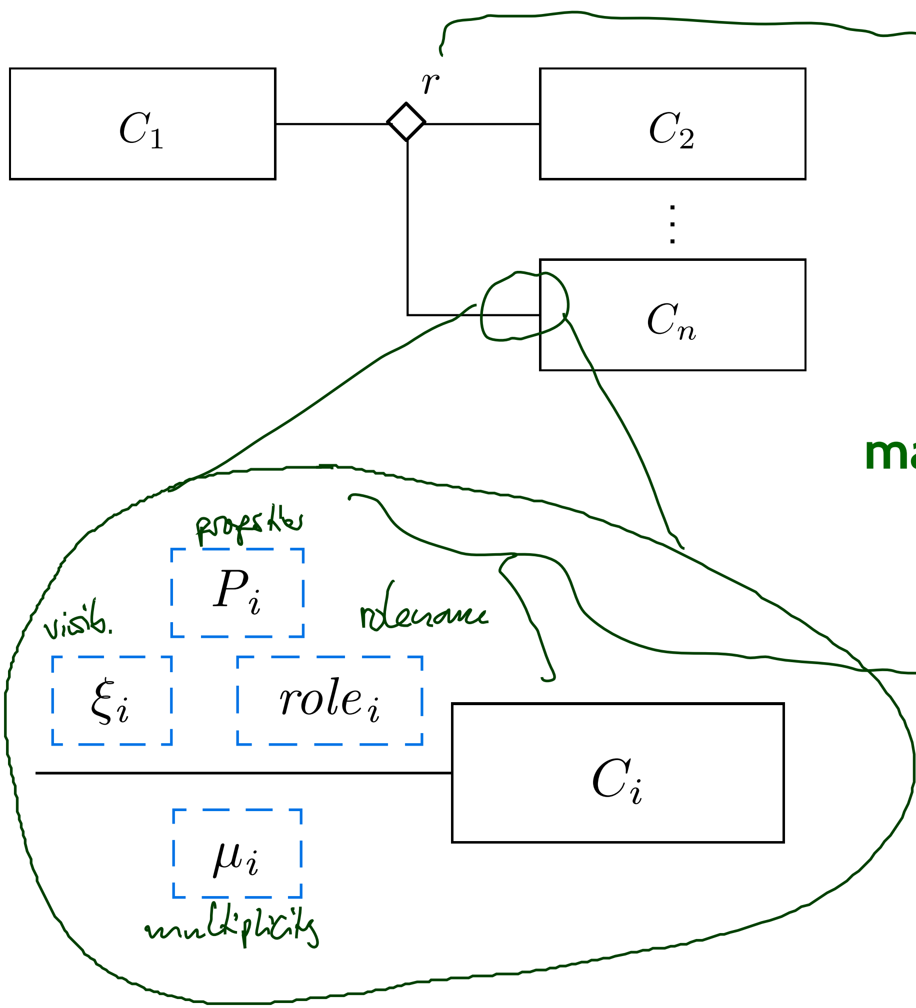
$(* := 0..*$
$N := N..N)$

In other words:

- only **basic type attributes “belong”** to a class (may appear in $atr(C)$),
- **associations** are not “owned” by a class (not in any $atr(C)$), but **“live on their own”**.

Associations in Class Diagrams

From Association Lines to Extended Signatures



maps to

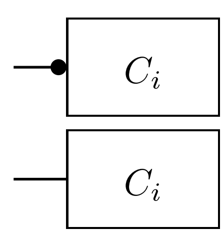
$$\langle r : \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1 \rangle$$

$$\vdots$$

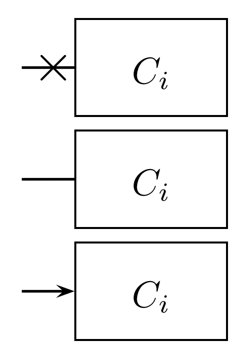
$$\langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n \rangle \rangle$$

navigability

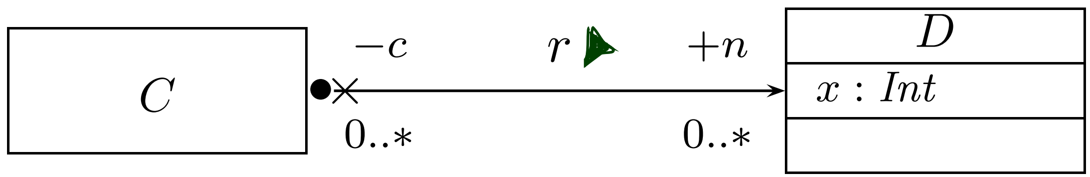
$$o_i = \begin{cases} 1 & \text{, if} \\ 0 & \text{, if} \end{cases}$$



$$\nu_i = \begin{cases} \times & \text{, if} \\ - & \text{, if} \\ > & \text{, if} \end{cases}$$



Association Example



Signature:

$$\mathscr{S} = (\{Int\}, \{C, D\}, \{\langle x : Int, +, \times, \emptyset \rangle,$$
$$\langle r : \langle n : D, *, \{unique\}, +, >, 0 \rangle,$$
$$\langle c : C, 0..*, \{unique\}, -, \times, 1 \rangle \rangle \},$$
$$\{C \mapsto \emptyset,$$
$$D \mapsto \{x\}\})$$

What If Things Are Missing?

Most components of associations or association end may be omitted.
For instance (OMG, 2011b, 17), Section 6.4.2, proposes the following rules:

- **Name**: Use

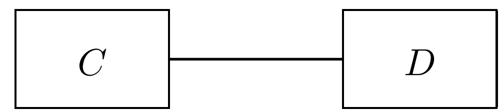
  $$A\_\langle C_1\rangle\_\cdots\_\langle C_n\rangle$$

  if the name is missing.

  Example:

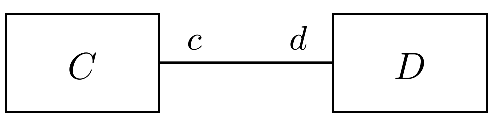
  for

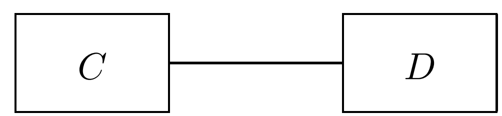
- **Reading Direction**: no default.

- **Role Name**: use the class name at that end in lower-case letters

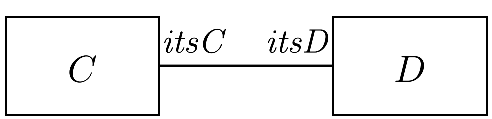
  Example:

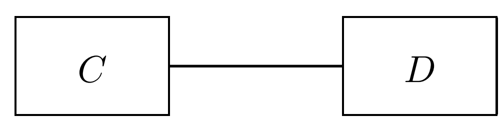
  for

  Other convention: (used e.g. by modelling tool Rhapsody)

  for

What If Things Are Missing?

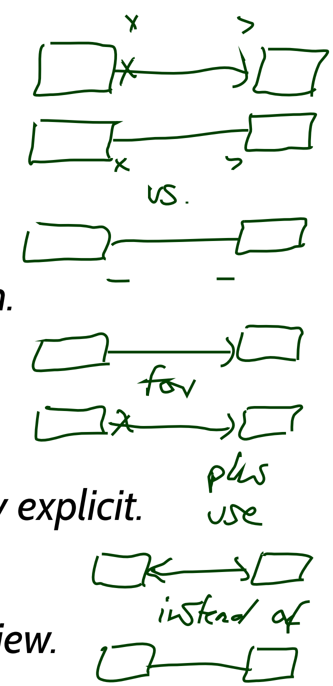
- **Multiplicity**: 1

  In my opinion, it's safer to assume $0..1$ or $*$ (for $0..*$)
  if there are no fixed, written, agreed conventions ("expect the worst").

- **Properties**: $\emptyset$ (in course: {unique})

- **Visibility**: public

- **Navigability and Ownership**: not so easy. (OMG, 2011b, 43)



"Various options may be chosen for showing navigation arrows on a diagram.

In practice, it is often convenient to suppress some of the arrows and crosses and just show exceptional situations:

- *Show all arrows and $\times$'s: Navigation and its absence are made completely explicit.*
- *Suppress all arrows and $\times$'s: No inference can be drawn about navigation.*

  This is similar to any situation in which information is suppressed from a view.

- *Suppress arrows for associations with navigability in both directions, and show arrows only for associations with one-way navigability.*

  In this case, the two-way navigability cannot be distinguished from situations where there is no navigation at all; however, the latter case occurs rarely in practice."

Wait, If Omitting Things...

- ...**is causing so much trouble** (e.g. leading to misunderstanding), why does the standard say "**In practice, it is often convenient**..."?

  Is it a good idea to trade **convenience** for **precision/unambiguity**?

  It depends.

  - Convenience as such is a **legitimate goal**.
  - In UML-As-Sketch mode, precision "**doesn't matter**", so convenience (for writer) can even be a primary goal.
  - In UML-As-Blueprint mode, **precision** is the **primary goal**. And misunderstandings are in most cases annoying.

    But: (even in UML-As-Blueprint mode)

    If all associations in your model have multiplicity $*$, then it's probably a good idea not to write all these $*$'s.

    So: tell the reader about your convention and leave out the $*$'s.

Associations: Semantics

Associations in General

Recall: We consider associations of the following form:

$$\langle r : \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1 \rangle, \dots, \langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n \rangle \rangle$$

Only these parts are relevant for extended system states:

$$\langle r : \langle role_1 : C_1, \_, P_1, \_, \_, \_ \rangle, \dots, \langle role_n : C_n, \_, P_n, \_, \_, \_ \rangle$$

(recall: we assume $P_1 = P_n = \{\mathtt{unique}\}$).

The UML standard "thinks" of associations as **n-ary relations** which "**live on their own**" in a system state.

That is, **links** (= association instances)

- **do not** belong (in general) to certain objects (in contrast to pointers, e.g.)
- are "first-class citizens" **next to objects**,
- are (in general) **not** directed (in contrast to pointers).

Links in System States

$$\langle r : \langle role_1 : C_1, \_, P_1, \_, \_, \_\rangle, \ldots, \langle role_n : C_n, \_, P_n, \_, \_, \_\rangle$$

Only for the course of lectures 7/8 / 9 we change the definition of system states:

Definition. Let $\mathscr{D}$ be a structure of the (extended) signature with associations $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$.

A system state of $\mathscr{S}$ wrt. $\mathscr{D}$ is a pair (σ, λ) consisting of

only basic type attributes here

- a type-consistent mapping (as before)

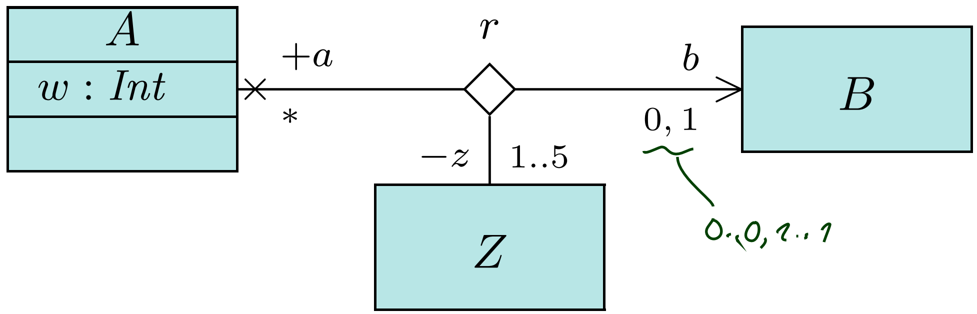
$$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (atr(\mathscr{C}) \nrightarrow \mathscr{D}(\mathscr{T})),$$

- a mapping λ which maps each association $\langle r : \langle role_1 : C_1\rangle, \ldots, \langle role_n : C_n\rangle\rangle \in V$ to a **relation**

$$\lambda(r) \subseteq \mathscr{D}(C_1) \times \cdots \times \mathscr{D}(C_n)$$

(i.e. a set of type-consistent n-tuples of identities).

Association / Link Example



Signature:

$$\mathscr{S} = (\{Int\}, \{A, Z, B\}, \{w : Int,$$
$$\langle r : \langle a : A, 0..*, +, \{unique\}, \times, 0\rangle,$$
$$\langle z : Z, 1..5, -, \{unique\}, -, 0\rangle,$$
$$\langle b : B, 0,1, +, \{unique\}, >, 0\rangle \rangle\},$$
$$\{A \mapsto \{w\}, Z \mapsto \emptyset, B \mapsto \emptyset\})$$

System state:

$$\sigma = \{1_A \mapsto \{w \mapsto 27\}, 2_A \mapsto \{w \mapsto 13\}, 4_Z \mapsto \emptyset, 3_B \mapsto \emptyset, 7_B \mapsto \emptyset, 8_B \mapsto \emptyset, 3_A \mapsto \emptyset\}$$

$$\in \mathscr{D}(A) \times \mathscr{D}(Z) \times \mathscr{D}(B)$$

$$\lambda = \{r \mapsto \{(1_A, 4_Z, 3_B), (1_A, 4_Z, 7_B), (1_A, 4_Z, 5_B), (2_A, 4_Z, 3_B)\}\}$$

$\approx$

a	z	b
1_A	4_Z	3_B
1_A	4_Z	7_B
1_A	4_Z	5_B
2_A	4_Z	3_B

NOT: $(4_Z, 3_B, 2_A)$

NOT: $(1_A, 4_Z)$

Associations and OCL

OCL and Associations: Syntax

Recall: OCL syntax as introduced in Lecture 3, interesting part:

$$
\begin{array}{llll}
expr ::= \dots & \mid r_1(expr_1) & : \tau_C \to \tau_D & r_1 : D_{0,1} \in atr(C) \\
 & \mid r_2(expr_1) & : \tau_C \to Set(\tau_D) & r_2 : D_* \in atr(C)
\end{array}
$$

Now becomes

$$
\begin{array}{llll}
expr ::= \dots & \mid role(expr_1) & : \tau_C \to \tau_D & \mu = 0..1 \text{ or } \mu = 1..1 \\
 & \mid role(expr_1) & : \tau_C \to Set(\tau_D) & \text{otherwise}
\end{array}
$$

if there is

$$\langle r : \dots, \langle role : D, \mu, \_, \_, \_, \_\rangle, \dots, \langle role' : C, \_, \_, \_, \_, \_\rangle, \dots \rangle \in V \text{ or}$$

$$\langle r : \dots, \langle role' : C, \_, \_, \_, \_, \_\rangle, \dots, \langle role : D, \mu, \_, \_, \_, \_\rangle, \dots \rangle \in V, \quad role \neq role'.$$

Note:

- Association name as such **does not occur** in OCL syntax, role names do.
- $expr_1$ has to denote an object of a class which "participates" in the association.

OCL and Associations: Semantics

Recall:

Assume $expr_1 : \tau_C$ for some $C \in \mathscr{C}$. Set $u_1 := I[\![expr_1]\!](\sigma, \beta) \in \mathscr{D}(T_C)$.

- $I[\![r_1(expr_1)]\!](\sigma, \beta) := \begin{cases} u & \text{, if } u_1 \in \mathrm{dom}(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \perp & \text{, otherwise} \end{cases}$
- $I[\![r_2(expr_1)]\!](\sigma, \beta) := \begin{cases} \sigma(u_1)(r_2) & \text{, if } u_1 \in \mathrm{dom}(\sigma) \\ \perp & \text{, otherwise} \end{cases}$

Now needed:

$$I[\![role(expr_1)]\!]((\sigma, \lambda), \beta)$$

- We cannot simply write $\sigma(u)(role)$.
  Recall: $role$ is (**for the moment**) not an attribute of object u (not in $atr(C)$).
- What we have is $\lambda(r)$ (with association name r, not with role name $role$!).

  $$\langle r : \ldots, \langle role : D, \mu, \_, \_, \_, \_\rangle, \ldots, \langle role' : C, \_, \_, \_, \_, \_\rangle, \ldots\rangle$$

  But it yields a set of n-tuples, of which **some** relate u and some instances of D.
- $role$ denotes the position of the D's in the tuples constituting the value of r.

OCL and Associations: Semantics Cont'd

Assume $expr_1 : \tau_C$ for some $C \in \mathscr{C}$. Set $u_1 := I[\![expr_1]\!]((\sigma, \lambda), \beta) \in \mathscr{D}(T_C)$.

$!$

- $I[\![role(expr_1)]\!]((\sigma, \lambda), \beta) := \begin{cases} u & \text{, if } u_1 \in \operatorname{dom}(\sigma) \text{ and } L(role)(u_1, \lambda) = \{u\} \\ \bot & \text{, otherwise} \end{cases}$

- $I[\![role(expr_1)]\!]((\sigma, \lambda), \beta) := \begin{cases} L(role)(u_1, \lambda) & \text{, if } u_1 \in \operatorname{dom}(\sigma) \\ \bot & \text{, otherwise} \end{cases}$

project onto i-th component

where

$$L(role)(u, \lambda) = \{(u_1, \ldots, u_n) \in \lambda(r) \mid u \in \{u_1, \ldots, u_n\}\} \downarrow i$$

if

$$\langle r : \langle role_1 : \_, \_, \_, \_, \_, \_\rangle, \ldots \langle role_n : \_, \_, \_, \_, \_, \_\rangle, \rangle, \quad role = role_i:$$

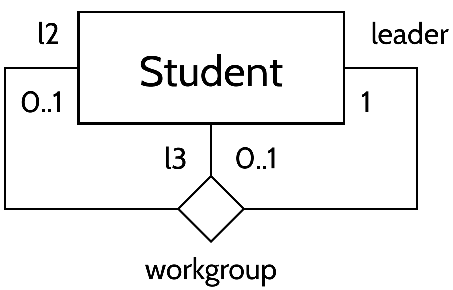
Given a set of n-tuples A,
$A \downarrow i$ denotes the element-wise projection onto the i-th component.

OCL and Associations Semantics: Example

$$I[\![role(expr_1)]\!]((\sigma,\lambda),\beta) := \begin{cases} u & \text{, if } u_1 \in \mathrm{dom}(\sigma) \text{ and } L(role)(u_1,\lambda) = \{u\} \\ \bot & \text{, otherwise} \end{cases}$$

$$I[\![role(expr_1)]\!]((\sigma,\lambda),\beta) := \begin{cases} L(role)(u_1,\lambda) & \text{, if } u_1 \in \mathrm{dom}(\sigma) \\ \bot & \text{, otherwise} \end{cases}$$

$$L(role)(u,\lambda) = \{(u_1,\dots,u_n) \in \lambda(r) \mid u \in \{u_1,\dots,u_n\}\} \downarrow i$$



$F :=$ allInstances$_{Student}$->
Exists$(s \mid s.l2 = s.l3)$

1. 2. 3.
leader l_2 l_3

$$\lambda(\text{workgroup}) = \{(1_S,2_S,3_S), (1_S,3_S,4_S), (5_S,1_S,1_S)\}$$

$I[\![F]\!]((\sigma,\lambda),\beta)$

$I[\![s.l2]\!]((\sigma,\lambda), \overbrace{\{s \mapsto 5_S\}}^{=:\beta_1}) = 1_S$

$u_1 = I[\![s]\!]((\sigma,\lambda),\beta_1) = \beta_1(s) = 5_S$

$L(l2)(u_1,\lambda) = \{(5_S,1_S,1_S)\} \downarrow 2 = \{1_S\}$

$I[\![s.l2]\!]((\sigma,\lambda), \overbrace{\{s \mapsto 1_S\}}^{=:\beta_2}) = \bot$

$u_1 = 1_S$

$(5_S,1_S,1_S)$

$L(l2)(u_1,\lambda) = (\{(1_S,2_S,3_S),(1_S,3_S,4_S)\}) \downarrow 2$

$= \{2_S, 3_S\}$ 1_S

Associations: The Rest

The Rest

Recapitulation: Consider the following association:

$$\langle r : \langle role_1 : C_1, \mu_1, P_1, \xi_1, \nu_1, o_1 \rangle, \dots, \langle role_n : C_n, \mu_n, P_n, \xi_n, \nu_n, o_n \rangle \rangle$$

- **Association name** r and **role names** / **types** $role_i$ / C_i induce extended system states (σ, λ).
- **Multiplicity** μ is considered in OCL syntax.
- **Visibility** ξ / **Navigability** ν: well-typedness (in a minute).

Now the rest:

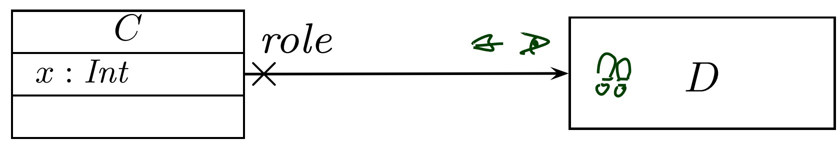
- **Multiplicity** μ: we propose to view them as constraints.
- **Properties** P_i: even more typing.
- **Ownership** o: getting closer to pointers/references.
- **Diamonds**: exercise.

Navigability

Navigability is treated similar to visibility:

Using names of non-navigable association ends ($\nu = \times$) are **forbidden**.

Example: Given

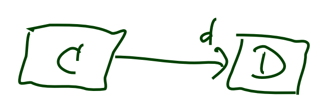


the following OCL expression is **not well-typed** wrt. navigability,

$$\text{context } D \text{ inv} : role.x > 0$$

The standard says: navigation is...

- '$-$': ...possible
- '$\times$': ...not possible
- '$>$': ...efficient



So: In general, UML associations **are different** from pointers / references in general!

But: Pointers / references **can faithfully** be modelled by UML associations.

Multiplicities as Constraints

Recall: Multiplicity is a term of the form $N_1..N_2,\ \dots,\ N_{2k-1}..N_{2k}$

where $N_i \leq N_{i+1}$ for $1 \leq i \leq 2k$, $\quad N_1, \dots, N_{2k-1} \in \mathbb{N}$, $\quad N_{2k} \in \mathbb{N} \cup \{*\}$.

Define $\mu^C_{\mathsf{OCL}}(role) :=$

$$\mathsf{context}\ C\ \mathsf{inv} : (N_1 \leq role \mathbin{\text{->}} \mathsf{size}() \leq N_2)\ \mathsf{or}\ \dots\ \mathsf{or}\ (N_{2k-1} \leq role \mathbin{\text{->}} \mathsf{size}() \underbrace{\leq N_{2k}}_{\text{omit if } N_{2k} = *})$$

for each $\langle r : \dots, \langle role : D, \mu, \_, \_, \_, \_\rangle, \dots, \langle role' : C, \_, \_, \_, \_, \_\rangle, \dots \rangle \in V$ or

$$\langle r : \dots, \langle role' : C, \_, \_, \_, \_, \_\rangle, \dots, \langle role : D, \mu, \_, \_, \_, \_\rangle, \dots \rangle \in V,$$

with $role \neq role'$, if $\mu \neq 0..1$, $\mu \neq 1..1$, and

$$\mu^C_{\mathsf{OCL}}(role) := \mathsf{context}\ C\ \mathsf{inv} : \mathsf{not}(\mathsf{oclIsUndefined}(role))$$

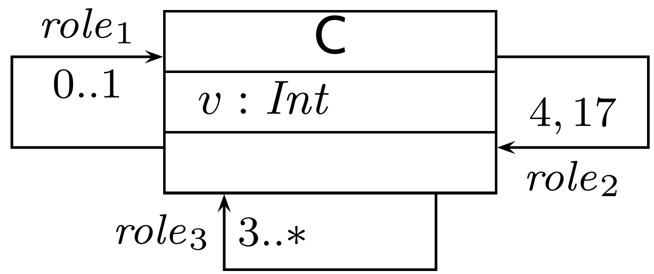
if $\mu = 1..1$.

Note: in n-ary associations with $n > 2$, there is redundancy.

Multiplicities as Constraints Example

$$\mu_{\mathsf{OCL}}^{C}(role) = \text{context } C \text{ inv}: \\ (N_1 \leq role \text{ -> size()} \leq N_2) \text{ or } \ldots \text{ or } (N_{2k-1} \leq role \text{ -> size()} \leq N_{2k})$$

$\mathcal{CD}$:



- $\{\text{context } C \text{ inv}: 4 \leq role_2 \text{ -> size()} \leq 4 \text{ or } 17 \leq role_2 \text{ -> size()} \leq 17\}$
  $= \{\text{context } C \text{ inv}: role_2 \text{ -> size()} = 4 \text{ or } role_2 \text{ -> size()} = 17\}$
- $\cup\ \{\text{context } C \text{ inv}: 3 \leq role_3 \text{ -> size()}\}$

Properties

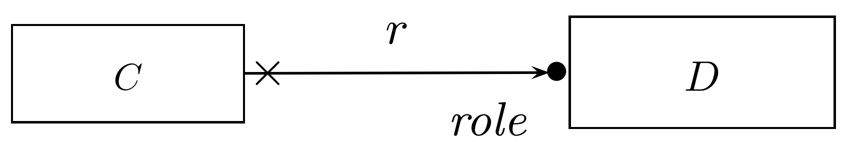
We don't want to cover association **properties** in detail,
only some observations (assume binary associations):

Property	Intuition	Semantical Effect
unique	one object has **at most one** r-link to a single other object	**current setting**
bag	one object may have **multiple** r-links to a single other object	have $\lambda(r)$ yield multi-sets
ordered, **sequence**	an r-link is a **sequence** of object identities (possibly including duplicates)	have $\lambda(r)$ yield sequences

Property	OCL Typing of expression $role(expr)$
unique	$T_D \to Set(T_C)$
bag	$T_D \to Bag(T_C)$
ordered, **sequence**	$T_D \to Seq(T_C)$

For **subsets**, **redefines**, **union**, etc. see (?, 127).

Ownership



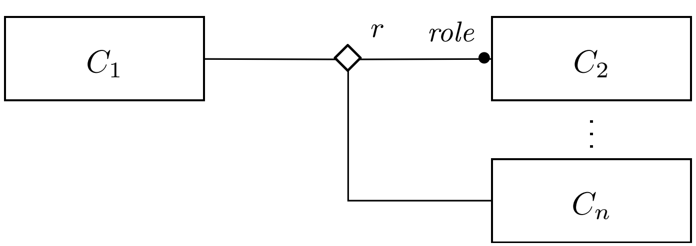
Intuitively it says:

> Association r is **not a "thing on its own"** (i.e. provided by λ),
> but association end '$role$' is **owned** by C (!).
> (That is, it's stored inside C object and provided by σ).

So: if multiplicity of $role$ is $0..1$ or $1..1$, then the picture above is very close to concepts of pointers/references.

Actually, ownership is seldom seen in UML diagrams. Again: if target platform is clear, one may well live without (cf. (OMG, 2011b, 42) for more details).

Not clear to me:



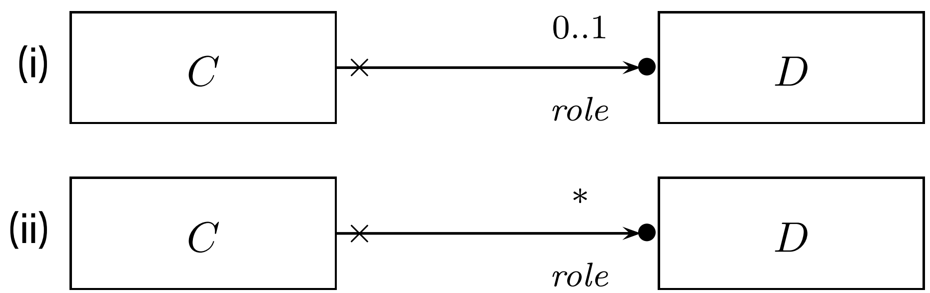
Back to the Main Track

Back to the main track:

Recall: on some earlier slides we said, the extension of the signature is **only** to study associations in "full beauty".
For the remainder of the course, we should look for something simpler...

Proposal:

- **from now on**, we only use associations of the form



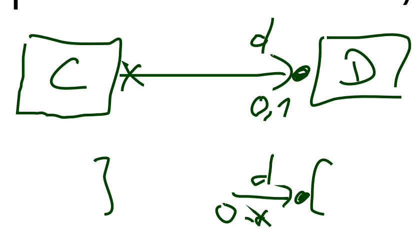
(And we may omit the non-navigability and ownership symbols.)

- Form (i) introduces $role : C_{0,1}$, and form (ii) introduces $role : C_*$ in V.
- In both cases, $role \in atr(C)$.
- We drop λ and go back to our nice σ with $\sigma(u)(role) \subseteq \mathscr{D}(D)$.

Tell Them What You've Told Them...

- From class diagrams with (general) **associations**, we obtain **extended signatures**.
- Links (instances of associations) "live on their own" in the λ in extended system states (σ, λ).
- OCL considers **role names**, the **semantics** is (more or less) **straightforward**.
- **The Rest**:
  - **navigability** is treated like visibility,
  - view **multiplicities** as shorthand for **constraints**,
  - properties, ownership, "diamonds": exist
- **Back to the main track**:

  For simplicity, let's restrict the following discussion to $C_{0,1}$ and C_* as before (now viewed as abbreviations for particular associations).



References

References

OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.