

Software Design, Modelling and Analysis in UML

Lecture 11: Core State Machines I

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Prof. Dr. Andreas Poddicki, Dr. Bernd Westphal
Albert-Ludwigs-Universität Freiburg, Germany

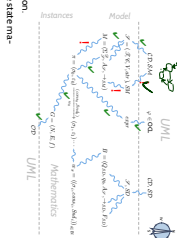
Content

- Recall: Basic Causality Model
- Event Pool
 - insert, remove, clear, ready
- System Configuration
 - implicit attributes: *initial*, *s.t.* and *friends*
 - system state plus event pool
- Actions
 - simple action language
 - transformer: effects of actions

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Readmap: Chronologically

- Syntax:**
- (I) UML State Machine Diagrams ✓
 - (II) Def: Signature with *sgf* ✓
 - (III) Def: Core state machine ✓
 - (IV) Map UML State Machine Diagrams to core state machines ✓
- Semantics:**
- (V) Def: Either (aka event pool) ✓
 - (VI) Def: System configuration ✓
 - (VII) Def: Transformer ✓
 - (VIII) Def: Transition system, computation ✓
 - (IX) Transition relation induced by core state machine ✓
 - (X) Def: step, run-to-completion step ✓
 - (XI) Later: Hierarchical state machines



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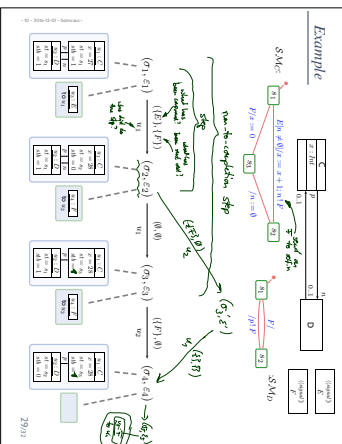
15.3.12 StateMachine (OMG, 2011b, 579)

- Event occurrences are detected, dispatched and then processed by the state machine, one at a time.
- The semantics of event occurrence processing is based on the run-to-completion assumption, interpreted as non-atomic processing.
- By the time an event is processed, the previous `runToCompletionStep` is fully completed.
- The processing of a single event occurrence by a state machine is known as a **run-to-completion step**.
- Before commencing on a run-to-completion step, the state machine must be in a **stable state configuration** with all necessary do-activities completed.
- The same conditions apply after the run-to-completion step is completed.
- Thus, an event occurrence will never be processed if in some intermediate and inconsistent situation.
- [OOV] The **run-to-completion step** is the package between two **stable configurations** of the state machine.
- The run-to-completion assumption since concurrency conflicts are avoided during the processing of event, allowing the step to safely complete its run-to-completion steps.
- The order of depending is **not defined**, since dependencies are resolved in different priority-based schemes.
- Run-to-completion may be implemented in **various ways**.

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Example



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Ether

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- The order of dequeuing is **not defined**, leaving open the possibility of modeling different (priority-based) schemes.



- The standard distinguishes (among ethers)
 - SignalEvent (OMG, 2011b, 450) and Reception (OMG, 2011b, 447).

On SignalEvents, it says

A signal event represents the receipt of an asynchronous signal instance. A signal event may, for example, cause a state machine to trigger a transition. (OMG, 2011b, 449) [1]

Semantic Variation Points

The means by which requests are transported to their target depend on the type of requesting action, the target, the properties of the communication medium, and numerous other factors.

In some cases, this is instantaneous and completely reliable while in others it may involve transmission delays of variable duration, loss of requests, reordering, or duplication.

(See also the discussion on page 421) (OMG, 2011b, 450)

- Our ether (→ in a minute) is a general representation of many possible choices. **Often seen minimal requirement: order of sending by one object is preserved.**

Definition. Let $\mathcal{S} = (\mathcal{S}^R, \mathcal{V}, \text{dir}, \mathcal{E})$ be a signature with signals and \mathcal{S} a structure.

We call a tuple $(Eh, \text{ready}, \oplus, [\cdot])$ an ether over \mathcal{S} and \mathcal{S} if and only if it provides

- a ready operation which yields a set of events (i.e. signal instances) that are ready for a given object, i.e.
- a operation to insert an event for a given object, i.e.
- a operation to remove an event, i.e.
- an operation to clear the ether for a given object, i.e.

$\text{ready} : Eh \times \mathcal{O}(\mathcal{E}) \rightarrow 2^{\mathcal{E}(\mathcal{E})}$
 $\oplus : Eh \times \mathcal{O}(\mathcal{E}) \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$
 $\ominus : Eh \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$
 $[\cdot] : Eh \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$

Example: FIFO Queue

A (single, global, shared, reliable) FIFO queue is an ether.

- $Eh = (\mathcal{D}(\mathcal{E}) \times \mathcal{D}(\mathcal{E}))^*$
 - the set of finite sequences of pairs $(a, b) \in \mathcal{D}(\mathcal{E}) \times \mathcal{D}(\mathcal{E})$
 - ready: $Eh \times \mathcal{O}(\mathcal{E}) \rightarrow 2^{\mathcal{E}(\mathcal{E})}$
 - $(\varepsilon, \text{in}) \mapsto \{(a, b)\}, \text{ if } \varepsilon = (a, b), \varepsilon$
 - $(\varepsilon, \text{in}) \mapsto \{\varepsilon\}, \text{ otherwise}$
 - $\oplus : Eh \times \mathcal{O}(\mathcal{E}) \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$
 - $(\varepsilon, a, b) \mapsto \varepsilon, (a, b, \varepsilon)$
 - $\ominus : Eh \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$
 - $(\varepsilon, a) \mapsto \{\varepsilon\}, \text{ if } \varepsilon = (a, b), b \in \mathcal{D}(\mathcal{E})$
 - $(\varepsilon, a) \mapsto \{\varepsilon\}, \text{ otherwise}$
 - $[\cdot] : Eh \times \mathcal{O}(\mathcal{E}) \rightarrow Eh$
- remove all (val) elements from the pipe. $\varepsilon := \varepsilon \circ \mathcal{D}(\mathcal{E})$

Other Examples

- One FIFO queue per active object is an ether.
 - $Eh = \mathcal{D}(\mathcal{E}) \rightarrow (\mathcal{D}(\mathcal{E}) \times \mathcal{O}(\mathcal{E}))^*$
- One-place buffer.
 - $Eh = \varepsilon \cup (\mathcal{D}(\mathcal{E}) \times \mathcal{D}(\mathcal{E}))$
- Priority queue.
 - ...
- Multi-queues (one per sender)
 - ...
- Inval example: sink, "back hole"
 - ...
- Lossy queue (\oplus needs to become a relation then).
 - ...

System Configuration

System Configuration

Definition. Let $\mathcal{S}_0 = (\mathcal{S}_0, \mathcal{R}_0, \mathcal{V}_0, \text{attr}_0, \mathcal{A}_0)$ be a signature with signals \mathcal{S}_0 , a structure of \mathcal{S}_0 , (E, H) , ready, $\mathcal{S}_0 \subseteq \{1, \dots, n\}$ an ether over \mathcal{S}_0 and \mathcal{S}_0 .
 Furthermore assume there is one core state machine M_C per class $C \in \mathcal{C}$.
 A system configuration over $\mathcal{S}_0, \mathcal{S}_0$ and (E, H) is a pair

$$(\sigma, \varepsilon) \in \Sigma_{\mathcal{S}_0}^{\mathcal{S}_0} \times E, H$$

where

- $\sigma = (\mathcal{S} \cup \{S, C\} \mid C \in \mathcal{C})$, \mathcal{S}_0
- $\mathcal{V}_0 \cup \{v \mid \text{stable} : \text{Bool}, \neg \text{true}(v)\}$
- $\cup \{(\text{param}_i : S, \text{val}_i + \text{st}(v)) \mid C \in \mathcal{C}\}$
- $\cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\}$
- $(C \mapsto \text{attr}(C))$
- $\cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\} \cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\} \cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\}$

and $\varepsilon = \mathcal{S}_0 \cup \{S, C\} \cup \{S, M_C\} \mid C \in \mathcal{C}\}$, and

- $\sigma(v) \cap \mathcal{S}_0(\mathcal{A}_0) = \emptyset$ for each $v \in \text{dom}(\sigma)$ and $r \in \mathcal{V}_0$.
- $\sigma(v) \cap \mathcal{S}_0(\mathcal{A}_0) = \emptyset$ for each $v \in \text{dom}(\sigma)$ and $r \in \mathcal{V}_0$.

Stability

Definition.
 Let (σ, ε) be a system configuration over some $\mathcal{S}_0, \mathcal{S}_0, E, H$.
 We call an object $u \in \text{dom}(\sigma) \cap \mathcal{S}_0(\mathcal{A}_0)$ stable in σ if and only if

$$\sigma(v) \cap \text{stable} = \text{true} \quad ?$$

And unstable otherwise.

System Configuration: Example

C	
E	0, 1, 1
S	1, 1, 1
M	1, 1, 1

$\mathcal{S}_0 = (\mathcal{S}_0, \mathcal{R}_0, \mathcal{V}_0, \text{attr}_0, \mathcal{A}_0)$ where
 $\mathcal{S}_0 = (\mathcal{S}_0 \cup \{S, C\} \mid C \in \mathcal{C})$
 $\mathcal{V}_0 \cup \{v \mid \text{stable} : \text{Bool}, \neg \text{true}(v)\} \cup \{(\text{param}_i : S, \text{val}_i + \text{st}(v)) \mid C \in \mathcal{C}\}$
 $\cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\} \cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\}$
 $(C \mapsto \text{attr}(C))$
 $\cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\} \cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\} \cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\}$

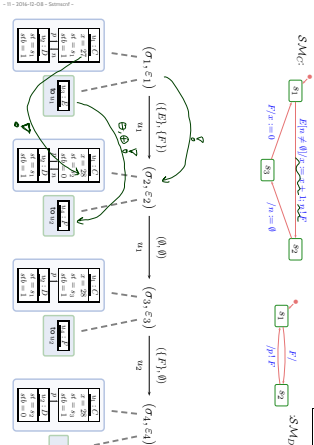
$\sigma = (\mathcal{S} \cup \{S, C\} \mid C \in \mathcal{C})$, \mathcal{S}_0
 $\cup \{(\text{param}_i : S, \text{val}_i + \text{st}(v)) \mid C \in \mathcal{C}\}$
 $\cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\}$
 $(C \mapsto \text{attr}(C))$
 $\cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\} \cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\} \cup \{(\text{stable}_i : \text{st}(v)) \mid C \in \mathcal{C}\}$

$\varepsilon = \mathcal{S}_0 \cup \{S, C\} \cup \{S, M_C\} \mid C \in \mathcal{C}\}$, and
 $\sigma(v) \cap \mathcal{S}_0(\mathcal{A}_0) = \emptyset$ for each $v \in \text{dom}(\sigma)$ and $r \in \mathcal{V}_0$.

$\mathcal{S} = \{S, C\}$
 $\mathcal{V} = \{v \mid \text{stable} : \text{Bool}, \neg \text{true}(v)\}$
 $\cup \{(\text{param}_i : S, \text{val}_i + \text{st}(v)) \mid C \in \mathcal{C}\}$
 $\cup \{(\text{param}_i : E, \text{val}_i + \text{st}(v)) \mid E \in \mathcal{E}_0\}$
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$\varepsilon = \mathcal{S}_0 \cup \{S, C\} \cup \{S, M_C\} \mid C \in \mathcal{C}\}$
 $\sigma(v) \cap \mathcal{S}_0(\mathcal{A}_0) = \emptyset$ for each $v \in \text{dom}(\sigma)$ and $r \in \mathcal{V}_0$.

Where are we?



System Configuration Step-by-Step

- We start with some signature with signals $\mathcal{S}_0 = (\mathcal{S}_0, \mathcal{R}_0, \mathcal{V}_0, \text{attr}_0, \mathcal{A}_0)$.
- A system configuration is a pair (σ, ε) which
- computes a system state σ wrt. \mathcal{S}_0 (not wrt. \mathcal{S}_0).
- Such a system state σ wrt. \mathcal{S}_0 provides, for each object $u \in \text{dom}(\sigma)$,
- values for the explicit attributes in \mathcal{V}_0 ,
- values for a number of implicit attributes, namely
- a stability flag, i.e. $\sigma(v) \cap \text{stable}$ is a boolean value.
- a current (state machine) state, i.e. $\sigma(v) \cap \text{st}$ denotes one of the states of core state machine M_C .
- a temporary association to access event parameters for each class, i.e. $\sigma(v) \cap \text{param}_i$ is defined for each $E \in \mathcal{E}_0$.
- For convenience require there is no link to an event except for param_i .

Transformer

Recall

- The (simplified) syntax of transition annotations
$$annot ::= [\langle event \rangle \ [\langle guard \rangle] \ [\langle action \rangle]]$$
- **Clear:** $\langle event \rangle$ is from \mathcal{E} of the corresponding signature.
- **But:** What are $\langle guard \rangle$ and $\langle action \rangle$?
- UML can be viewed as being parametrised in **expression language** (providing $\langle guard \rangle$) and **action language** (providing $\langle action \rangle$).
- **Examples:**
 - **Expression language**
 - OCL
 - Java C++... expressions
 - ...
 - **Action language:**
 - UML Action Semantics, "Executable UML"
 - Java C++... statements (plus some event send action)
 - ...
 - \rightarrow

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Needed: Semantics

- In the following, we assume that we're given
 - an **expression language** $Expr$ for guards, and
 - an **action language** Act for actions,and that we're given
- a semantics for boolean expressions in form of a **partial function**

$$\llbracket \cdot \rrbracket (\cdot, \cdot) : Expr \times \Sigma_{\mathcal{S}}^{\mathcal{S}} \times \mathcal{D}(\mathcal{V}) \rightarrow \mathbb{B}$$

which evaluates expressions in a given system configuration. Assuming f to be partial is a way to treat "undefined" during runtime. If f is not defined (for instance because of dangling-reference navigation or division-by-zero), we want to go to a designated "error" system configuration.

- a **transformer** for each action, for each $act \in Act$, we assume to have

$$t_{act} \subseteq \mathcal{D}(\mathcal{V}) \times (\Sigma_{\mathcal{S}}^{\mathcal{S}} \times EIn) \times (\Sigma_{\mathcal{S}}^{\mathcal{S}} \times EIn)$$

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Transformer

Definition.
Let $\Sigma_{\mathcal{S}}^{\mathcal{S}}$ the set of system configurations over some $\mathcal{F}_0, \mathcal{S}_0, EIn, EOut$.
We call a relation
$$t \subseteq (\mathcal{D}(\mathcal{V}) \times (\Sigma_{\mathcal{S}}^{\mathcal{S}} \times EIn)) \times (\Sigma_{\mathcal{S}}^{\mathcal{S}} \times EIn)$$

a **system configuration transformer**.

- **Example**
- $t_{[u_2]}(\alpha, \varepsilon) \subseteq \Sigma_{\mathcal{S}}^{\mathcal{S}} \times EIn$ is
- the set (l) of the system configurations
- which may result from object u_2 .
- **executing transformer** t .
- $t_{[obj]}(u_2)(\alpha, \varepsilon) = \{(\alpha, \varepsilon)\}$
- $t_{create}(u_2)(\alpha, \varepsilon)$: add a previously non-existent object to σ / id , $new-obj$, $act(obj)$.

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Observations

- In the following, we assume that
 - each application of a transformer t
 - to some system configuration (α, ε)
 - for object u_2
- is associated with a set of
- observations**
-
- $$Obs_{[u_2]}(u_2)(\alpha, \varepsilon) \in \mathcal{P}^{Obs(\mathcal{O})} \cup (\{*\} \times \mathcal{P}(\mathcal{V}))$$
-
- $$(u_2, u_{id}) \in Obs_{[u_2]}(u_2)(\alpha, \varepsilon)$$
- An **observation**
- represents the information that, as a "side effect" of object
- u_2
- , executing
- t
- in system configuration
- (α, ε)
- , the event
- u_2
- has been sent to
- u_{id}
- .
-
- Special cases:**
- creation
- $(*)$
- / destruction
- $(+)$

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A Simple Action Language

In the following we use

$$Act, \mathcal{S} = \{act, \sharp\}$$

- $\cup \{update(\langle expr_1, v_1, \langle expr_2 \rangle \mid \langle expr_1, \langle expr_2 \rangle \in Expr_{\mathcal{S}}, v \in \mathcal{V} \}$
 - $\cup \{send(\langle Expr_1, \dots, \langle expr_n \rangle, \langle expr_{id} \rangle \mid \langle expr_1, \langle expr_{id} \rangle \in Expr_{\mathcal{S}}, B \in \mathcal{E} \}$
 - $\cup \{create(\langle C, \langle expr \rangle, v \rangle \mid C \in \mathcal{C}, \langle expr \rangle \in Expr_{\mathcal{S}}, v \in \mathcal{V} \}$
 - $\cup \{destroy(\langle expr \rangle \mid \langle expr \rangle \in Expr_{\mathcal{S}} \}$
- and OCL expressions over
- \mathcal{S}
- (with partial interpretation) as
- $Expr_{\mathcal{S}}$
- .

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Transformer Examples: Presentation

abstract syntax	concrete syntax
op	
intuitive semantics	
well-typedness	
semantics	$((\alpha, \varepsilon), (\alpha', \varepsilon')) \in t_{op}(u_2)$ iff ...
observables	$t_{sp}(u_2)(\alpha, \varepsilon) = \{(\alpha', \varepsilon') \mid \text{where ...}\}$
(error) conditions	$Obs_{sp}(u_2) = \{\dots\}$ Not defined if ...

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Transformer: Skip

abstract syntax skip	concrete syntax skip
inuitive semantics	do nothing
well-typedness	/
semantics	$\text{skip}[\sigma] = \{\sigma\}$
observables	$\text{Obs}_{\text{skip}}[\sigma] = \emptyset$
(error) conditions	

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Transformer: Update

abstract syntax update(e , v)	concrete syntax e, v
inuitive semantics update contains v in the object denoted by e to the value denoted by v .	
well-typedness $e, v : T$ and $\sigma : T \in \text{dom}(C)$	
semantics $\text{update}(\sigma, e, v) = \{\sigma[\alpha \mapsto v]\}$	
observables $\text{Obs}_{\text{update}(\sigma, e, v)} = \text{Obs}_{\sigma}$	
(error) conditions Not defined if $\text{Err}[\sigma, e]$ or $\text{Err}[\sigma, v]$ not defined	

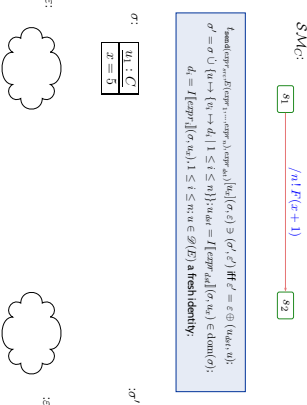
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Transformer: Send

abstract syntax send(E , e_1, \dots, e_n)	concrete syntax e_1, \dots, e_n
inuitive semantics Object $e_1 : C$ sends event E to object e_2 . e_1 creates a fresh signal instance, fill in its attributes and place it in the ether.	
well-typedness $E \in \text{Event}(C)$, $e_1, \dots, e_n \in \text{dom}(C)$	
semantics all expressions obey visibility and navigability in C	
observables $\text{Obs}_{\text{send}(e_1, \dots, e_n)} = \{e_1, \dots, e_n\}$	
(error) conditions Not defined for any $e_i \in \text{Err}[\sigma, e_i]$	

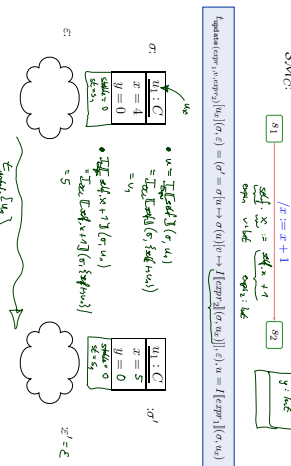
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Send Transformer Example



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Update Transformer Example



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Sequential Composition of Transformers

- Sequential composition $t_1 \circ t_2$ of transformers t_1 and t_2 is canonically defined as

$$(t_2 \circ t_1)(\sigma) = t_2(t_1(\sigma))$$
 with observation

$$\text{Obs}_{(t_2 \circ t_1)(\sigma)} = \text{Obs}_{t_1(\sigma)} \cup \text{Obs}_{t_2(t_1(\sigma))}$$
- Clear not defined if one the two intermediate "micro steps" is not defined

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Observation: our transformers are in principle the **denotational semantics** of the actions/action sequences. The trivial case, to be precise:

Note with the previous examples, we can capture

- empty statements, skips
- assignments
- conditionals (by normalisation and auxiliary variables),
- create/destroy (then),

but not possibly **diverging loops**.

Our (Simple) Approach... if the action language is, e.g. Java then **practically** forbid loops and calls of recursive functions.

Other Approach: use full blown denotational semantics.

No show-stopper, because loops in the action annotation can be converted into transition cycles in the state machine.

- A **ether** is an abstract representation of different possible "event pools" like
 - FIFO queues (shared or per sender),
 - One-place buffers,
 - ...

- A **system configuration** consists of
 - an event pool (pending message),
 - a **system state** over a signature with **input and outputs** for
 - current state
 - stability,
 - etc.

• Transitions are labelled with **actions**; the effect of actions is explained by **transformers** (transformers may modify **system state** and **ether**).

References

References

OMG (2011a). Unified modeling language: Infrastructure version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure version 2.4.1. Technical Report formal/2011-08-06.