Software Design, Modelling and Analysis in UML

Lecture 14: Hierarchical State Machines I

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- **Putting It All Together** (Again)
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- **Hierarchical State Machines**
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    - pseudo-states, regions, …
  - **(Legal) System Configurations**
  - **Abstract Syntax**: Transitions
  - **Enabledness of Fork/Join Transitions**
    - scope, priority, maximality, …

Putting It All Together

Initial States

Recall: a labelled transition system is $(S, A, \to, S_0)$.

We **have**

- S: system configurations (σ, ε)
- $\to$: labelled transition relation $(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$.

Wanted: initial states S_0.

Proposal:

Require a (finite) set of **object diagrams** $\mathscr{OD}$ as part of a UML model

$$(\mathscr{CD}, \mathscr{SM}, \mathscr{OD}).$$

And set

$$S_0 = \{(\sigma, \varepsilon) \mid \sigma \in G^{-1}(\mathcal{OD}), \quad \mathcal{OD} \in \mathscr{OD}, \quad \varepsilon \text{ empty}\}.$$

Other Approach: (used by Rhapsody tool) multiplicity of classes (plus initialisation code).

We can read that as an abbreviation for an object diagram.

Semantics of UML Model (So Far)

The **semantics** of the **UML model**

$$\mathcal{M} = (\mathscr{CD}, \mathscr{SM}, \mathscr{OD})$$

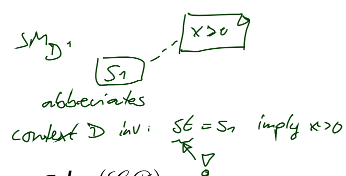
where

- some classes in $\mathscr{CD}$ are stereotyped as 'signal' (standard), some signals and attributes are stereotyped as 'external' (non-standard),
- there is a 1-to-1 relation between classes and state machines,
- $\mathscr{OD}$ is a set of object diagrams over $\mathscr{CD}$,

is the **transition system** $(S, A, \to, S_0)$ constructed on the previous slide(s).

The **computations of** $\mathcal{M}$ are the computations of $(S, A, \to, S_0)$.

OCL Constraints and Behaviour

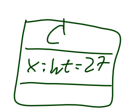


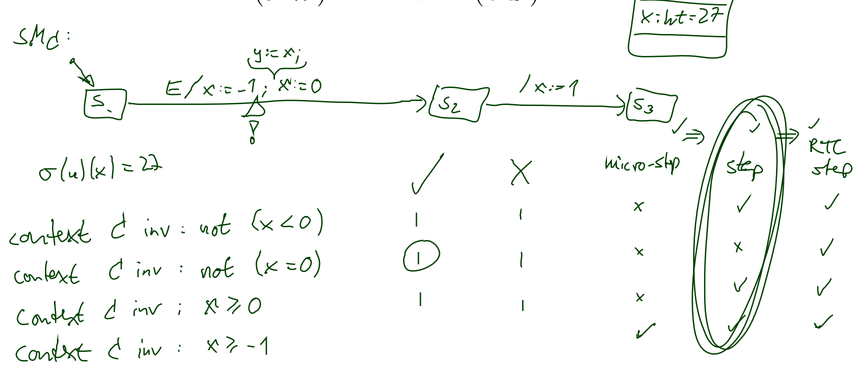
- Let $\mathcal{M} = (\mathscr{CD}, \mathscr{SM}, \mathscr{OD})$ be a UML model.
- We call $\mathcal{M}$ **consistent** iff, for each OCL constraint $expr \in Inv(\mathscr{CD})$,

  $\sigma \models expr$ for each "reasonable point" (σ, ε) of computations of $\mathcal{M}$.

  (Cf. tutorial for discussion of "reasonable point".)

Note: we could define *Inv*$(\mathscr{SM})$ similar to *Inv*$(\mathscr{CD})$.





Last Missing Piece: Create and Destroy Transformer

Transformer: Create

abstract syntax	**concrete syntax**
`create`$(C, expr, v)$	expr.v := new C

intuitive semantics

Create an object of class C and assign it to attribute v of the object denoted by expression $expr$.

well-typedness

$$expr : T_D, v \in atr(D),$$
$$atr(C) = \{\langle v_i : T_i, expr_i^0 \rangle \mid 1 \leq i \leq n\}$$

semantics

$$\ldots$$

observables

$$\ldots$$

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined.

x = (new C).y + (new D).z;

can be written as

tmp₁ := new C;

tmp₂ := new D;

x = tmp₁.y + tmp₂.z;

Transformer: Create

abstract syntax	**concrete syntax**
$\mathtt{create}(C, expr, v)$	

intuitive semantics

Create an object of class C and assign it to attribute v of the object denoted by expression $expr$.

well-typedness

$$expr : T_D, v \in atr(D),$$
$$atr(C) = \{\langle v_1 : T_1, expr_i^0 \rangle \mid 1 \leq i \leq n\}$$

semantics

$$\ldots$$

observables

$$\ldots$$

(error) conditions

$I[\![expr]\!](\sigma, \beta)$ not defined.

- We use an "and assign"-action for simplicity – it doesn't add or remove expressive power, but moving creation to the expression language raises all kinds of other problems since then expressions would need to modify the system state.
- Also for simplicity: no parameters to construction ($\sim$ parameters of constructor). Adding them is straightforward (but somewhat tedious).

How To Choose New Identities?

- **Re-use**: choose any identity that is not alive **now**, i.e. not in $\mathrm{dom}(\sigma)$.
  - Doesn't depend on history.
  - May "undangle" dangling references – may happen on some platforms.

- **Fresh**: choose any identity that has not been alive **ever**, i.e. not in $\mathrm{dom}(\sigma)$ and any predecessor in current run.
  - Depends on history.
  - Dangling references remain dangling – could mask "dirty" effects of platform.

Transformer: Create

abstract syntax — **concrete syntax**

$\texttt{create}(C, expr, v)$

intuitive semantics

Create an object of class C and assign it to attribute v of the object denoted by expression $expr$.

well-typedness

$$expr : T_D,\, v \in atr(D),$$
$$atr(C) = \{\langle v_1 : T_1, expr_i^0\rangle \mid 1 \le i \le n\}$$

semantics

$$((\sigma, \varepsilon), (\sigma', \varepsilon')) \in t_{\texttt{create}(C,expr,v)}[u_x]$$

iff

$$\sigma' = \underbrace{\sigma[u_0 \mapsto \sigma(u_0)[v \mapsto u]]}_{\text{similar to update}} \cup \underbrace{\{u \mapsto \{v_i \mapsto d_i \mid 1 \le i \le n\}\}}_{\text{similar to send}},$$

$\varepsilon' = [u](\varepsilon)$; $\quad u \in \mathscr{D}(C)$ fresh, i.e. $u \notin \mathrm{dom}(\sigma)$;

$u_0 = I[\![expr]\!](\sigma, u_x)$; $d_i = I[\![expr_i^0]\!](\sigma, \emptyset)$ if $expr_i^0 \neq \emptyset$ and arbitrary value from $\mathscr{D}(T_i)$ otherwise.

clean others

observables

creation

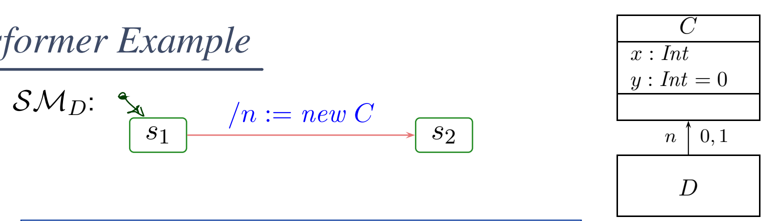
$$Obs_{\texttt{create}}[u_x] = \{(*, u)\}$$

(error) conditions

$expr_i^0$

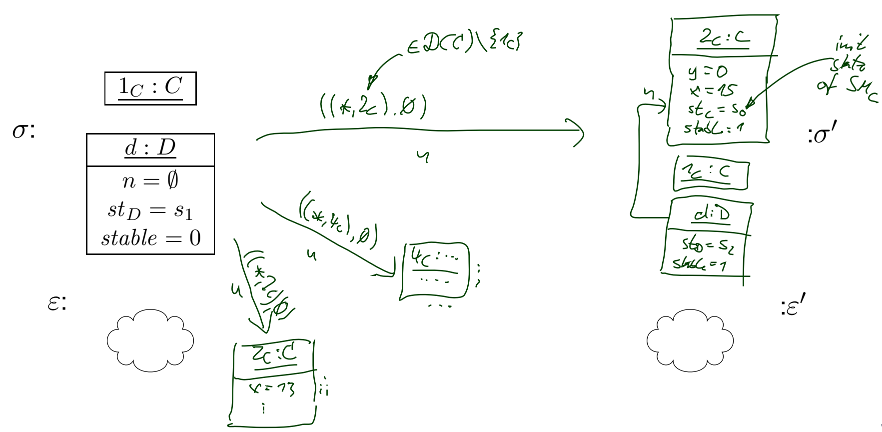
$I[\![expr]\!](\sigma, u_x)$ not defined.

Create Transformer Example



$\texttt{create}(C, expr, v)$

$t_{\texttt{create}(C,expr,v)}[u_x](\sigma, \varepsilon) = \ldots$



Transformer: Destroy

abstract syntax	concrete syntax
$\texttt{destroy}(expr)$	
intuitive semantics	
Destroy the object denoted by expression $expr$.	
well-typedness	
$expr : T_C, C \in \mathscr{C}$	
semantics	
$\dots$	
observables	
$Obs_{\texttt{destroy}}[u_x] = \{(u_x, \bot, (+, \emptyset), u)\}$	
(error) conditions	
$I[\![expr]\!](\sigma, \beta)$ not defined.	

What to Do With the Remaining Objects?

Assume object u_0 is destroyed…

- object u_1 may still refer to it via association r:
  - allow dangling references?
  - or remove u_0 from $\sigma(u_1)(r)$?
- object u_0 may have been the last one linking to object u_2:
  - leave u_2 alone?
  - or remove u_2 also? (garbage collection)
- Plus: (temporal extensions of) OCL may have dangling references.

Our choice: Dangling references and no garbage collection!

This is in line with "expect the worst", because there are target platforms which don't provide garbage collection – and models shall (in general) be correct without assumptions on target platform.

But: the more "dirty" effects we see in the model, the more expensive it often is to analyse. Valid proposal for simple analysis: monotone frame semantics, no destruction at all.

Transformer: Destroy

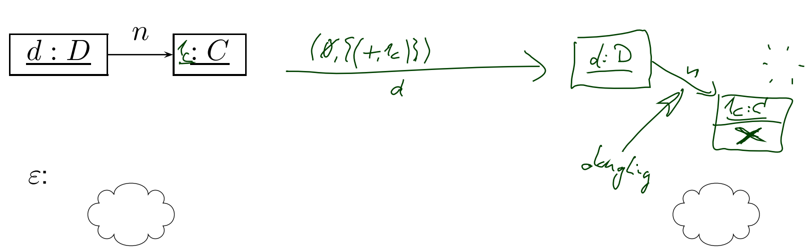
abstract syntax	concrete syntax
$\texttt{destroy}(\mathit{expr})$	
intuitive semantics	
Destroy the object denoted by expression expr.	
well-typedness	
$\mathit{expr} : T_C, C \in \mathscr{C}$	
semantics	
$t_{\texttt{destroy}(\mathit{expr})}[u_x](\sigma, \varepsilon) = \{(\sigma', \varepsilon')\}$ $\quad \varepsilon' = [u](\varepsilon)$	
where $\sigma' = \sigma\|_{\mathrm{dom}(\sigma)\setminus\{u\}}$ with $u = I[\![\mathit{expr}]\!](\sigma, u_x)$.	
function restriction	
observables	
$\mathit{Obs}_{\texttt{destroy}(\mathit{expr})}[u_x] = \{(+, u)\}$	
(error) conditions	
$I[\![\mathit{expr}]\!](\sigma, u_x)$ not defined.	

Destroy Transformer Example

$\mathcal{SM}_C$:

$$\texttt{destroy}(\mathit{expr})$$
$$t_{\texttt{destroy}(\mathit{expr})}[u_x](\sigma, \varepsilon) = \ldots$$

σ: $\qquad$:σ'

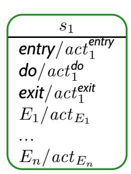
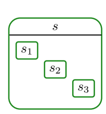
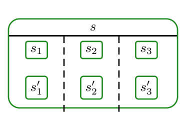


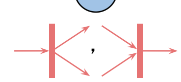
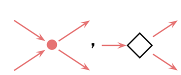
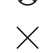
ε: $\qquad$:ε'

Hierarchical State-Machines

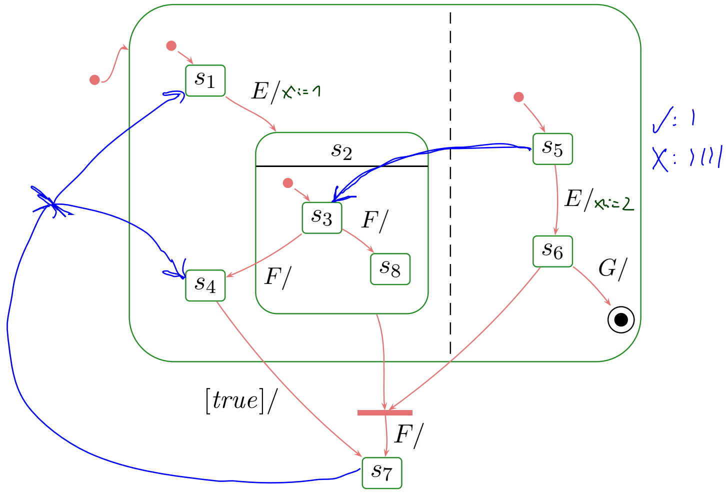
The Full Story

UML distinguishes the following **kinds of states**:

	example
simple state	
final state	
composite state	
OR	
AND	

	example
pseudo-state	
initial	
(shallow) history	H
deep history	H*
fork/join	
junction, choice	
entry point	
exit point	
terminate	
submachine state	$S : s$

Blessing or Curse. . . ?



Blessing or Curse. . . ?

Plan:

States / Syntax:

- What is the abstract syntax of a diagram?

States / Semantics:

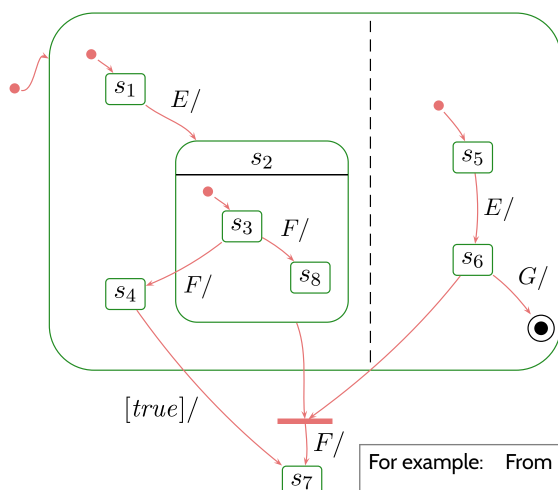
- what is the type of the implicit st attribute?
- what are **legal system configurations**?

Transitions / Syntax:

- what are **legal** / well-formed transitions?

Transitions / Semantics:

- when is a legal transition enabled?
- which effects do transitions have?



For example: From s_1, s_5,

- what may happen on E?
- what may happen on $\underline{E}, F$?
- can $\underline{E}, G$ kill the object?
- ...

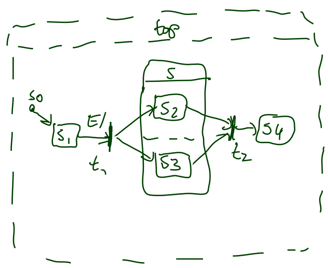
- **So far**:

init. state

source state — trigger — guard — action — dst. state

$$(S, s_0, \to), \quad s_0 \in S, \quad \to \ \subseteq S \times (\mathscr{E} \cup \{\_\}) \times Expr_{\mathscr{S}} \times Act_{\mathscr{S}} \times S$$

set of states — edges



kind: init

kind: st

s_0

$\{s, s_1, \dots, s_4, top\}$,

$\{t_1, t_2\}$,

$\{t_1 \mapsto (\{s_1\}, \{s_2, s_3\}), \dots\}$

$\{t_1 \mapsto (E, true, skip)\}$

$s \mapsto \{ \{s_2\}, \ \{s_3\} \}$,

$s_2 \mapsto \emptyset$

$top \mapsto \{ \{s_0, s_1, s, s_4\} \}$

Representing All Kinds of States

- **So far**:

$$(S, s_0, \to), \quad s_0 \in S, \quad \to \ \subseteq S \times (\mathscr{E} \cup \{\_\}) \times Expr_{\mathscr{S}} \times Act_{\mathscr{S}} \times S$$

- **From now on**: (**hierarchical**) **state machines**

$$(S, kind, region, \to, \psi, annot)$$

where

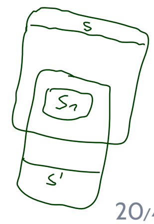
- $S \supseteq \{top\}$ is a finite set of states **(new: top)**,
- $kind : S \to \{$*st*, *init*, *fin*, *shist*, *dhist*, *fork*, *join*, *junc*, *choi*, *ent*, *exi*, *term*$\}$ is a function which labels states with their **kind**, **(new)**
- $region : S \to 2^{2^S}$ is a function which characterises the **regions** of a state, **(new)**
- $\to$ is a set of transitions, **(changed)**
- $\psi : (\to) \to 2^S \times 2^S$ is an **incidence function**, and **(new)**
- $annot : (\to) \to (\mathscr{E} \cup \{\_\}) \times Expr_{\mathscr{S}} \times Act_{\mathscr{S}}$ provides an annotation for each transition. **(new)**

(s_0 is then redundant – replaced by proper state (!) of kind '*init*'.)

Well-Formedness: Regions

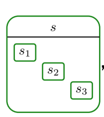
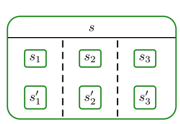
	$\in S$	$kind$	$region \subseteq 2^S, S_i \subseteq S$	$child \subseteq S$
final state	s	*fin*	$\emptyset$	$\emptyset$
pseudo-state	s	*init*, …	$\emptyset$	$\emptyset$
simple state	s	*st*	$\emptyset$	$\emptyset$
composite state	s	*st*	$\{S_1, \dots, S_n\}, n \geq 1$	$S_1 \cup \dots \cup S_n$
implicit top state	top	*st*	$\{S_1\}$	S_1

- Final and pseudo states **must not comprise** regions.
- States $s \in S$ with $kind(s) =$ *st* **may comprise** regions.
  Naming conventions can be defined based on regions:
  - No region: simple state.
  - One region: OR-state.
  - Two or more regions: AND-state.
- Each state (except for top) **must** lie in exactly one region.
- **Note**: The region function induces a **child** function.
- **Note**: Diagramming tools (like Rhapsody) can ensure well-formedness.



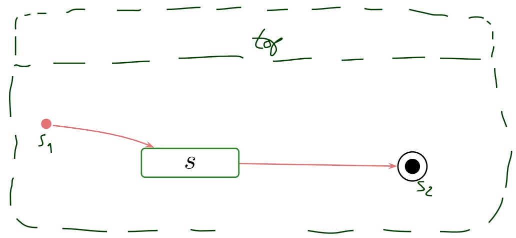
From UML to Hierarchical State Machine: By Example

$(S, kind, region, \to, \psi, annot)$

	example	$\in S$	$kind$	$region$
simple state	$\boxed{s}$	s	st	$\emptyset$
final state	◉ q	q	fin	$\emptyset$
composite state				
OR		s	st	$\{\{s_1, s_2, s_3\}\}$
				region
AND		s	st	$\{\{s_1, s_1'\}, \{s_2, s_2'\}, \{s_3, s_3'\}\}$
submachine state	(later)			
pseudo-state	q •, …	q	init, …	

$\underbrace{\hphantom{\in S \quad kind}}_{(s, kind(s)) \text{ for short}}$

From UML to Hierarchical State Machine: By Example



... denotes $(S, kind, region, \to, \psi, annot)$ with

- $S = \{top, s_1, s, s_2\}$
- $kind = \{top \mapsto$ **st**, $s_1 \mapsto$ **init**, $s \mapsto$ **st**, $s_2 \mapsto$ **fin**$\}$
- or $(S, kind) = \{(top, \textbf{st}), (s_1, \textbf{init}), (s, \textbf{st}), (s_2, \textbf{fin})\}$
- $region = \{top \mapsto \{\{s_1, s, s_2\}\}, s_1 \mapsto \emptyset \quad , s \mapsto \emptyset \quad , s_2 \mapsto \emptyset \quad \}$
- $\to, \psi, annot$: in a minute.

– 14 – 2016-12-22 – Shiersyn –

Recall

Plan:

States / Syntax:

- What is the abstract syntax of a diagram? ✓

States / Semantics:

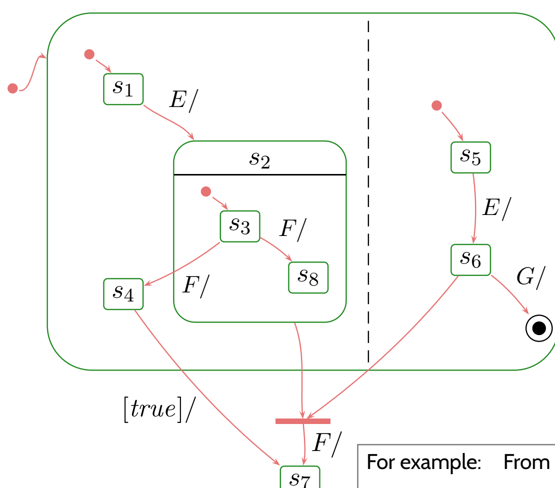
- what is the type of the implicit st attribute?
- what are **legal system configurations**?

Transitions / Syntax:

- what are **legal** / well-formed transitions?

Transitions / Semantics:

- when is a legal transition enabled?
- which effects do transitions have?



For example: From s_1, s_5,

- what may happen on E?
- what may happen on $\underline{E}, F$?
- can $\underline{E}, G$ kill the object?
- ...

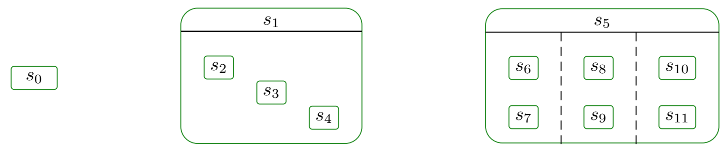
– 14 – 2016-12-22 – Shiersyn –

Semantics: State Configuration

- The type of (implicit attribute) st is from now on **a set of** states, i.e. $\mathscr{D}(S_{M_C}) = 2^S$
- A set $S_1 \subseteq S$ is called (**legal**) **state configuration** if and only if
  - $top \in S_1$, and
  - for each region R of a state in S_1,
    exactly one (non pseudo-state) element of R is in S_1, i.e.

$$\forall\, s \in S_1\; \forall\, R \in region(s) \bullet |\{s \in R \mid kind(s) \in \{\mathsf{st}, \mathsf{fin}\}\} \cap S_1| = 1.$$

- **Examples**:



$S_1 = \{s_0\}$ ✗
$S_2 = \{s_0, top\}$ ✓
$S_3 = \{s_1, top\}$ ✗
$S_4 = \{s_1, top, s_3, s_4\}$ ✗
$S_5 = \{s_1, top, s_4\}$ ✓
$S_6 = \{s_5, top, s_6, s_9\}$ ✗
$S_7 = \{s_5, top, s_6, s_9, s_{10}\}$ ✓
$S_8 = \{top, s_0, s_1, s_2\}$ ✗

Recall

Plan:

States / Syntax:
- What is the abstract syntax of a diagram? ✓

States / Semantics:
- what is the type of the implicit st attribute?
- what are **legal system configurations**? ✓

Transitions / Syntax:
- what are **legal** / well-formed transitions?

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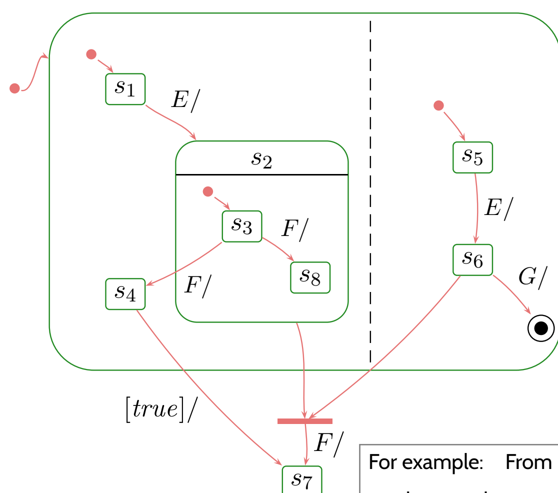


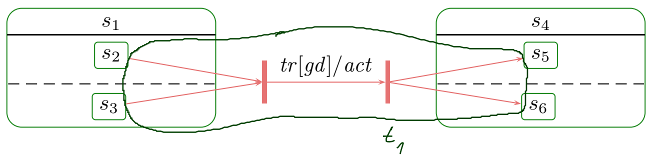
For example: From s_1, s_5,
- what may happen on E?
- what may happen on $\underline{E}, F$?
- can $\underline{E}, G$ kill the object?
- ...

Transitions Syntax: Fork/Join

- For simplicity, we consider transitions with (possibly) multiple sources and targets, i.e.

$$\psi : (\rightarrow) \rightarrow (2^S \setminus \emptyset) \times (2^S \setminus \emptyset)$$

- For instance,



translates to

$$(S, kind, region, \underbrace{\{t_1\}}_{\rightarrow}, \underbrace{\{t_1 \mapsto (\{s_2, s_3\}, \{s_5, s_6\})\}}_{\psi}, \underbrace{\{t_1 \mapsto (tr, gd, act)\}}_{annot})$$

- Naming convention: $\psi(t) = (source(t), target(t))$.

Orthogonal States

- Two states $s_1, s_2 \in S$ are called **orthogonal**, denoted $s_1 \perp s_2$, if and only if
  - they "live" in different regions of **one** AND-state, i.e.

$$\exists\, s, region(s) = \{S_1, \dots, S_n\}, 1 \leq i \neq j \leq n : s_1 \in child(S_i) \wedge s_2 \in child(S_j),$$

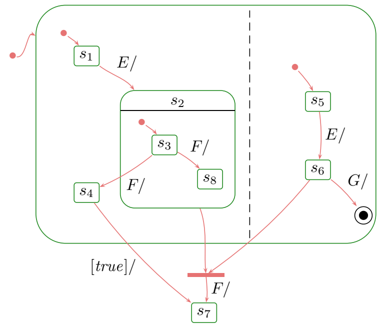
Legal Transitions

A hierarchical state-machine $(S, kind, region, \rightarrow, \psi, annot)$ is called **well-formed** if and only if for all transitions $t \in \rightarrow$,

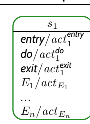
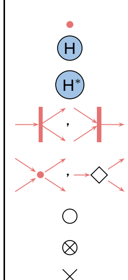
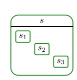
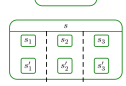
- source (and destination) states are pairwise orthogonal, i.e.
  - $\forall\, s, s' \in source(t)\ (\in target(t)) \bullet s \perp s'$,
- the top state is neither source nor destination, i.e.
  - $top \notin source(t) \cup source(t)$.

Recall: final states are not sources of transitions.

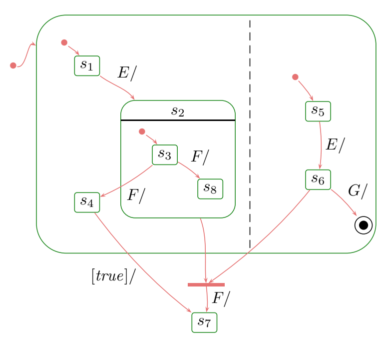
Example:



Plan

	example		example
simple state		**pseudo-state**	
		initial	
		(shallow) history	
		deep history	
		fork/join	
final state		junction, choice	
composite state		entry point	
OR		exit point	
AND		terminate	
		submachine state	$S : s$

- Transitions involving non-pseudo states.
- Initial pseudostate, final state.
- Entry/do/exit actions, internal transitions.
- History and other pseudostates, the rest.



- For the **Create Action**, we have two main choices:
  - **re-use** identities ("nasty semantics"),
  - use **fresh** identities ("clean semantics", depends on history).

  Similar for **Destroy**.

- **Hierarchical State Machines** introduce **Regions**.
  - Thereby, **states** can lie within **states** as **children**.
  - The implicit variable st becomes set-valued.
- **Transitions** may now have
  - **multiple** source states, **multiple** destination states,
  - but need to adhere to **well-formedness conditions**.
- **Enabledness** of a set (!) of transitions
  is **a bit tricky to define** ($\rightarrow$ scope, priority, maximality).
- **Steps** are a proper generalisation of core state machines.

References

References

OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.