Software Design, Modelling and Analysis in UML

Lecture 19: Live Sequence Charts III

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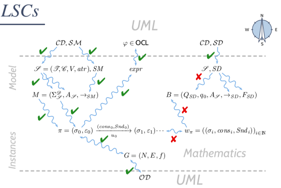
Content

Live Sequence Charts — Semantics

TBA-based Semantics of LSCs



Plan:

(i) Given an LSC $\mathscr{L}$ with body

$$((L, \preceq, \sim), \mathcal{I}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv}, \Theta),$$

(ii) construct a TBA $\mathcal{B}_{\mathscr{L}}$, and

(iii) define language $\mathcal{L}(\mathscr{L})$ of $\mathscr{L}$ in terms of $\mathcal{L}(\mathcal{B}_{\mathscr{L}})$, in particular taking activation condition and activation mode into account.

(iv) define language $\mathcal{L}(\mathcal{M})$ of a UML model.

- Then $\mathcal{M} \models \mathscr{L}$ (**universal**) if and only if $\mathcal{L}(\mathcal{M}) \subseteq \mathcal{L}(\mathscr{L})$.
  And $\mathcal{M} \models \mathscr{L}$ (**existential**) if and only if $\mathcal{L}(\mathcal{M}) \cap \mathcal{L}(\mathscr{L}) \neq \emptyset$.

Live Sequence Charts — TBA Construction

Formal LSC Semantics: It's in the Cuts!

Definition.
Let $((L, \preceq, \sim), \mathcal{I}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv}, \Theta)$ be an LSC body.
A non-empty set $\emptyset \neq C \subseteq L$ is called a **cut** of the LSC body iff

- it is **downward closed**, i.e. $\forall l, l' \bullet l' \in C \wedge l \preceq l' \implies l \in C$,
- it is **closed** under **simultaneity**, i.e.

$$\forall l, l' \bullet l' \in C \wedge l \sim l' \implies l \in C, \text{ and}$$

- it comprises at least **one location per instance line**, i.e.

$$\forall i \in I \; \exists l \in C \bullet i_l = i.$$

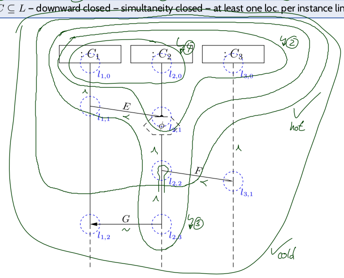
The **temperature function** is extended to cuts as follows:

$$\Theta(C) = \begin{cases} \text{hot} & , \text{if } \exists l \in C \bullet (\nexists l' \in C \bullet l \prec l') \wedge \Theta(l) = \text{hot} \\ \text{cold} & , \text{otherwise} \end{cases}$$

that is, C is **hot** if and only if at least one of its maximal elements is hot.

Cut Examples

$\emptyset \neq C \subseteq L$ – ~~downward closed – simultaneity closed – at least one loc. per instance line~~



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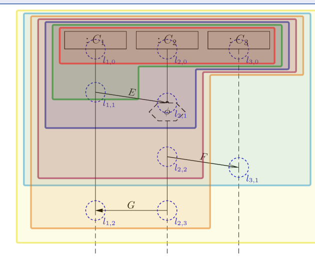


A Successor Relation on Cuts

The partial order "$\preceq$" and the simultaneity relation "$\sim$" of locations induce a **direct successor relation** on cuts of an LSC body as follows:

Definition.
Let $C \subseteq L$ bet a cut of LSC body $((L, \preceq, \sim), \mathcal{I}, \mathsf{Msg}, \mathsf{Cond}, \mathsf{LocInv}, \Theta)$.

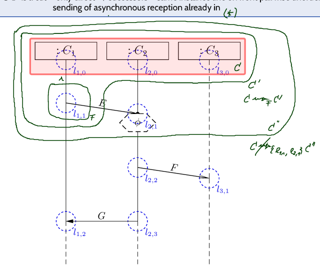
A set $\emptyset \neq F \subseteq L$ of locations is called fired-set F of cut C if and only if

- $C \cap F = \emptyset$ and $C \cup F$ is a cut, i.e. F is closed under simultaneity,
- all locations in F are direct $\prec$-successors of the front of C, i.e.
$$\forall\, l \in F\, \exists\, l' \in C \bullet l' \prec l \wedge (\nexists\, l'' \in C \bullet l' \prec l'' \prec l),$$
- locations in F, that lie on the same instance line, are pairwise unordered, i.e.
$$\forall\, l \neq l' \in F \bullet (\exists\, I \in \mathcal{I} \bullet \{l, l'\} \subseteq I) \implies l \not\preceq l' \wedge l' \not\preceq l,$$
- for each asynchronous (!) message reception in F, the corresponding sending is already in C,
$$\forall\, (l, E, l') \in \mathsf{Msg} \bullet l' \in F \implies l \in C.$$

The cut $C' = C \cup F$ is called direct successor of C via F, denoted by $C \rightsquigarrow_F C'$.

Successor Cut Example

$C \cap F = \emptyset$ – $C \cup F$ is a cut – only direct $\prec$-successors – same instance line on front pairwise unordered – sending of asynchronous reception already in (+)



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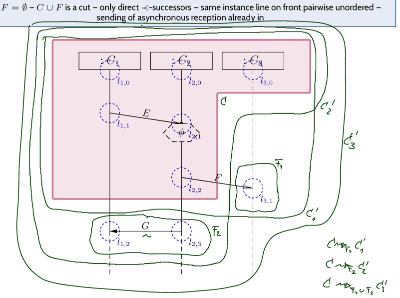
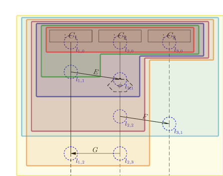
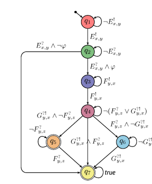


Language of LSC Body: Example





The TBA $\mathcal{B}_{\mathscr{L}}$ of LSC $\mathscr{L}$ ~~over Φ and ℰ~~ is $(Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ with

- Q is **the set of cuts** of $\mathscr{L}$, q_{ini} is the **instance heads** cut,
- $Expr_{\mathcal{B}}(X) = Expr_{\mathscr{S}}(\mathscr{E}, X)$ (for considered signature $\mathscr{S}$),
- $\rightarrow$ consists of **loops**, **progress transitions** (by $\rightsquigarrow_F$), and **legal exits** (cold cond./local inv.),
- $Q_F = \{C \in Q \mid \Theta(C) = \text{cold} \vee C = L\}$ is the set of cold cuts and the maximal cut.

Signal and Attribute Expressions

- Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$ be a signature and X a set of logical variables,
- The signal and attribute expressions $Expr_{\mathscr{S}}(\mathscr{E}, X)$ are defined by the grammar:

$$\psi ::= \textit{true} \mid \cancel{\boxed{\psi}} \; E^!_{x,y} \mid E^?_{x,y} \mid \neg\psi \mid \psi_1 \vee \psi_2 \mid expr,$$

where $expr : Bool \in Expr_{\mathscr{S}}$, $E \in \mathscr{E}$, $x, y \in X$ (or keyword env).

- We use

$$\mathscr{E}_{!?}(X) := \{E^!_{x,y}, E^?_{x,y} \mid E \in \mathscr{E}, x, y \in X\}$$

to denote the set of **event expressions** over $\mathscr{E}$ and X.

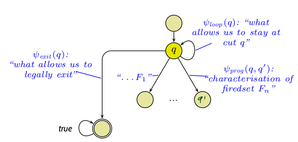
TBA Construction Principle

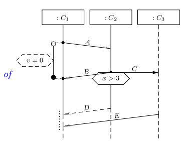
Recall: The TBA $\mathcal{B}(\mathscr{L})$ of LSC $\mathscr{L}$ is $(Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ with

- Q is **the set of cuts** of $\mathscr{L}$, q_{ini} is the **instance heads** cut,
- $Expr_{\mathcal{B}} = \Phi \,\dot\cup\, \mathscr{E}_{!?}(X)$,
- $\rightarrow$ consists of **loops**, **progress transitions** (from $\rightsquigarrow_F$), and **legal exits** (cold cond./local inv.),
- $F = \{C \in Q \mid \Theta(C) = \text{cold} \vee C = L\}$ is the set of cold cuts.

So in the following, we "only" need to construct the transitions' labels:

$$\rightarrow = \{(q, \psi_{loop}(q), q) \mid q \in Q\} \cup \{(q, \psi_{prog}(q, q'), q') \mid q \rightsquigarrow_F q'\} \cup \{(q, \psi_{exit}(q), L) \mid q \in Q\}$$





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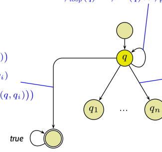
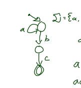
$$\psi_{loop}(q) = \overbrace{\psi^{\mathsf{Msg}}(q) \wedge \psi^{\mathsf{LocInv}}_{\mathsf{hot}}(q)}^{=:\psi^{\mathsf{hot}}_{loop}(q)} \wedge \psi^{\mathsf{LocInv}}_{\mathsf{cold}}(q)$$

$$\psi_{exit}(q) = \left(\psi^{\mathsf{hot}}_{loop}(q) \wedge \neg\psi^{\mathsf{LocInv}}_{\mathsf{cold}}(q)\right) \vee \bigvee_{1 \le i \le n} \left(\psi^{\mathsf{hot}}_{prog}(q, q_i) \wedge \left(\neg\psi^{\mathsf{LocInv},\bullet}_{\mathsf{cold}}(q, q_i) \vee \neg\psi^{\mathsf{Cond}}_{\mathsf{cold}}(q, q_i)\right)\right)$$

$$\psi_{prog}(q, q_n) = \overbrace{\psi^{\mathsf{Msg}}(q, q_n) \wedge \psi^{\mathsf{Cond}}_{\mathsf{hot}}(q, q_n) \wedge \psi^{\mathsf{LocInv},\bullet}_{\mathsf{hot}}(q, q_n)}^{=:\psi^{\mathsf{hot}}_{prog}(q, q_n)} \wedge \psi^{\mathsf{Cond}}_{\mathsf{cold}}(q, q_n) \wedge \psi^{\mathsf{LocInv},\bullet}_{\mathsf{cold}}(q, q_n)$$



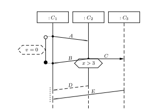


$\Sigma = \{a, b, c\}$

$a^* bc$

$aacb$

$aab\cancel{c}$



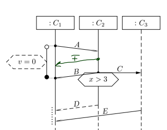
Loop Condition

$$\psi_{loop}(q) = \psi^{\mathsf{Msg}}(q) \wedge \psi^{\mathsf{LocInv}}_{\mathsf{hot}}(q) \wedge \psi^{\mathsf{LocInv}}_{\mathsf{cold}}(q)$$

- $\psi^{\mathsf{Msg}}(q) = \neg \bigvee_{1 \le i \le n} \psi^{\mathsf{Msg}}(q, q_i) \wedge \underbrace{\left(strict \implies \bigwedge_{\psi \in \mathsf{Msg}(L)} \neg\psi\right)}_{=:\psi_{\mathsf{strict}}(q)}$
- $\psi^{\mathsf{LocInv}}_{\theta}(q) = \bigwedge_{\ell = (l, \iota, \phi, l', \iota') \in \mathsf{LocInv},\ \Theta(\ell) = \theta,\ \ell \text{ active at } q} \phi$

  A location l is called **front location** of cut C if and only if $\nexists\, l' \in L \bullet l \prec l'$.

  Local invariant $(l_o, \iota_0, \phi, l_1, \iota_1)$ is **active** at cut (!) q
  if and only if $l_0 \preceq l \prec l_1$ for some front location l of cut q ~~or $l_1 \in q \wedge \iota_1 = \bullet$.~~
- $\mathsf{Msg}(F) = \{E^!_{x_l, x_{l'}} \mid (l, E, l') \in \mathsf{Msg},\ l \in F\} \cup \{E^?_{x_l, x_{l'}} \mid (l, E, l') \in \mathsf{Msg},\ l' \in F\}$
- $x_l \in X$ is the logical variable associated with the instance line I which includes l, i.e. $l \in I$.
- $\mathsf{Msg}(F_1, \ldots, F_n) = \bigcup_{1 \le i \le n} \mathsf{Msg}(F_i)$



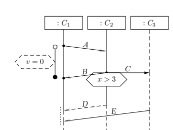
Progress Condition

$$\psi^{\mathsf{hot}}_{prog}(q, q_i) = \psi^{\mathsf{Msg}}(q, q_n) \wedge \psi^{\mathsf{Cond}}_{\mathsf{hot}}(q, q_n) \wedge \psi^{\mathsf{LocInv},\bullet}_{\mathsf{hot}}(q_n)$$

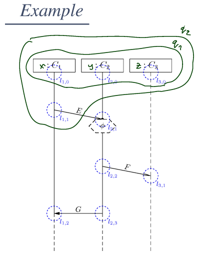
- $\psi^{\mathsf{Msg}}(q, q_i) = \bigwedge_{\psi \in \mathsf{Msg}(q_i \setminus q)} \psi \wedge \bigwedge_{j \ne i} \bigwedge_{\psi \in \mathsf{Msg}(q_j \setminus q) \setminus \mathsf{Msg}(q_i \setminus q)} \neg\psi$
  $\wedge \underbrace{\left(strict \implies \bigwedge_{\psi \in \mathsf{Msg}(L) \setminus \mathsf{Msg}(F_i)} \neg\psi\right)}_{=:\psi_{\mathsf{strict}}(q, q_i)}$
- $\psi^{\mathsf{Cond}}_{\theta}(q, q_i) = \bigwedge_{\gamma = (L, \phi) \in \mathsf{Cond},\ \Theta(\gamma) = \theta,\ L \cap \underbrace{(q_i \setminus q)}_{\text{fired-set}} \ne \emptyset} \phi$
- $\psi^{\mathsf{LocInv},\bullet}_{\theta}(q, q_i) = \bigwedge_{\lambda = (l, \iota, \phi, l', \iota') \in \mathsf{LocInv},\ \Theta(\lambda) = \theta,\ \lambda\ \bullet\text{-active at } q_i} \phi$

  Local invariant $(l_0, \iota_0, \phi, l_1, \iota_1)$ is **$\bullet$-active** at q if and only if
  - $l_0 \prec l \prec l_1$, or
  - $l = l_0 \wedge \iota_0 = \bullet$, or
  - $l = l_1 \wedge \iota_1 = \bullet$

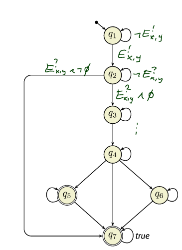
  for some front location l of cut (!) q.



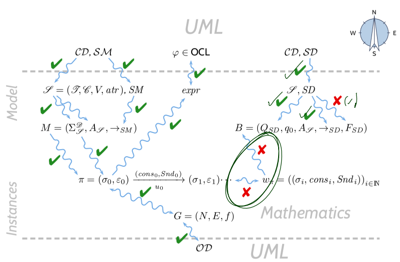
Example



Using logical variables x, y, z for the instances lines (from left to right).

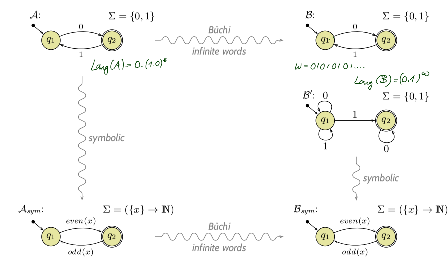


Course Map



Excursion: Büchi Automata

From Finite Automata to Symbolic Büchi Automata



Symbolic Büchi Automata

Definition. A **Symbolic Büchi Automaton** (TBA) is a tuple

$$\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$$

where

- X is a set of logical variables,
- $Expr_{\mathcal{B}}(X)$ is a set of Boolean expressions over X,
- Q is a finite set of **states**,
- $q_{ini} \in Q$ is the initial state,
- $\rightarrow \subseteq Q \times Expr_{\mathcal{B}}(X) \times Q$ is the **transition relation**. Transitions (q, ψ, q') from q to q' are labelled with an expression $\psi \in Expr_{\mathcal{B}}(X)$.
- $Q_F \subseteq Q$ is the set of **fair** (or accepting) states.

Word

Definition. Let X be a set of logical variables and let $Expr_{\mathcal{B}}(X)$ be a set of Boolean expressions over X.

A set $(\Sigma, \cdot \models . \cdot)$ is called an **alphabet** for $Expr_{\mathcal{B}}(X)$ if and only if

- for each $\sigma \in \Sigma$,
  - for each expression $expr \in Expr_{\mathcal{B}}$, and
    - for each valuation $\beta : X \to \mathscr{D}(X)$ of logical variables,

$$\textbf{either} \quad \sigma \models_\beta expr \quad \textbf{or} \quad \sigma \not\models_\beta expr.$$

(σ **satisfies** (or does not satisfy) $expr$ under valuation β)

An **infinite sequence**

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in \Sigma^\omega$$

over $(\Sigma, \cdot \models . \cdot)$ is called **word** (for $Expr_{\mathcal{B}}(X)$).

Run of TBA over Word

Definition. Let $\mathcal{B} = (Expr_{\mathcal{B}}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ be a TBA and

$$w = \sigma_1, \sigma_2, \sigma_3, \ldots$$

a word for $Expr_{\mathcal{B}}(X)$. An infinite sequence

$$\varrho = q_0, q_1, q_2, \ldots \in Q^\omega$$

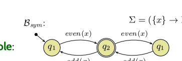
is called **run of $\mathcal{B}$ over w** under valuation $\beta : X \to \mathscr{D}(X)$ if and only if

- $q_0 = q_{ini}$,
- for each $i \in \mathbb{N}_0$ there is a transition

$$(q_i, \psi_i, q_{i+1}) \in \rightarrow$$

such that $\sigma_i \models_\beta \psi_i$.

Example:



The Language of a TBA

Definition.
We say TBA $\mathcal{B} = (Expr_\mathcal{B}(X), X, Q, q_{ini}, \rightarrow, Q_F)$ **accepts** the word

$$w = (\sigma_i)_{i \in \mathbb{N}_0} \in (Expr_\mathcal{B} \rightarrow \mathbb{B})^\omega$$

if and only if $\mathcal{B}$ **has** a run

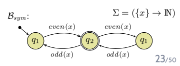
$$\varrho = (q_i)_{i \in \mathbb{N}_0}$$

over w such that fair (or accepting) states are **visited infinitely often** by ϱ,
i.e., such that

$$\forall\, i \in \mathbb{N}_0\, \exists\, j > i : q_j \in Q_F.$$

We call the set $\mathcal{L}(\mathcal{B}) \subseteq (Expr_\mathcal{B} \rightarrow \mathbb{B})^\omega$ of words that are accepted by $\mathcal{B}$ the **language of** $\mathcal{B}$.

Example:



References

References

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OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.