

Content

- Inheritance
  - Abstract syntax
  - UML Substitution Principle
  - Well-posedness with inheritance
  - Subset-semantics vs. uplink-semantics
- Meta-Modelling
  - Idea
  - Experiment: can we model classes?
  - Revisit the UML 2.x standard
  - Model Semantics (MS)
  - Model Semantics (MO)
  - The principle illustrated (once again)

- And That's It!
  - The map - in hindsight
  - Educational objectives - useful questions
- Any open questions?

Inheritance

Abstract Syntax

A signature with inheritance is a tuple

$$\mathcal{S} = (\mathcal{S}, \mathcal{V}, \text{attr}, \mathcal{F}, \text{meth}, \leq)$$

where

- $(\mathcal{S}, \mathcal{V}, \text{attr}, \mathcal{F})$  is a signature with signals and behavioural features
- $(\mathcal{F}, \text{meth})$  are methods, analogous to  $V/\text{attr}$  attributes, and
- is a **substitution** relation, i.e.  $C \leq C'$  for no  $C \in \mathcal{C}$ .

$$\leq \subseteq (\mathcal{C} \times \mathcal{C}) \cup (\mathcal{F} \times \mathcal{F})$$

is a **substitution** relation, i.e.  $C \leq C'$  for no  $C \in \mathcal{C}$ .

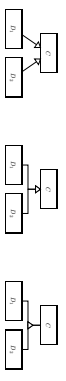
In the following (for simplicity), we assume that all attribute/method names are of the form  $C \leq C'$  and  $C \leq F$  for some  $C \in \mathcal{C}$  or  $F \in \mathcal{F}$  (fully qualified names).

- Read  $C \leq D$  as:
  - $D$  inherits from  $C$ .
  - $C$  is a generalization of  $D$ .
  - $C$  is a **super-class** of  $D$ .
  - $D$  is a **sub-class** of  $C$ .

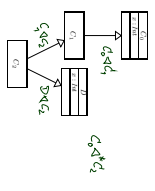


Inheritance: Concrete Syntax

Common graphical representations for  $\leq = ((C, D), (C, D_2))$ :



Mapping Concrete to Abstract Syntax by Example:



Note: we can have multiple inheritance

Desired Semantics of Specialisation: Subtyping

There is a classical description of what one expects from sub-types, which is closely related to inheritance in object-oriented approaches:

The principle of **type substitutability**.  
Liskov's Substitution Principle (LSP) Liskov (1988), Liskov and Wing (1994).

### Desired Semantics of Specialisation: Subtyping

There is a classical description of what one expects from sub-types, which is closely related to inheritance in object-oriented approaches.

The principle of **type substitutability**:

Liskov Substitution Principle (LSP) Liskov (1988); Liskov and Wing (1994).

If for each object  $o_S$  of type  $S$

there is an object  $o_T$  of type  $T$

such that for all programs  $P$  defined in terms of  $T$

the behavior of  $P$  is unchanged when  $o_S$  is substituted for  $o_T$

then  $S$  is a subtype of  $T$ .

In other words: **Richard and Wehner (2000)**

"An instance of the sub-type shall be usable

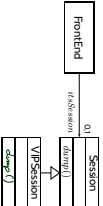
whenever an instance of the supertype was expected

without a client being able to tell the difference"

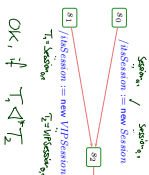


6/11

### Static Sub-Typing



In FrontEnd's state machine:



7/11

### System States with Inheritance

Wanted: a formal representation of "If  $C \leq T$ ,  $D$  then  $D$  is a  $C$ " that is:

(i)  $D$  has the same attributes and behavioral features as  $C$ ; and

(ii)  $D$  objects (identified) can replace  $C$  objects.

Two approaches to semantics

• Domain-inclusion Semantics

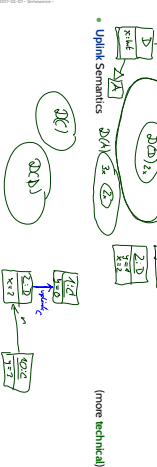
for  $u, v \in \mathcal{O}(C_i)$ :

$dom \sigma(D) = \bigcup_{i \in I} \sigma(D_i)$

(more theoretical)

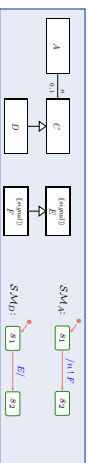
• Uplink Semantics

(more technical)



9/11

### Inheritance and State-Machines: Example



SMC.A	m1	no
SMC.A	m2	no
SMC.C	m1	no
SMC.C	m2	no
SMC.D	m1	no
SMC.D	m2	no
SMC.SMC	m1	yes
SMC.SMC	m2	yes

(4)  $(\text{inv}, P, \text{inv}) \rightarrow \text{yes}$

$\mathcal{O}(C) \rightarrow \mathcal{O}(D)$

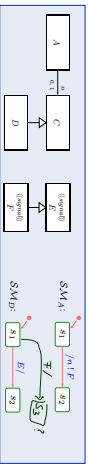
$\mathcal{O}(D) \rightarrow \mathcal{O}(SMC)$

$\sigma = \epsilon$

$\sigma = \epsilon$

10/11

### Domain Inclusion vs. Uplink Semantics



SMC.A	m1	no
SMC.A	m2	no
SMC.C	m1	no
SMC.C	m2	no
SMC.D	m1	no
SMC.D	m2	no
SMC.SMC	m1	yes
SMC.SMC	m2	yes

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$\mathcal{O}(C) \rightarrow \mathcal{O}(D)$

$\mathcal{O}(D) \rightarrow \mathcal{O}(SMC)$

$\sigma = \epsilon$

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10/11

(ii) Dispatch

$(\sigma, \xi) \xrightarrow{\text{consumes } \xi} (\sigma', \xi')$   
 if  

- $w \in \text{dom}(\sigma) \cap \text{dom}(\xi) \wedge \exists \text{type} \in \text{dom}(\xi) : w \in \text{mod}(\xi, w)$
- $w$  is stable and its action  $\text{mod}(\xi, w)$  is  $\text{act}(w)$  and  $\text{act}(\sigma)(\sigma) = s_w$
- a transition is enabled, i.e.  
 $\exists (k, F, \text{expr}, \text{act}, \delta) \in \text{SSM}(\xi) : F \subseteq \xi \wedge \text{[expr]}(\sigma, w) = 1$   
 $\delta = \text{of\_type } w \text{ mod } \xi \rightarrow w$   
 $\xi' = \xi \setminus \text{[consumes } \xi]$

 and  

- $(\sigma', \xi')$  results from applying  $\text{act}$  to  $(\sigma, \xi)$  and removing  $w$  from the ethic.  
 $\sigma' = (\sigma \setminus \{w, \text{act}\}) \cup \{s_w, \text{mod}(\xi, w) \rightarrow s_w\}$  **action  $s_w$**   
 $\xi' = \xi \setminus \{w\}$
- where  $\delta$  depends (see (i))
- Consumption of  $w$  and the side-effects of the action are observed, i.e.  
 $\text{cons} = \{w, \delta\}$ ,  $\text{Side} = \{s_w, \text{mod}(\xi, w)\}$

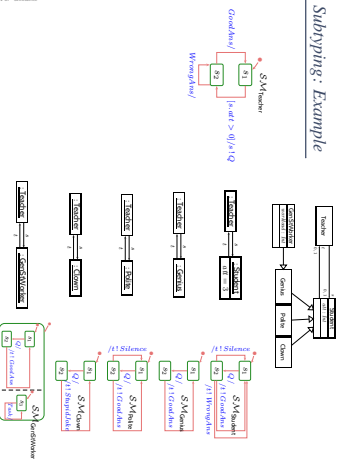
Recall: Subtyping

There is a classical description of what one expects from sub-types, which is closely related to inheritance in object-oriented approaches:

**The principle of type substitutability** Liskov (1988), Liskov and Wing (1994)

If for each object  $o$  of type  $S$   
 there is an object  $o'$  of type  $T$   
 such that for all programs  $P$  defined in terms of  $T$   
 the behavior of  $P$  is unchanged when  $o$  is substituted for  $o'$   
 then  $S$  is a subtype of  $T$ .  
 In other words: Fichtel and Wehrlein (2000)  
 "An instance of the sub-type shall be usable  
 whenever an instance of the supertype was expected,  
 without a client being able to tell the difference."

Subtyping: Example



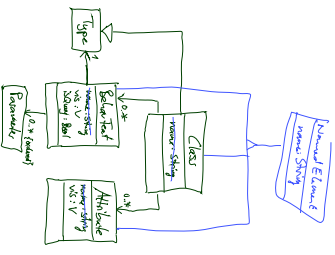
Meta-Modelling: Idea

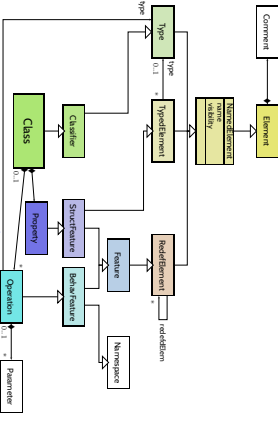
Meta-Modelling: Why and What

- Meta-Modelling is one major prerequisite for understanding the standard documents OMG (UML, B, and the MDA ideas of the OMG)
- The idea is somewhat simple:
- if a modeling language is about modeling things
- and if UML models are things
- then why not describe (or model) the set of all UML models using a modeling language?

Meta-Modelling: Example

- For example, let's consider a class.
- A class has (among others)
  - a name,
  - any number of attributes,
  - any number of behavioural features
- Each of the latter two has
- a name and
  - a visibility
- Behavioural features in addition have
- a boolean attribute isOidernary,
  - any number of parameters,
  - a return type.
- Can we model this (in UML, for a start)?





7/46

The UML 2x Standard Revisited

18/46

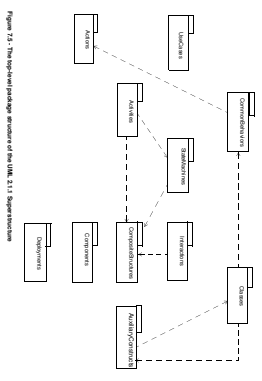
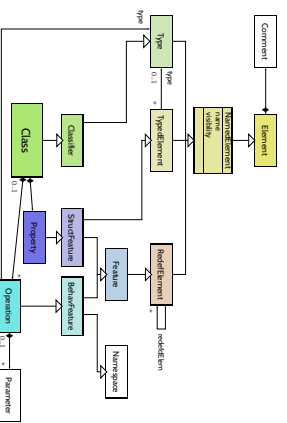


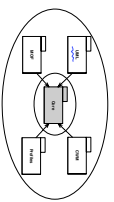
Figure 21: The superstructure packages of the UML 2.1.1 Superstructure

20/46

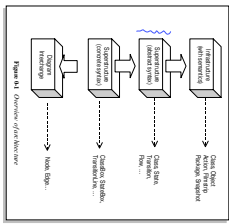


21/46

- Meta-modelling has already been used for UML 1x
- For UML 2.0 the request for a separation of concerns, Infrastructure and Superstructure
- One reason: sharing with MOF (see later) and, e.g. CWM



19/46



19/46

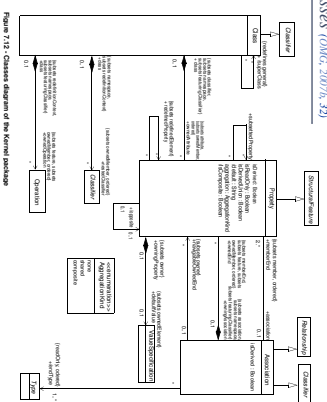


Figure 7.13: Classes diagram of the kernel package

22/46

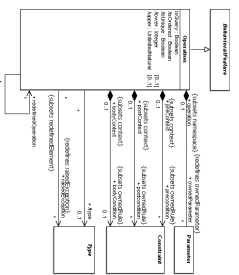


Figure 27: Operation diagram of the kernel package

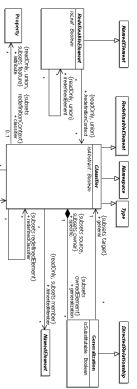


Figure 28: Classifiers diagram of the kernel package

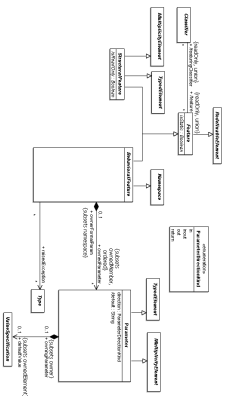


Figure 29: Relation diagram of the kernel package

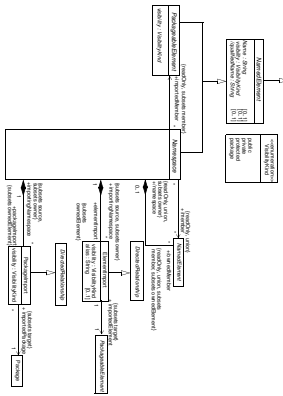


Figure 31: Namespace diagram of the kernel package

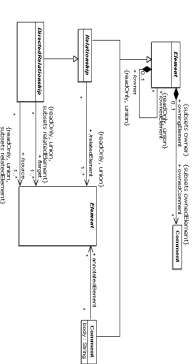
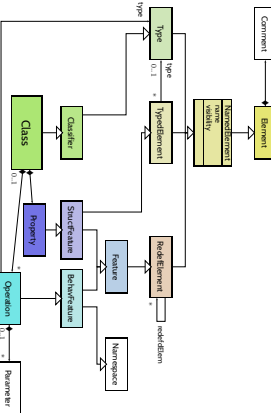


Figure 33: Root diagram of the kernel package



The slide contains a table of contents with sections like 'Introduction', 'Motivation', 'Goals', 'Prerequisites', 'Structure', 'References', and 'Appendix'. Below the table, there is introductory text about the course's focus on modeling and the use of the Staple2 framework.

The slide features a diagram of the Staple2 architecture. It shows a central 'Staple2' component connected to 'Modeling' and 'Code Generation'. Below this, there are sections for 'Introduction', 'Motivation', 'Goals', 'Prerequisites', 'Structure', 'References', and 'Appendix', similar to the first slide but with updated content.

The slide features a diagram of the Staple2 architecture, similar to the previous slides, showing the relationship between 'Staple2', 'Modeling', and 'Code Generation'. It also includes a table of contents and introductory text.

Meta Object Facility (MOF)

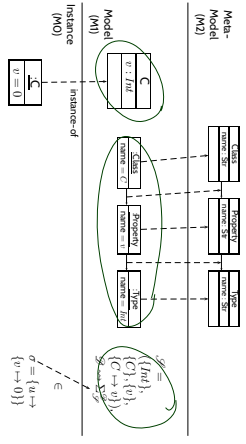
Open Questions...

- Now you've been "tricked"...
- We didn't tell what the modelling language for meta-modelling is
- Idea: have a minimal object-oriented core comprising the notions of **class**, **association**, **inheritance**, etc. with **self**, **external** & **signature**
- This is Meta Object Facility (MOF), which forms one of the core services with UML Infrastructure (UML INFRA)
- So things on meta level
- MO are object diagrams/system states
- MI are words of the language UML
- M2 are words of the language MOF
- M3 are words of the language MOF

Benefits

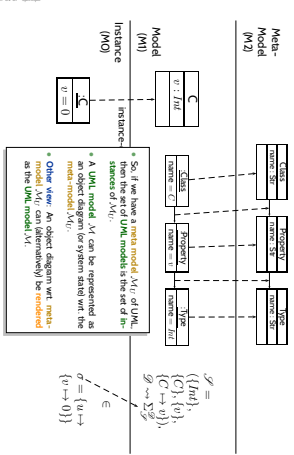
- In particular:
- Benefits for Modelling Tools
- Benefits for Language Design
- Benefits for Code Generation and MDA

### Meta-Modelling: Principle



34

### Modelling vs. Meta-Modelling



35

### Well-Formedness as Constraints in the Meta-Model

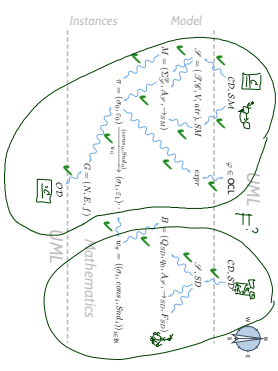
- The set of well-formed UML models can be defined as the set of object diagrams satisfying all constraints of the meta-model.
  - Constant example:
    - Generalization hierarchies must be declared logical. A class  $C$  is a valid object diagram of  $M_1$  iff (i.e. satisfies all constraints from  $hw(M_1)$ ) then  $M_1$  is a well-formed UML model.
    - not self: allParent() -> includesSelf [OMG, 2007b, 53]
  - The other way round:
    - Given a UML model  $M_1$ , unfold it into an object diagram  $O_1$  wrt.  $M_1$ .
    - If  $O_1$  is a valid object diagram of  $M_1$  (i.e. satisfies all constraints from  $hw(M_1)$ ), then  $M_1$  is a well-formed UML model.
- That is: if we have an object diagram validity checker for the meta-modelling language, then we have a well-formedness checker for UML models.

36

And That's It!

37

### Modelling vs. Meta-Modelling



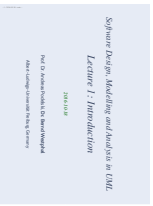
### The Map

38



Content

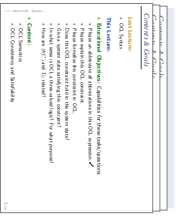
- Lecture 1 Introduction



39/41

Content

- Lecture 1 Introduction
- Lecture 2 Semantical Model
- Lecture 3 Object Constraint Language (OCL)
- Lecture 4 OCL Semantics



39/41

Content

- Lecture 1 Introduction
- Lecture 2 Semantical Model



39/41

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- Lecture 3 Object Constraint Language (OCL)
- Lecture 4 OCL Semantics
- Lecture 5 Object Diagrams



39/41

Content

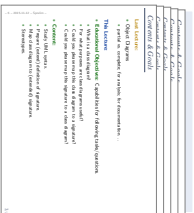
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- Lecture 5 Object Diagrams
- Lecture 6 Class Diagrams



39/41

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39/41





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- Lecture 13 Core State Machines III
- Lecture 14 Hierarchical State Machines I
- Lecture 15 Hierarchical State Machines II
- Lecture 16 Hierarchical State Machines III
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- Lecture 18 Live Sequence Charts II
- Lecture 19 Live Sequence Charts III
- Lecture 20 Live Sequence Charts IV
- Lecture 21 MDE Eshelance
- Lecture 22 MDE Modeling

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