Software Design, Modelling and Analysis in UML

Lecture 2: Semantical Model

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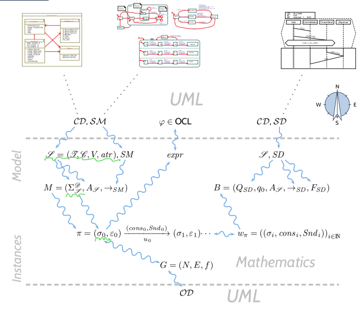
Course Map



Content

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  - typed attributes,
  - attribute mapping.
- **Basic Object System Structure**
  - objects / object identities,
  - domains of basic and derived types.
- **System State**
  - concrete and symbolic,
  - dangling references,
- **A Complete Example**

Semantical Foundation

Basic Object System Signature

Definition. A (Basic) Object System Signature is a quadruple

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr) \qquad C_1, C_2$$

where

- $\mathscr{T}$ is a set of (basic) types,
- $\mathscr{C}$ is a finite set of classes,
- V is a finite set of typed attributes, i.e., each $v \in V$ has a type
  - $\tau \in \mathscr{T}$, or
  - $C_{0,1}$ or C_*, where $C \in \mathscr{C}$

  (written $v : \tau$ or $v : C_{0,1}$ or $v : C_*$).
- $atr : \mathscr{C} \to 2^V$ maps each class to its set of attributes.

Class name total functions Powerset of V

Note: Inspired by OCL 2.0 standard OMG (2006), Annex A.

Basic Object System Signature Example

- $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where
- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$ (both finite),
- typed attributes V, τ from $\mathscr{T}$, or $C_{0,1}$ or C_*, for some $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

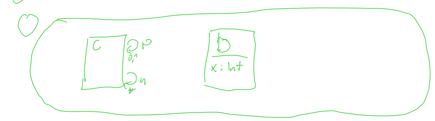
Example:

$\mathscr{T}$ set of basic type Attributes V atr

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

set of classes $\mathscr{C}$

atr(C) = {p, n}
atr(D) = {x}



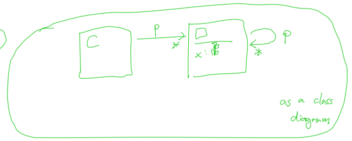
Basic Object System Signature Another Example

$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$ (both finite),
- typed attributes V, τ from $\mathscr{T}$, or $C_{0,1}$ or C_*, for some $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

Example:

$\mathscr{S}_1 = (\{ ✿, MyType\}, \{C, D\}, \{ ~~x : Int~~, x : ✿, p : D_*, q : D_{0,1}\},$
$\{C \mapsto \{p\}, D \mapsto \{x, p\}\})$



Basic Object System Structure

Definition. A Basic Object System Structure of $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ is a domain function $\mathscr{D}$ which assigns to each type a domain, i.e.

- $\tau \in \mathscr{T}$ is mapped to $\mathscr{D}(\tau)$. $\mathscr{D}(\mathscr{T})$ is a set
- $C \in \mathscr{C}$ is mapped to an infinite set $\mathscr{D}(C)$ of (object) identities.
  Note: Object identities only have the "=" operation.
- Sets of object identities for different classes are disjoint, i.e.
  $$\forall C, D \in \mathscr{C} : C \neq D \to \mathscr{D}(C) \cap \mathscr{D}(D) = \emptyset.$$
- C_* and $C_{0,1}$ for $C \in \mathscr{C}$ are mapped to $2^{\mathscr{D}(C)}$. $\mathscr{D}(C_*) := 2^{\mathscr{D}(C)}$, $\mathscr{D}(C_{0,1}) := 2^{\mathscr{D}(C)}$

We use $\mathscr{D}(\mathscr{C})$ to denote $\bigcup_{C \in \mathscr{C}} \mathscr{D}(C)$; analogously $\mathscr{D}(\mathscr{C}_*)$.

$\mathscr{D}(\mathscr{C}_*) = \bigcup_{C \in \mathscr{C}} \mathscr{D}(C_*)$

Note: We identify objects and object identities, because both uniquely determine each other (cf. OCL 2.0 standard).

Basic Object System Structure Example

Wanted: a structure for signature

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

$\mathscr{D}$ needs to map:

- $\tau \in \mathscr{T}$ to **some** $\mathscr{D}(\tau)$,
- $C \in \mathscr{C}$ to **some** set of identities $\mathscr{D}(C)$ (infinite, disjoint for different classes),
- C_* and $C_{0,1}$ for $C \in \mathscr{C}$: always mapped to $\mathscr{D}(C_*) = \mathscr{D}(C_{0,1}) = 2^{\mathscr{D}(C)}$.

$$\mathscr{D}(Int) = \mathbb{Z}$$
$$\mathscr{D}(C) = \mathbb{N}^+ \times \{C\} \cong \{1_C, 2_C, 3_C, 4_C, \ldots\}$$
$$\mathscr{D}(D) = \mathbb{N}^+ \times \{D\} \cong \{1_D, 2_D, 3_D, \ldots\}$$
$$\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) = 2^{\mathscr{D}(C)}$$
$$\mathscr{D}(D_{0,1}) = \mathscr{D}(D_*) = 2^{\mathscr{D}(D)}$$

$\mathscr{D}(Int) = \{-2, -1, 0, 1, 2\}$
$\mathscr{D}(C) = \{a, aa, aaa, \ldots\}$
$\mathscr{D}(D) = \{b, bb, bbb, \ldots\}$

System State

Definition. Let $\mathscr{D}$ be a structure of $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$.

A system state of $\mathscr{S}$ wrt. $\mathscr{D}$ is a type-consistent mapping

$$\sigma : \underbrace{\mathscr{D}(\mathscr{C})}_{\text{set of all class identities}} \nrightarrow (V \underbrace{\nrightarrow}_{\text{partial fnc.}} \underbrace{(\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*))}_{\text{union of all domains } \mathscr{D}(\tau) \text{ and all subset of object identities}}).$$

That is, for each $u \in \mathscr{D}(C)$, $C \in \mathscr{C}$, if $u \in \mathrm{dom}(\sigma)$

- $\mathrm{dom}(\sigma(u)) = atr(C)$
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$
- $\sigma(u)(v) \in \mathscr{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathscr{C}$

type-consistent

We call $u \in \mathscr{D}(\mathscr{C})$ alive in σ if and only if $u \in \mathrm{dom}(\sigma)$.

We use $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ to denote the set of all system states of $\mathscr{S}$ wrt. $\mathscr{D}$.

System State Example

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$
$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, \ldots\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, \ldots\}$$

Wanted: $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that (i) $\mathrm{dom}(\sigma(u)) = atr(C)$, and
(ii) $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$, (iii) $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ with $D \in \mathscr{C}$.

$\sigma_0 = \emptyset$ ("empty function")
alive in σ_0: none

$\sigma_1 = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{1_C, 5_C\}\}, 5_C \mapsto \{p \mapsto \{1_C\}, n \mapsto \emptyset\}, 3_D \mapsto \{x \mapsto 3\}\}$

$\sigma_2 = \{1_D \mapsto \{x \mapsto 27\}, 2_D \mapsto \{x \mapsto 27\}, 17_D \mapsto \{x \mapsto 0\}\}$

System State Example

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$
$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, \ldots\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, \ldots\}$$

Wanted: $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that (i) $\mathrm{dom}(\sigma(u)) = atr(C)$, and
(ii) $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$, (iii) $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ with $D \in \mathscr{C}$.

Two options:

- **Concrete, explicit** identities:
  $$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}.$$
  $1_C \mapsto \emptyset$
- **Alternative**: **symbolic** system state.
  $$\sigma = \{c_1 \mapsto \{p \mapsto \emptyset, n \mapsto \{c_2\}\}, c_2 \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, d \mapsto \{x \mapsto 23\}\}$$
  assuming $c_1, c_2 \in \mathscr{D}(C)$, $d \in \mathscr{D}(D)$, $c_1 \neq c_2$.

System State: Spot the 10 (?) Mistakes

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$
$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, ...\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, ...\}$$

Wanted: $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that (i) $\mathrm{dom}(\sigma(u)) = atr(C)$, and
(ii) $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$, (iii) $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ with $D \in \mathscr{C}$.

- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, \quad 5_C \mapsto \{p \mapsto \emptyset, n \mapsto 1_C\}, \quad 1_D \mapsto \{x \mapsto 2.3\}\}.$
  (iii) $\notin \mathscr{D}(C_*)$ (ii) $\notin \mathscr{D}(Int)$
- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, \quad 5_C \mapsto \{p \mapsto 1_C, n \mapsto \emptyset\}, \quad 1_D \mapsto \{x \mapsto 23\}\}.$
- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{1_D\}\}, \quad 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, \quad 1_D \mapsto \{x \mapsto 22\}\}.$
  (iii) $\notin \mathscr{D}(C_*)$
- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, \quad 5_C \mapsto \{n \mapsto \emptyset\}, \quad 1_D \mapsto \{x \mapsto 1, p \mapsto \{1_C\}\}\}.$
  (i) $p \notin \mathrm{dom}\,\sigma(5_C)$
- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, \quad 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \{9_C\}\}\}$
  (iii): $\{9_C\} \in \mathscr{D}(C_*)$

Dangling References

Definition. Let $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$ be a system state.
We say attribute $v \in V_{0,1,*}$, i.e. $v : C_{0,1}$ or $v : C_*$, in object $u \in \mathrm{dom}(\sigma)$ has a dangling reference if and only if the attribute's value comprises an object which is not alive in σ, i.e. if

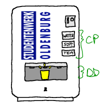
$$\sigma(u)(v) \not\subseteq \mathrm{dom}(\sigma).$$

We call σ closed if and only if no attribute has a dangling reference in any object alive in σ.

Example:

- $\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}\}$

A Complete Example: Vending Machine Structure



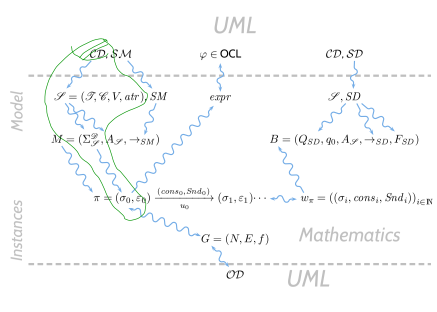
$\mathscr{S} = (\{Int, Bool\},$
$\{VM, CP, DD\},$
$\{cp : CP_*, dd : DD_{0,1}, win : Int, wen : Bool\},$
$\{VM \mapsto \{cp, dd\}$
$CP \mapsto \{wen\}$
$DD \mapsto \{win\} \})$

$\mathscr{D}(Bool) = \{true, false\}$ $\mathscr{D}(Int) = \mathbb{Z}$
$\mathscr{D}(VM) = \{1_{VM}, 2_{VM}, ...\}$
$\mathscr{D}(DD) = ...$

$\sigma = \{ 1_{VM} \mapsto \{dd \mapsto \{1_{DD}\}, cp \mapsto \{42_{CP}, 3_{CP}\}\},$
$1_{DD} = \{win \mapsto 10\}, \quad 42_{CP} = \{wen \mapsto true\}, \quad 3_{CP} = \{wen \mapsto false\} \}$

You Are Here.

Course Map



Tell Them What You've Told Them...

- We can directly use **object system signatures** to model the structure of systems.
  → We don't **need** diagrams, they will be more pleasent to read.
- We introduce
  - **basic types** and **classes**,
  - basic type and derived type **attributes**, and
  - assign to each class a set of attributes.
- **Object system structures** provide domains for basic and derived types.
- An **object system signature** $\mathscr{S}$ and an **object system structure** $\mathscr{D}$ uniquely define the set $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ of **system states**.
- **Outlook**:
  - **Object system signatures** will be used to capture the **abstract syntax** of class diagrams.
  - **OCL expressions** will be evaluated on **system states**.
  - **State machines** will define sequences of **system configurations** (consisting of a **system state** and an **ether**).

References

References

OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.

OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

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