# Software Design, Modelling and Analysis in UML

# Lecture 15: Hierarchical State Machines II

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## Content







## Recall





## Blessing or Curse. . . ?

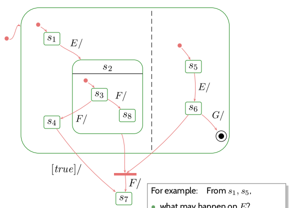
**Plan:**

**States** / Syntax:
- What is the abstract syntax of a diagram?

**States** / Semantics:
- what is the type of the implicit $st$ attribute?
- what are **legal system configurations**?

**Transitions** / Syntax:
- what are **legal** / well-formed transitions?

**Transitions** / Semantics:
- when is a legal transition enabled?
- which effects do transitions have?




For example: From $s_1, s_5$,
- what may happen on $E$?
- what may happen on $\underline{E}, F$?
- can $\underline{E}, G$ kill the object?
- ...





## Representing All Kinds of States

- **So far**:

$$(S, s_0, \to), \quad s_0 \in S, \quad \to \subseteq S \times (\mathscr{E} \cup \{\_\}) \times Expr_\mathscr{S} \times Act_\mathscr{S} \times S$$

- **From now on**: (**hierarchical**) **state machines**

$$(S, kind, region, \to, \psi, annot)$$

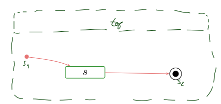
where
- $S \supseteq \{top\}$ is a finite set of states (new: $top$),
- $kind : S \to \{$*st*, *init*, *fin*, *shist*, *dhist*, *fork*, *join*, *junc*, *choi*, *ent*, *exi*, *term*$\}$ is a function which labels states with their **kind**, (new)
- $region : S \to 2^{2^S}$ is a function which characterises the **regions** of a state, (new)
- $\to$ is a set of transitions, (changed)
- $\psi : (\to) \to 2^S \times 2^S$ is an **incidence function**, and (new)
- $annot : (\to) \to (\mathscr{E} \cup \{\_\}) \times Expr_\mathscr{S} \times Act_\mathscr{S}$ provides an annotation for each transition. (new)

($s_0$ is then redundant – replaced by proper state (!) of kind '*init*'.)





## From UML to Hierarchical State Machine: By Example




... denotes $(S, kind, region, \to, \psi, annot)$ with
- $S = \{top, s_1, s, s_2\}$
- $kind = \{top \mapsto$ *st*$, s_1 \mapsto$ *init*$, s \mapsto$ *st*$, s_2 \mapsto$ *fin*$\}$
- or $(S, kind) = \{(top,$ *st*$), (s_1,$ *init*$), (s,$ *st*$), (s_2,$ *fin*$)\}$
- $region = \{top \mapsto \{\{s_1, s, s_2\}\}, s_1 \mapsto \emptyset \quad , s \mapsto \emptyset \quad , s_2 \mapsto \emptyset \quad \}$
- $\to, \psi, annot$: in a minute.

## Recall

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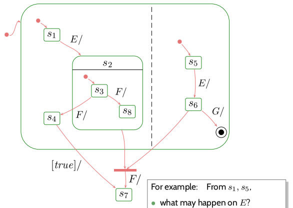


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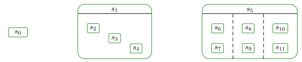
## Semantics: State Configuration

- The type of (implicit attribute) $st$ is from now on a set of states, i.e. $\mathscr{D}(S_{\mathcal{M}_C}) = 2^S$
- A set $S_1 \subseteq S$ is called **(legal) state configuration** if and only if
  - $top \in S_1$, and
  - for each region $R$ of a state in $S_1$, exactly one (non pseudo-state) element of $R$ is in $S_1$, i.e.

$$\forall\, s \in S_1\; \forall\, R \in region(s) \bullet |\{s \in R \mid kind(s) \in \{st, fin\}\} \cap S_1| = 1.$$

- **Examples**:




$S_1 = \{s_0\}$ ✗
$S_2 = \{s_0, top\}$ ✓

$S_3 = \{s_1, top\}$ ✗
$S_4 = \{s_1, top, s_3, s_4\}$ ✗
$S_5 = \{s_1, top, s_4\}$ ✓

$S_6 = \{s_5, top, s_6, s_9\}$ ✗
$S_7 = \{s_5, top, s_6, s_9, s_{10}\}$ ✓
$S_8 = \{top, s_0, s_1, s_2\}$ ✗



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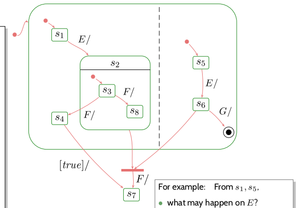


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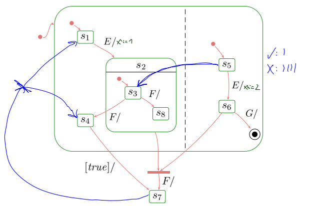




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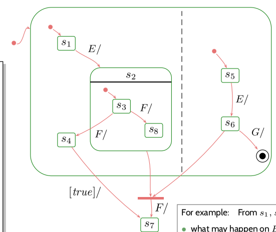


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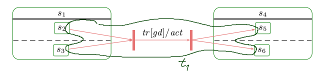
## Transitions Syntax: Fork/Join

- For simplicity, we consider transitions with (possibly) multiple sources and targets, i.e.

$$\psi : (\to) \to (2^S \setminus \emptyset) \times (2^S \setminus \emptyset)$$

No:

- For instance,




translates to

$$(S, kind, region, \underbrace{\{t_1\}}_{\to}, \underbrace{\{t_1 \mapsto (\{s_2, s_3\}, \{s_5, s_6\})\}}_{\psi}, \underbrace{\{t_1 \mapsto (tr, gd, act)\}}_{annot})$$

- Naming convention: $\psi(t) = (source(t), target(t))$.

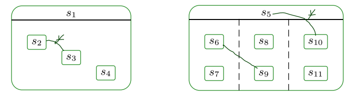
## *Orthogonal States*

- Two states $s_1, s_2 \in S$ are called **orthogonal**, denoted $s_1 \perp s_2$, if and only if
  - they "live" in different regions of **one** AND-state, i.e.

$$\exists\, s, region(s) = \{S_1, \dots, S_n\}, 1 \le i \ne j \le n : s_1 \in child^*(S_i) \wedge s_2 \in child^*(S_j),$$

$region(s_5) = \{ \underbrace{\{s_6, s_7\}}_{S_i}, \underbrace{\{s_8, s_9\}}_{S_j}, \{s_{10}, s_{11}\} \}$

$s_6 \in child^*(S_i)$, $s_9 \in child^*(S_j)$





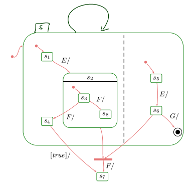

## *Legal Transitions*

A hierarchical state-machine $(S, kind, region, \to, \psi, annot)$ is called **well-formed** if and only if for all transitions $t \in \to$,

- source (and destination) states are pairwise orthogonal, i.e.
  - $\forall\, s \cancel{\ne} s' \in source(t)\ (\in target(t)) \bullet s \perp s'$,
- the top state is neither source nor destination, i.e.
  - $top \notin source(t) \cup \cancel{source}(t)$. target

**Recall**: final states are not sources of transitions.

**Example**:

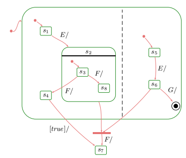




## *Plan*

| | example | | example |
|---|---|---|---|
| simple state | | pseudo-state | |
| | | initial | |
| | | (shallow) history | H |
| | | deep history | H* |
| final state | | fork/join | |
| composite state | | | |
| OR | | junction, choice | |
| | | entry point | |
| AND | | exit point | |
| | | terminate | |
| | | submachine state | $S : s$ |

- Transitions involving non-pseudo states.
- Initial pseudostate, final state.
- Entry/do/exit actions, internal transitions.
- History and other pseudostates, the rest.





## *Scope*

- The **scope** ("set of possibly affected states") of a transition $t$ is the **least common region** of
$$source(t) \cup target(t).$$

- Two transitions $t_1, t_2$ are called **consistent** if and only if their scopes are disjoint.



## *A Partial Order on States*

The substate- (or **child**-) relation **induces** a **partial order on states**:

- $top \le s$, for all $s \in S$,
- $s \le s'$, for all $s' \in child(s)$,
- transitive, reflexive, antisymmetric,
- $s' \le s$ and $s'' \le s$ implies $s' \le s''$ or $s'' \le s'$.

OR $s \ge s'$ iff $s \in child^*(s')$



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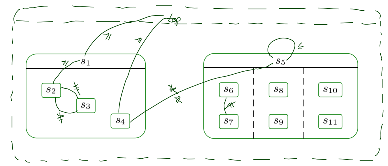
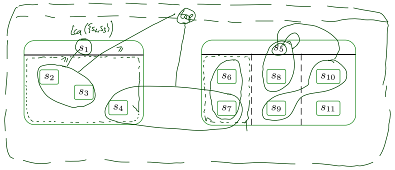


## Least Common Ancestor

- The **least common ancestor** is the function $lca : 2^S \to S$ such that
  - ① The states in $S_1$ are (transitive) children of $lca(S_1)$, i.e.
$$lca(S_1) \leq s, \text{ for all } s \in S_1 \subseteq S,$$
  - ② $lca(S_1)$ is maximal, i.e. if $\hat{s} \leq s$ for all $s \in S_1$ then $\hat{s} \leq lca(S_1)$ $\overbrace{(\forall s \in S_1 \bullet \hat{s} \leq s)}^{①}$
- **Note**: $lca(S_1)$ exists for all $S_1 \subseteq S$ (last candidate: $top$). $\implies \hat{s} \in lca(S_1)$






## Scope

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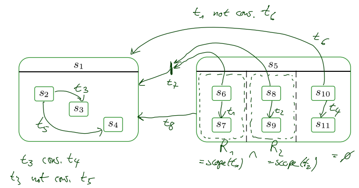
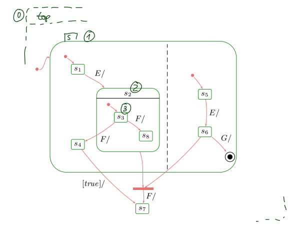





## Priority and Depth

- The **priority** of transition $t$ is the depth of its innermost source state, i.e.
$$prio(t) := \max\{depth(s) \mid s \in source(t)\}$$
where
  - $depth(top) = 0$,
  - $depth(s') = depth(s) + 1$, for all $s' \in child(s)$

**Example**:






## Enabledness in Hierarchical State-Machines

- A set of transitions $T \subseteq \to$ is **enabled** for an object $u$ in $(\sigma, \varepsilon)$ if and only if
  - $T$ is **consistent**,
  - for all $t \in T$, the **source states are active**, i.e.
$$source(t) \subseteq \sigma(u)(st) \; (\subseteq S).$$
  - all transitions in $T$ **have the same trigger** $tr$ and
    - $tr = \_$ and $u$ is **unstable**, or
    - $tr = E$ and there is an $E$ ready for $u$ in $\varepsilon$,
  - the guards of all transitions in $T$ are satisfied in $\tilde{\sigma}$ wrt. $u$, and

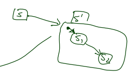
A set $T$ of **enabled transitions** is called **maximal** wrt.

- **extension** if and only if there is no transition $t' \notin T$ such that $T \cup \{t'\}$ is enabled.
- **priority** if and only if for each $t \in T$, there is no $t' \in \to$ such that
  - $prio(t') > prio(t)$,
  - $(T \setminus \{t\}) \cup \{t'\}$ is enabled, and
  - $st' \geq st$ for some $st' \in source(t')$ and $st \in source(t)$.



## Transitions in Hierarchical State-Machines

- Let $T$ be a maximal (extension and priority) set of transitions enabled for $u$ in $(\sigma, \varepsilon)$.
- Then $(\sigma, \varepsilon) \xrightarrow{(cons, Snd)}_u (\sigma', \varepsilon')$ if
  - $\sigma'(u)(st)$ consists of the target states of $T$,
    i.e. for **simple states** the **simple states themselves**,
    for **composite states** the **initial states**,
  - $\sigma'$, $\varepsilon'$, $cons$, and $Snd$ are the effect of firing each transition $t \in T$ **one by one**, **in any order**, i.e. for each $t \in T$,
    - the exit action transformer ($\to$ later) of all affected states, highest depth first,
    - the transformer of $t$,
    - the entry action transformer ($\to$ later) of all affected states, lowest depth first.




$\rightsquigarrow$ adjust Rules (i), (ii), (iii), (v) accordingly.

(For state machines with only simple states, and no trigger, guard, or action on transitions originating at initial states: **Same behaviour as before**.)

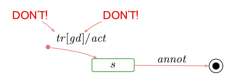


## Additional Well-Formedness Constraints

- Each non-empty region has **exactly one** initial pseudo-state and **at least one** transition from there to a state of the region, i.e.
  - for each $s \in S$ with $region(s) = \{S_1, \dots, S_n\}$,
  - for each $1 \leq i \leq n$, there exists exactly one initial pseudo-state $(s_1^i, init) \in S_i$ and at least one transition $t \in \to$ with $s_1^i$ as source,
- Initial pseudo-states are not targets of transitions.

**For simplicity:**

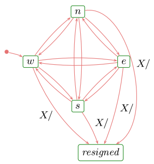
- The target of a transition with initial pseudo-state source in $S_i$ is (also) in $S_i$.
- Transitions from initial pseudo-states have no trigger or guard, i.e. $t \in \to$ from $s$ with $kind(s) = st$ implies $annot(t) = (\_, true, act)$.
- Final states are not sources of transitions.



## An Intuition for “Or-States”

- In a sense, composite states are about
  - **abbreviation**,
  - **structuring**, and
  - **avoiding redundancy**.

- **Idea:** instead of

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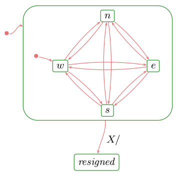




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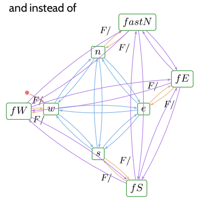
and instead of

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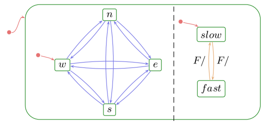




## References



## References


OMG (2011a). Unified modeling language: Infrastructure, version 2.4.1. Technical Report formal/2011-08-05.

OMG (2011b). Unified modeling language: Superstructure, version 2.4.1. Technical Report formal/2011-08-06.