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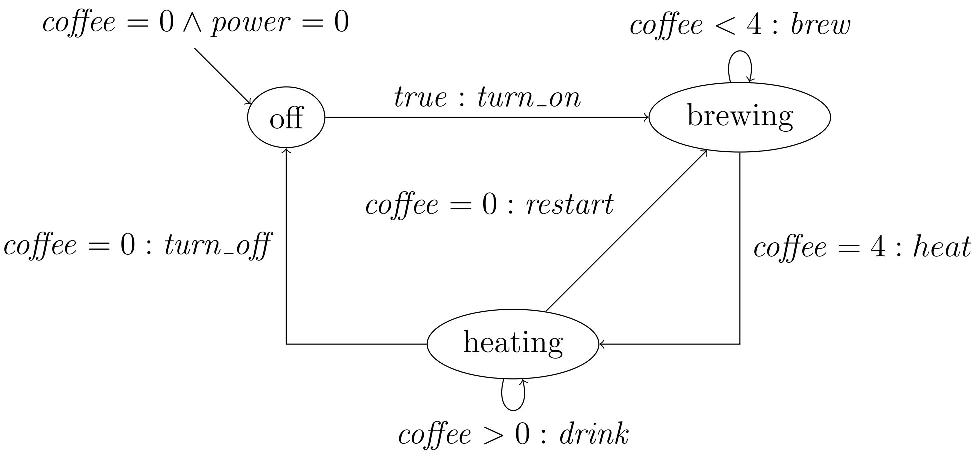
Hand in until October 27th, 2017
11:59 via the post boxes
Discussion: November 6th, 2017

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 2

Exercise 1: Coffee machine

The following program graph describes a simple coffee machine:



The effect of the operations is given by:

$$\begin{aligned}
\mathsf{Effect}(turn_on, \eta) &= \eta[power := 1] \\
\mathsf{Effect}(turn_off, \eta) &= \eta[power := 0] \\
\mathsf{Effect}(brew, \eta) &= \eta[coffee := coffee + 1] \\
\mathsf{Effect}(drink, \eta) &= \eta[coffee := coffee - 1] \\
\mathsf{Effect}(restart, \eta) &= \eta \\
\mathsf{Effect}(heat, \eta) &= \eta
\end{aligned}$$

(a) Draw the transition system corresponding to the program graph.

(b) Check the following properties. Label the transition system with the corresponding atomic propositions given in parentheses.

(i) If the machine is turned off ($power = 0$) it contains no coffee ($coffee = 0$).

(ii) If there are two cups of coffee ($coffee = 2$) there are either three or four cups of coffee in the next step ($coffee = 3$, $coffee = 4$).

(iii) There are always at most four cups of coffee ($coffee \leq 4$).

(iv) The coffee machine will be eventually turned off.

(v) If there is no coffee ($coffee = 0$), there will be coffee after at most three steps.

Exercise 2: Set Notation for Evaluations
Let $AP = \{a_1, \dots, a_n\}$ be a set of atomic propositions.
An evaluation $\mu : AP \to \{0, 1\}$ can be represented by the set $A_\mu = \{a \in AP \mid \mu(a) = 1\}$.

(a) Give a description of all evaluations μ such that $\mu \models \phi$,
once expressed in terms of functions and once in terms of sets, with

- $\phi = a_1 \wedge \dots \wedge a_i$
- $\phi = a_1 \vee \dots \vee a_i$

where i is some number smaller than n, i.e., $i \leq n$.

(b) Let $AP = \{a_1, a_2, a_3, a_4, a_5\}$. Find a formula ϕ such that μ_A with $A = \{a_2, a_3\}$ is the unique satisfying evaluation for ϕ.

Exercise 3: Propositional Logic
Alice, Bob and Claire want to attend the CPS I lecture. The exercise groups are almost full, only group 1 and group 2 have places left.

(a) If Alice joins group 1, the tutor refuses to accept Bob because they always talk.

(b) At least one of Bob and Claire cannot go to group 1, as they lead a chess group together that meets at the same time.

(c) Claire hates Alice and doesn't want to be in the same group.

(d) Alice wants to submit the solutions with either Bob or Claire and thus needs to be in a group with this person.

Model the above statements in propositional logic where the atomic propositions a (Alice), b (Bob), c (Claire) are assigned the value `true` if the corresponding person joins group 1, and `false` else.
Which persons join which group? Use a truth table to find out.