Prof. Dr. Andreas Podelski
Tanja Schindler

Hand in until December 22th, 2017
11:59 via the post boxes
Discussion: January 8th, 2018

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 9

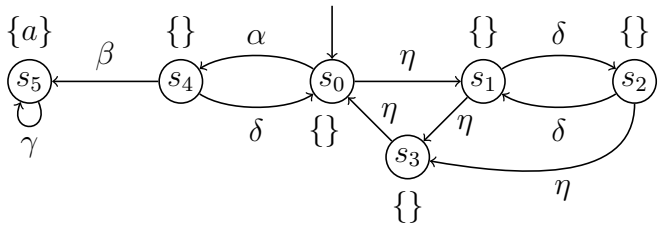
Please tell us the time you spend for the CPS course outside the classes, i.e., for working on the exercises and reading the text. Please be accurate in tracking the time.

We will explain the goal of this exercise sheet and of each single exercise (so that you are able to see why you should do the exercise, i.e., why it is not a waste of time). We want to give credit to Nico for being at the origin of this idea.
The overall goal of this exercise sheet is to understand fairness. Now, one may ask why it is important to understand fairness. We don't want to go down this path forever (a path which ends with the question of the meaning of life), but we still want to say so much:

- To model the behavior of a transition system (i.e., a graph) is fine when one is interested in safety properties but it is not fine when one is interested in liveness properties. Remember that a counterexample to a liveness property must be represented by an infinite path, but not every infinite path is realistic. We use fairness to single out realistic paths. In other words, our model is given by a graph plus a fairness assumption.
- It is rewarding intellectually to be able to reason about infinite objects, such as infinite paths in a graph. We don't have much intuition about infinite paths, so it is all the more important to be able to reason formally about infinite paths. Taking into account fairness trains us in reasoning formally about infinite paths.

Exercise 1: Strength of Fairness Assumptions

Consider the following transition system:



Under which fairness assumptions $\mathcal{F}_i$ does the system satisfy the property "eventually a"? Justify your answer.

(a) $\mathcal{F}_1 = (\{\{\gamma\}\}, \emptyset, \emptyset)$

(b) $\mathcal{F}_2 = (\{\{\alpha\}, \{\gamma\}\}, \emptyset, \emptyset)$

(c) $\mathcal{F}_3 = (\{\{\alpha, \gamma\}\}, \emptyset, \emptyset)$

(d) $\mathcal{F}_4 = (\emptyset, \{\{\beta\}\}, \emptyset)$

(e) $\mathcal{F}_5 = (\emptyset, \{\{\alpha\}, \{\beta\}\}, \emptyset)$

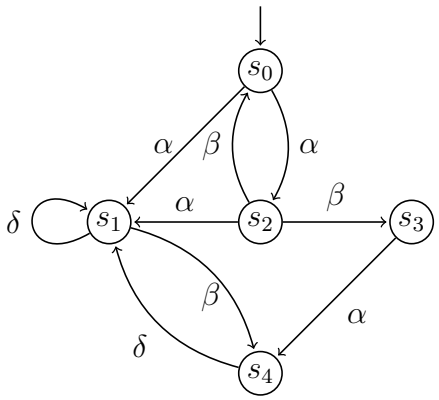
(f) $\mathcal{F}_6 = (\emptyset, \{\{\alpha\}, \{\beta\}, \{\eta\}\}, \emptyset)$

(g) $\mathcal{F}_7 = (\emptyset, \emptyset, \{\{\alpha\}, \{\beta\}, \{\eta\}\})$

(h) $\mathcal{F}_8 = (\emptyset, \{\{\alpha\}, \{\beta\}\}, \{\{\eta\}\})$

Motivation: The goal is to work with fairness assumptions and to see fairness assumptions "in action"; i.e., to see the effect of "assuming" a fairness assumption on the validity of a liveness property. Another goal is to get a feeling for the difference between the three kinds of fairness assumptions, and to see when one fairness assumption is stronger or weaker than another (what does "stronger" and "weaker" mean here?).

Exercise 2: Realizable Fairness Assumptions
Consider the following transition system TS with actions $\{\alpha, \beta, \delta\}$ (and without atomic propositions):



Decide which of the following fairness assumptions $\mathcal{F}_i$ are realizable for TS. Justify your answers!

(a) $\mathcal{F}_1 = (\{\{\alpha\}\}, \{\{\delta\}\}, \{\{\alpha, \beta\}\})$

(b) $\mathcal{F}_2 = (\{\{\alpha, \delta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$

(c) $\mathcal{F}_3 = (\{\{\alpha, \delta\}, \{\beta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$

Motivation: The goal is to get a grasp on the notion of a realizable fairness assumption, and to see why SCCs are relevant for reasoning about whether a given fairness assumption is realizable. Another goal is to be able to read fairness assumptions; a fairness assumption is a triple of sets, where each of the three sets contains one or more sets, and each of those sets contains ... (what?); sometimes a set "means" the disjunction of its elements, sometimes the conjunction (example: $\{\{\alpha, \delta\}, \{\beta\}\}$ corresponds to "α or δ, and β"). The goal is also to get across the insight that strong fairness assumptions are always realizable (same for weak fairness assumptions; unconditional fairness assumptions, however, are not always realizable).
As an aside, the situation is different when we specify fairness assumptions by states and not by actions.

Exercise 3: Feedback
Please answer the following questions:

- Why do you come to the lecture, in case you do?
- Why don't you come to the lecture, in case you don't?
- Do you appreciate that we give the motivation for the exercises as we do above?
- Do you understand the motivation?
- Why is the material that you have learned so far, useful?
- Why is the material that you have learned so far, interesting?