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Hand in until January 26th, 2018
11:59 via the post boxes
Discussion: January 29th, 2018

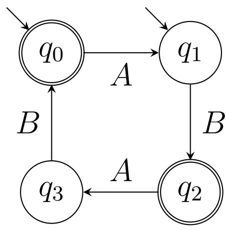
Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 12

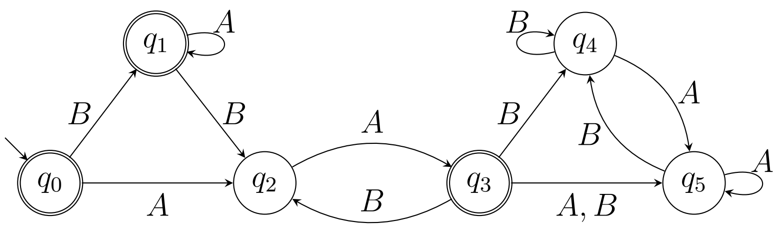
The overall goal of this sheet is to get a deeper understanding of Nondeterministic Büchi Automata (NBA).
One aspect is analyzing Büchi automata to determine the corresponding ω-regular language. Another aspect is to see what changes compared to finite automata with regard to nondeterminism. For this purpose, we will have a closer look on Deterministic Büchi Automata (DBA). Finally, we will see that a variant of NBAs, the Generalized Nondeterministic Büchi Automata (GNBA), can be more convenient than NBAs.

Exercise 1: From NBA to ω-regular expressions
Construct for each of the following NBA an ω-regular expression that describes the same language. The alphabet of each automaton is $\Sigma = \{A, B\}$.

(a) Apply the construction described in Theorem 4.32 in the book (see also the subsequent lemmas!). You do not have to write down each NFA $\mathcal{A}_{q,p}$ that you construct, but you have to write down the regular expression for each NFA that you construct.

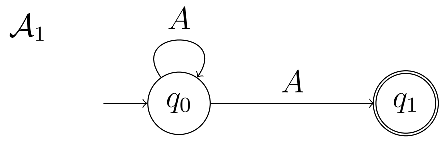


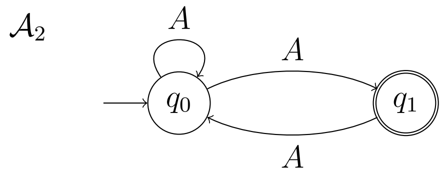
(b) This time you need not apply the construction from the book. You may use optimizations like omitting automata whose language is empty.



Motivation: The goal of this exercise is to improve the ability to "read" Büchi automata, i.e., to determine the ω-regular language accepted by a Büchi automaton. Part (a) aims at understanding the proof and consequences of Theorem 4.32, in particular at seeing how to *systematically* determine the language recognized by a Büchi automaton.

Exercise 2: Powerset Construction and Büchi Automata
Consider NBA $\mathcal{A}_1$ and $\mathcal{A}_2$ depicted below. Show that the powerset construction applied to $\mathcal{A}_1$ and $\mathcal{A}_2$ (viewed as NFA) yields the same deterministic automaton, while $\mathcal{L}_\omega(\mathcal{A}_1) \neq \mathcal{L}_\omega(\mathcal{A}_2)$.





Motivation: The goal of this exercise is to see how going from finite to infinite changes things.

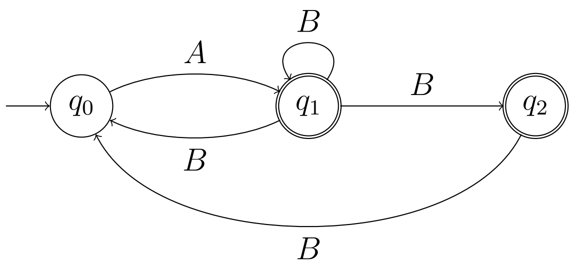
Exercise 3: Expressive Power of DBA
Is there a DBA that accepts the language described by the ω-regular expression $(A + B)^*(AB + BA)^\omega$? Justify your answer.

Motivation: The goal is to fully understand the idea of the proof of Theorem 4.50 by carrying over the idea from one example to another. You are not required to spell out a full proof; just give the essentials details.

Exercise 4: DBA and Complementation
Show that the class of languages that are accepted by DBAs is not closed under complementation.

Motivation: The goal is to experience the "Oh, yes, of course!" experience. Neuroscientists have determined that the "Oh, yes, of course!" experience causes an endorphin rush (in other words, solving the exercise has an effect similar to running a marathon or eating a bar of chocolate).

Exercise 5: From GNBA to NBA
Consider the GNBA outlined below with acceptance sets $F_1 = \{q_1\}$ and $F_2 = \{q_2\}$. Construct an NBA that accepts the same language.



Motivation: The goal of this exercise is to understand the connection between GNBAs and NBAs and to see why GNBAs can be more convenient than NBAs.