Real-Time Systems

Lecture 4: Duration Calculus II

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  - **syntactic substitution**
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  - **for some subinterval** / **for all subintervals**
- **Validity, Satisfiability, Realisability**
  - **realisability** / **validity** from 0
- Proving design ideas correct: **Method**
  - Example: **gas burner**

Duration Calculus: Formulae

Duration Calculus: Overview

We will introduce **four syntactical categories** (and **abbreviations**):

(i) **Symbols:**

$$\overbrace{true, false, =, <, >, \leq, \geq}^{p,q}, \quad f, g, \quad X, Y, Z, \quad d, \quad x, y, z,$$

(ii) **State Assertions:**

$$P ::= 0 \mid 1 \mid X = d \mid \neg P_1 \mid P_1 \wedge P_2$$

(iii) **Terms:**

$$\theta ::= x \mid \ell \mid \textstyle\int P \mid f(\theta_1, \dots, \theta_n)$$

(iv) **Formulae:**

chop operator

$$F ::= p(\theta_1, \dots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

(v) **Abbreviations:**

$$\lceil\,\rceil, \quad \lceil P \rceil, \quad \lceil P \rceil^t, \quad \lceil P \rceil^{\leq t}, \quad \Diamond F, \quad \Box F$$

Formulae: Syntax

- The set of **DC formulae** is defined by the following grammar:

$$F ::= p(\theta_1, \dots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

  where p is a predicate symbol, θ_i are terms, and x is a global variable.

- **chop operator**: ‘;’
- **atomic formula**: $p(\theta_1, \dots, \theta_n)$
- **rigid formula**: all terms are rigid (no ℓ, no $\int P$)
- **chop free**: ‘;’ doesn't occur
- usual notion of **free** and **bound** (global) variables

$\forall x \bullet \int P > x \wedge y < 10$

bound free

- Note: quantification only over (**first-order**) global variables, not over (**second-order**) state variables.

↳ $\exists X \bullet \int X > 3$ NOT

Formulae: Priority Groups

- To avoid parentheses, we define the following five **priority groups** from highest to lowest priority (or precedence):
  - $\neg$ **(negation)**
  - **;** **(chop)**
  - $\wedge, \vee$ **(and/or)**
  - $\implies, \iff$ **(implication/equivalence)**
  - $\exists, \forall$ **(quantifiers)**

Examples:

- $\neg F \,;\, F \vee G$
  - $(\neg(F \,;\, F)) \vee G$ —
  - $\underbrace{(\underbrace{(\neg F)} \,;\, F)} \vee G$ 卌 || ✓
  - $(\neg F) \,;\, (F \vee G)$ ||

- $\forall x \bullet \quad F \quad \wedge \quad G$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

- We use

$$F[x := \theta]$$

to denote the formula that results from performing the following steps:

(i) transform F into $\tilde{F}$ by (consistently) **renaming bound variables** such that **no free occurrence** of x in $\tilde{F}$ appears within a **quantified subformula** $\exists\, z \bullet G$ or $\forall\, z \bullet G$ for some z **occurring in term** θ,

(ii) textually replace all free occurrences of x in $\tilde{F}$ by θ.

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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$

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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$
- $\theta_2 := \ell + z, \quad F \qquad\quad = (x \qquad \geq y \implies \exists\, z \bullet z \geq 0 \wedge x \qquad = y + z)$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

- We use

$$F[x := \theta]$$

to denote the formula that results from performing the following steps:

(i) transform F into $\tilde{F}$ by (consistently) **renaming bound variables** such that **no free occurrence** of x in $\tilde{F}$ appears within a **quantified subformula** $\exists\, z \bullet G$ or $\forall\, z \bullet G$ for some z **occurring in term** θ,

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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$

suddenly bound

- $\theta_2 := \ell + z, \quad F[x := \theta_2] = (\ell + z \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell + z = y + z)$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

- We use

$$F[x := \theta]$$

to denote the formula that results from performing the following steps:

(i) transform F into $\tilde{F}$ by (consistently) **renaming bound variables** such that **no free occurrence** of x in $\tilde{F}$ appears within a **quantified subformula** $\exists\, z \bullet G$ or $\forall\, z \bullet G$ for some z **occurring in term** θ,

(ii) textually replace all free occurrences of x in $\tilde{F}$ by θ.

Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$ ✓
- $\theta_2 := \ell + z, \quad F[x := \theta_2] = (\ell + z \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell + z = y + z)$ ✗
- $F[x := \theta_2] = \ell + z \geq y \implies \exists\, \tilde{z} \bullet \tilde{z} \geq 0 \wedge \ell + z = y + \tilde{z})$ ✓

Formulae: Semantics

- The **semantics** of a **formula** is a function

$$\mathcal{I}[\![F]\!] : \mathsf{Val} \times \mathsf{Intv} \to \{\mathsf{tt}, \mathsf{ff}\}$$

$\mathcal{I}[\![F]\!](\mathcal{V}, [b, e])$: truth value of F under interpretation $\mathcal{I}$ and valuation $\mathcal{V}$ in the interval $[b, e]$.

- $\mathcal{I}[\![F]\!](\mathcal{V}, [b, e])$ is defined **inductively** over the structure of F:

base step

$$\mathcal{I}[\![p(\theta_1, \ldots, \theta_n)]\!](\mathcal{V}, [b, e]) = \hat{p}(\,\mathcal{I}[\![\theta_1]\!](\mathcal{V}, [b, e]),\ \ldots,\ \mathcal{I}[\![\theta_n]\!](\mathcal{V}, [b, e])\,),$$

$$\mathcal{I}[\![\neg F_1]\!](\mathcal{V}, [b, e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b, e]) = \mathsf{ff},$$

$$\mathcal{I}[\![F_1 \wedge F_2]\!](\mathcal{V}, [b, e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_i]\!](\mathcal{V}, [b, e]) = \mathsf{tt}, i \in \{1, 2\},$$

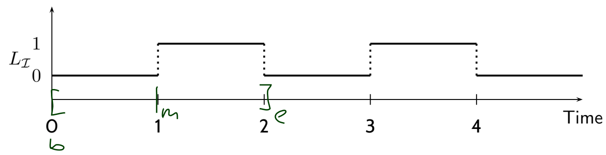
function symbol

$$\mathcal{I}[\![\forall\, x \bullet F_1]\!](\mathcal{V}, [b, e]) = \mathsf{tt} \text{ iff for all } a \in \mathbb{R},\ \mathcal{I}[\![F_1[x := a]]\!](\mathcal{V}, [b, e]) = \mathsf{tt}$$

$$\mathcal{I}[\![F_1\,;\,F_2]\!](\mathcal{V}, [b, e]) = \text{ iff there is an } m \in [b, e] \text{ such that } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b, m]) = \mathsf{tt} \text{ and } \mathcal{I}[\![F_2]\!](\mathcal{V}, [m, e]) = \mathsf{tt}.$$

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0,2]) = \mathrm{tt}$

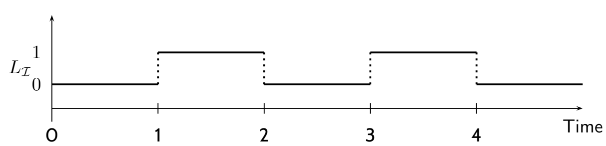
  Proof:

  - Choose $m = 1$ as **chop point**.

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$

$$\equiv ((\int L) = 0) \;\; ; \;\; ((\int L) = 1) \quad \equiv \quad = (\, (\int L), 0 \,) \;\; ; \;\; = (\, (\int L), 1 \,)$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0,2]) = \mathrm{tt}$

  Proof:

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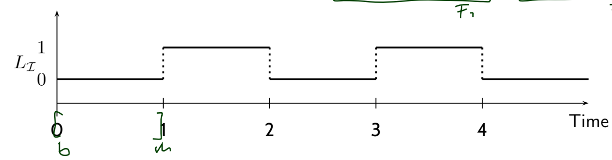
Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$

$$\equiv ((\textstyle\int L) = 0) \;\;;\;\; ((\textstyle\int L) = 1) \quad \equiv \quad \underbrace{=(\,(\textstyle\int L), 0\,)}_{F_1} \;;\; \underbrace{=(\,(\textstyle\int L), 1\,)}_{F_2}$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0,2]) = \text{tt}$

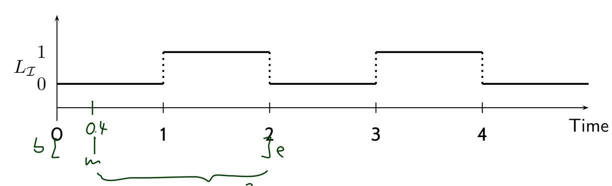
  Proof:

  - Choose $m = 1$ as **chop point**. Then
  - $\mathcal{I}[\![=((\int L), 0)]\!](\mathcal{V}, [0,1]) \;= \hat{=}(\,\mathcal{I}[\![\int L]\!](\mathcal{V}, [0,1]),\, \mathcal{I}[\![0]\!](\mathcal{V}, [0,1])\,)$
    $= \hat{=}\left(\int_0^1 L_\mathcal{I}(t)\, dt, \quad \hat{0}\right) \;= \hat{=}(0,0) = \text{tt},$

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$

$$\equiv ((\textstyle\int L) = 0) \;\;;\;\; ((\textstyle\int L) = 1) \quad \equiv \quad =(\,(\textstyle\int L), 0\,) \;;\; =(\,(\textstyle\int L), 1\,)$$



$\int_{0.4}^{2} L_\mathcal{I}(t)\, dt = 1$

- $\mathcal{I}[\![F]\!](\mathcal{V}, [0,2]) = \text{tt}$

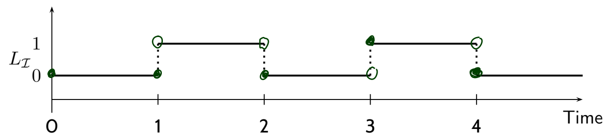
  Proof:

  - Choose $m = 1$ as **chop point**. Then
  - $\mathcal{I}[\![=((\int L), 0)]\!](\mathcal{V}, [0,1]) \;= \hat{=}(\,\mathcal{I}[\![\int L]\!](\mathcal{V}, [0,1]),\, \mathcal{I}[\![0]\!](\mathcal{V}, [0,1])\,)$
    $= \hat{=}\left(\int_0^1 L_\mathcal{I}(t)\, dt, \quad \hat{0}\right) \;= \hat{=}(0,0) = \text{tt},$
  - and $\mathcal{I}[\![=(\;(\int L),\;\; 1\;)]\!](\mathcal{V}, [1,2])$
    $= \hat{=}(\;\mathcal{I}[\![\int L]\!](\mathcal{V}, [1,2]),\;\; \mathcal{I}[\![1]\!](\mathcal{V}, [1,2])\;) = \hat{=}(1,1) = \text{tt},$ $\square$

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$

$$\equiv ((\int L) = 0) \;\; ; \;\; ((\int L) = 1) \quad \equiv \quad =((\int L), 0) \;;\; =((\int L), 1)$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0, 2]) =$ tt

  Proof:

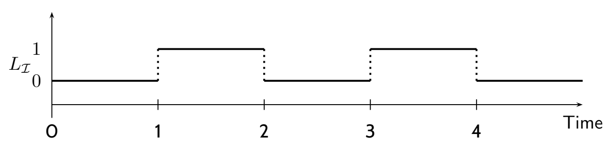
  - Choose $m = 1$ as **chop point**. Then
  - $\mathcal{I}[\![=((\int L), 0)]\!](\mathcal{V}, [0, 1]) \;=\hat{=}(\, \mathcal{I}[\![\int L]\!](\mathcal{V}, [0, 1]),\, \mathcal{I}[\![0]\!](\mathcal{V}, [0, 1])\,)$
    $= \hat{=}\left(\int_0^1 L_\mathcal{I}(t)\, dt, \quad \hat{0} \right) \; = \hat{=}(0, 0) =$ tt,
  - and $\mathcal{I}[\![=(\quad (\int L), \quad 1 \quad)]\!](\mathcal{V}, [1, 2])$
    $= \hat{=}(\quad \mathcal{I}[\![\int L]\!](\mathcal{V}, [1, 2]), \quad \mathcal{I}[\![1]\!](\mathcal{V}, [1, 2]) \quad) = \hat{=}(1, 1) =$ tt, $\square$

- Is the **chop point** m **unique**?

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$

$$\equiv ((\int L) = 0) \;\; ; \;\; ((\int L) = 1) \quad \equiv \quad =((\int L), 0) \;;\; =((\int L), 1)$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0, 2]) =$ tt

  Proof:

  - Choose $m = 1$ as **chop point**. Then
  - $\mathcal{I}[\![=((\int L), 0)]\!](\mathcal{V}, [0, 1]) \;=\hat{=}(\, \mathcal{I}[\![\int L]\!](\mathcal{V}, [0, 1]),\, \mathcal{I}[\![0]\!](\mathcal{V}, [0, 1])\,)$
    $= \hat{=}\left(\int_0^1 L_\mathcal{I}(t)\, dt, \quad \hat{0} \right) \; = \hat{=}(0, 0) =$ tt,
  - and $\mathcal{I}[\![=(\quad (\int L), \quad 1 \quad)]\!](\mathcal{V}, [1, 2])$
    $= \hat{=}(\quad \mathcal{I}[\![\int L]\!](\mathcal{V}, [1, 2]), \quad \mathcal{I}[\![1]\!](\mathcal{V}, [1, 2]) \quad) = \hat{=}(1, 1) =$ tt, $\square$

- Is the **chop point** m **unique**?

NO, all $m \in [0, 1]$ are proper chop points (and only those)

- $\mathcal{I}[\![\int \neg L < 1 ; \int L < 1]\!](\mathcal{V}, [0, ?]) = $ ff

- Would the **chop point** for formula $\int \neg L = 1 \,;\, \int L = 1$ be **unique**?

Formulae: Remarks

$x + y > 3 \,;\, x + y \geq 5$

- **rigid formula**: all terms are rigid
- **rigid term**: no length or integral operators
- **chop free**: ';' doesn't occur

Remark 2.10. [*Rigid and chop-free*] Let F be a duration formula, $\mathcal{I}$ an interpretation, $\mathcal{V}$ a valuation, and $[b, e] \in$ Intv.

- If F is **rigid**, then

$$\forall\, [b', e'] \in \mathsf{Intv} : \mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \mathcal{I}[\![F]\!](\mathcal{V}, [b', e']).$$

- If F is **chop-free** or θ is **rigid**, then in the calculation of the semantics of F, every occurrence of θ denotes the same value.

Substitution Lemma

Lemma 2.11. [*Substitution*]
Consider a formula F, a global variable x, and a term θ such that F is **chop-free** or θ is **rigid**.
Then for all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and intervals $[b, e]$,

$$\mathcal{I}[\![F[x := \theta]]\!](\mathcal{V}, [b, e]) = \mathcal{I}[\![F]\!](\mathcal{V}[x := a], [b, e])$$

where $a = \mathcal{I}[\![\theta]\!](\mathcal{V}, [b, e])$.

- **Negative Example**: $F := (\ell = x) ; (\ell = x) \implies (\ell = 2 \cdot x)$ $\quad \theta := \ell$
  - $\mathcal{I}[\![F[x := \ell]]\!](\mathcal{V}, [b, e]) = \mathcal{I}[\![(\ell = \ell) ; (\ell = \ell) \implies (\ell = 2 \cdot \ell)]\!](\mathcal{V}, [b, e])$
    ↳ yields ff for $b < e$
  - $\mathcal{I}[\![F]\!](\mathcal{V}[x := a], [b, e]) = tt$ (even valid)

Duration Calculus: Overview

We will introduce **four syntactical categories** (and **abbreviations**):

(i) **Symbols:**

$$\overbrace{true, false, =, <, >, \leq, \geq}^{p,q}, \quad f, g, \quad X, Y, Z, \quad d, \quad x, y, z,$$

(ii) **State Assertions:**

$$P ::= 0 \mid 1 \mid X = d \mid \neg P_1 \mid P_1 \wedge P_2$$

(iii) **Terms:**

$$\theta ::= x \mid \ell \mid \textstyle\int P \mid f(\theta_1, \ldots, \theta_n)$$

(iv) **Formulae:**

$$F ::= p(\theta_1, \ldots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

(v) **Abbreviations:**

$$\lceil\,\rceil, \quad \lceil P \rceil, \quad \lceil P \rceil^t, \quad \lceil P \rceil^{\leq t}, \quad \Diamond F, \quad \Box F$$

Duration Calculus Abbreviations

Abbreviations

- $\lceil\rceil := \ell = 0$ — **(point interval)**
- $\lceil P \rceil := (\int P = \ell) \wedge (\ell > 0)$ — **(almost everywhere)**

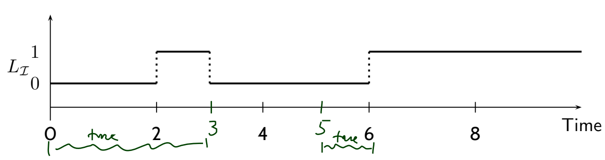
  state assertion
- $\lceil P \rceil^t := \lceil P \rceil \wedge \ell = t$ — **(for time t)**
- $\lceil P \rceil^{\leq t} := \lceil P \rceil \wedge \ell \leq t$ — **(up to time t)**
- $\Diamond F := \mathit{true}\,;F\,;\mathit{true}$ — **(for some subinterval)**

  diamond
- $\Box F := \neg\Diamond\neg F$ — **(for all subintervals)**

  box

• $\lceil P \rceil$ not satisfied on any point interval

Abbreviations: Examples



$(\int(\neg L)) = \ell \wedge \ell > 0$

true; $\lceil L \rceil$; true

$\mathcal{I}[\![\ (\int L) = 0\ ]\!](\mathcal{V},\ [0,2]\ ) = tt$

$\mathcal{I}[\![\ \int L = 1\ ]\!](\mathcal{V},\ [2,6]\ ) = tt$

$\mathcal{I}[\![\ \int L = 0\,;\int L = 1\ ]\!](\mathcal{V},\ [0,6]\ ) = tt$, $m = 2$

$\mathcal{I}[\![\ \lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,2]\ ) = tt$

$\mathcal{I}[\![\ \lceil L \rceil\ ]\!](\mathcal{V},\ [2,3]\ ) = tt$

$\mathcal{I}[\![\ \lceil \neg L \rceil\,;\lceil L \rceil\ ]\!](\mathcal{V},\ [0,3]\ ) = tt$, $m = 2$

$\mathcal{I}[\![\ \lceil \neg L \rceil\,;\lceil L \rceil\,;\lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) = tt$, $m_1 = 2$, $m_2 = 3$

$\mathcal{I}[\![\ \Diamond\lceil L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) = tt$ $m_1 = 2$ $m_2 = 3$

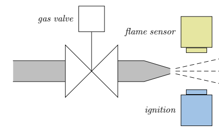
$\mathcal{I}[\![\ \Diamond\lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) =$

$\mathcal{I}[\![\ \Diamond\lceil \neg L \rceil^2\ ]\!](\mathcal{V},\ [0,6]\ ) = tt$ $m_1 = 3$ or 0 $m_2 = 5$ or 2

$\mathcal{I}[\![\ \Diamond\lceil \neg L \rceil^2\,;\lceil \cancel{tt} L \rceil^1\,;\lceil \neg L \rceil^3\ ]\!](\mathcal{V},\ [0,6]\ ) = tt$ $m_1 = 2$ $m_2 = 3$

Duration Calculus: Preview

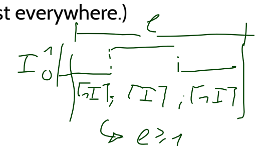
- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an (**implicitly given**) interval.



- $G, F, I, H : \{0, 1\}$
- Define $L : \{0, 1\}$ as $G \wedge \neg F$.

Strangest operators: $\lceil \text{Form} \rceil$

- **almost everywhere** – Example: $\lceil G \rceil$
  (Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere.)
- **chop** – Example: $(\lceil \neg I \rceil \,;\, \lceil I \rceil \,;\, \lceil \neg I \rceil) \implies \ell \geq 1$
  (Ignition phases last at least one time unit.)



- **integral** – Example: $\ell \geq 60 \implies \int L \leq \frac{\ell}{20}$
  (At most 5% leakage time within intervals of at least 60 time units.)

– 2 – 2017-10-19 – Sdcpreview –

– 4 – 2017-11-02 – main –

Content

– 4 – 2017-11-02 – Scontent –

DC Validity, Satisfiability, Realisability

Validity, Satisfiability, Realisability

Let $\mathcal{I}$ be an interpretation, $\mathcal{V}$ a valuation, $[b, e]$ an interval, and F a DC formula.

- $\mathcal{I}, \mathcal{V}, [b, e] \models F$ (read: F **holds** in $\mathcal{I}$, $\mathcal{V}$, $[b, e]$) iff $\mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \text{tt}$.
- F is called **satisfiable** iff it **holds** in some $\mathcal{I}$, $\mathcal{V}$, $[b, e]$.
- $\mathcal{I}, \mathcal{V} \models F$ (read: $\mathcal{I}$ and $\mathcal{V}$ **realise** F) iff $\forall\, [b, e] \in \text{Intv} : \mathcal{I}, \mathcal{V}, [b, e] \models F$.
- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ **realise** F.
- $\mathcal{I} \models F$ (read: $\mathcal{I}$ **realises** F) iff $\forall\, \mathcal{V} \in \text{Val} : \mathcal{I}, \mathcal{V} \models F$.
- $\models F$ (read: F is **valid**) iff $\forall\, \mathcal{I} : \mathcal{I} \models F$.

Validity vs. Satisfiability vs. Realisability

Remark 2.13. For all DC formulae F,

- F is satisfiable if and only if $\neg F$ is not valid,
  F is valid if and only if $\neg F$ is not satisfiable.
- If F is valid then F is realisable, but not vice versa.
- If F is realisable then F is satisfiable, but not vice versa.

Examples: Valid? Realisable? Satisfiable?

- $\ell \geq 0$
- $\ell = \int 1$
- $\ell = 30 \iff \ell = 10 \,;\, \ell = 20$
- $((F \,;\, G) \,;\, H) \iff (F \,;\, (G \,;\, H))$
- $\int L \leq x$
- $\ell = 2$

Initial Values

- $\mathcal{I}, \mathcal{V} \models_0 F$ (read: $\mathcal{I}$ and $\mathcal{V}$ **realise** F **from** 0) iff

$$\forall\, t \in \text{Time} : \mathcal{I}, \mathcal{V}, [0, t] \models F.$$

- F is called **realisable from** 0 iff some $\mathcal{I}$ and $\mathcal{V}$ **realise** F from 0.

- **Intervals** of the form $[0, t]$ are called **initial intervals**.

- $\mathcal{I} \models_0 F$ (read: $\mathcal{I}$ **realises** F **from** 0) iff $\forall\, \mathcal{V} \in \text{Val} : \mathcal{I}, \mathcal{V} \models_0 F$.

- $\models_0 F$ (read: F is **valid from** 0) iff $\forall\, \mathcal{I} : \mathcal{I} \models_0 F$.

Initial or not Initial...

Remark. For all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and DC formulae F,

(i) $\mathcal{I}, \mathcal{V} \models F$ implies $\mathcal{I}, \mathcal{V} \models_0 F$,

(ii) if F is realisable then F is realisable from 0, but not vice versa,

(iii) F is valid iff F is valid from 0.

Content

- **Formulae**
  - **syntax, priority groups**
  - **syntactic substitution**
  - **semantics**
  - **well-definedness**
  - **remarks, substitution lemma**
- **DC Abbreviations**
  - **point interval, almost everywhere**
  - **for some subinterval** / **for all subintervals**
- **Validity, Satisfiability, Realisability**
  - **realisability** / **validity** from 0
- Proving design ideas correct: **Method**
  - Example: **gas burner**

Specification and Semantics-based Correctness Proofs of Real-Time Systems with DC

Methodology *(in an ideal world)*

In order to **prove** a controller design **correct** wrt. a **specification**:

(i) Choose **observables** 'Obs'.

(ii) Formalise the **requirements** 'Spec'
as a conjunction of DC formulae (over 'Obs').

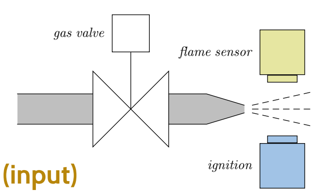
(iii) Formalise a **controller design** 'Ctrl'
as a conjunction of DC formulae (over 'Obs').

(iv) We say 'Ctrl' is **correct** (wrt. 'Spec') iff

$$\models_0 \mathsf{Ctrl} \implies \mathsf{Spec},$$

so "just" prove $\models_0 \mathsf{Ctrl} \implies \mathsf{Spec}$.

Gas Burner Revisited



(i) Choose **observables**:

- $F : \{0, 1\}$: value 1 models "flame sensed now" **(input)**
- $G : \{0, 1\}$: value 1 models "gas valve is open now" **(output)**
- define $L := G \wedge \neg F$ to model **leakage**

(ii) Formalise the **requirement**:

$$\mathsf{Req} := \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

"in each interval of length at least 60 time units, at most 5% of the time leakage"

(iii) Formalise **controller design ideas**:

- $\mathsf{Des\text{-}1} := \Box(\lceil L \rceil \implies \ell \leq 1)$
  "leakage phases last for at most one time unit"
- $\mathsf{Des\text{-}2} := \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$
  "non-leakage phases between two leakage-phases last at least 30 time units"

(iv) Prove **correctness**, i.e. prove $\models (\mathsf{Des\text{-}1} \wedge \mathsf{Des\text{-}2} \implies \mathsf{Req})$.
(Or do we want "$\models_0$"...?)

Content

Tell Them What You've Told Them…

- **Duration Calculus Formulae**
  - using, e.g., the **chop operator**

  are **evaluated** for **intervals** and **valuations**.

  The **semantics** of a **DC formula** is a **truth value**.
- The following **abbreviations** are sometimes useful
  - **point interval** ($\lceil\rceil$), **almost everywhere** ($\lceil P\rceil$),
  - **for some subinterval** ($\Diamond F$), **for all subintervals** ($\Box F$)
- **DC Formulae** have notions of
  - **satisfiability** and **validity** (as usual),
  - **realisability** ("for all subintervals")
  - also: from 0
- Outlook on next lecture:
  proving design ideas correct wrt. requirements.

References

References

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.

EXAM

– oral / written

– DATE
(mid / late March)

↳ Tue ⇄ fix on