

Real-Time Systems

Lecture 5: Duration Calculus

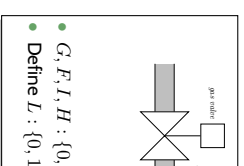
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Dr. Bernd Westphal

Albert-Ludwigs-Universität Freiburg, Germany

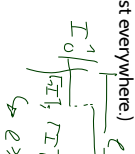
Duration Calculus: Preview

- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an **(implicitly given) interval**.



Strangest operators: $\lceil \cdot \rceil_{\text{True}}$

- **almost everywhere** – Example: $\lceil G \rceil$
(Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere)
- **chop** – Example: $\lceil \neg I \rceil : \lceil I \rceil : \lceil \neg I \rceil \Rightarrow \ell \geq 1$
(Ignition phases last at least one time unit)
- **integral** – Example: $\ell \geq 60 \Rightarrow \int L \leq \frac{\ell}{20}$
(At most 5% leakage time within intervals of at least 60 time units)



Content

Introduction

- **Observables and Evolutions**
- **Duration Calculus (DC)** ✓
- **Semantical Correctness Proofs** \mathcal{S}
- **DC Decidability** \mathcal{S}/\mathcal{Z}
- **DC Implementables**
- **PLC-Automata**
- **Timed Automata (TA)**, Uppaal
- **Networks of Timed Automata**
- **Region/Zone-Abstraction**
- **TA model-checking**
- **Extended Timed Automata**
- **Undecidability Results**

$obs : \text{Time} \rightarrow \mathcal{S}(obs)$

$(obs_0, \nu_0), t_0 \xrightarrow{\lambda_0} (obs_1, \nu_1)$

- **Automatic Verification...**
...whether a TA satisfies a DC formula, observer-based
- **Recent Results:**
- **Timed Sequence Diagrams, or Quasi-equal Clocks,**
or **Automatic Code Generation**, or ...

Content

- **Semantics-based Correctness Proofs**
- **Example: Gas Burner Controller**
- **Theorem 2.16:** Des-1 and Des-2 is a correct design wrt. Req
- **Lemma 2.19:** Des-1 and Des-2 imply a simplified requirement Req-1
- **Some Laws of the DC Integral Operator**
- **Lemma 2.17:** Req-1 implies Req
- **Obstacles (in a Non-Ideal World)**
- requirements may be **unrealisable** without considering plant assumptions
- **intermediate design levels**
- **different observables**
- **proving correctness** may be difficult
- If time permits:
A Calculus for DC

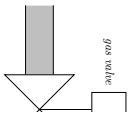
Specification and Semantics-based Correctness 1 of Real-Time Systems with DC

Methodology (in an ideal world)

In order to **prove** a controller design **correct** wrt. a **specification**:

- (i) Choose **observables** 'Obs';
 - (ii) Formalise the **requirements** 'Req'
as a conjunction of DC formulae (over 'Obs').
 - (iii) Formalise a **controller design** 'Ctrl'
as a conjunction of DC formulae (over 'Obs').
 - (iv) We say 'Ctrl' is **correct** (wrt. 'Req') iff
$$\models_0 \text{Ctrl} \implies \text{Req},$$
- so "just" prove $\models_0 \text{Ctrl} \implies \text{Req}$.

Gas Burner Revisited



(i) Choose **observables**:

- $F : \{0, 1\}$: value 1 models "flame sensed now" (input)
- $G : \{0, 1\}$: value 1 models "gas valve is open now" (output)
- define $L := G \wedge \neg F$ to model **leakage**

(ii) Formalise the **requirement**:

$$\text{Req} := \Box(\ell \geq 60 \implies 20 \cdot \int L \leq \ell)$$

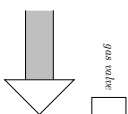
"in each interval of length at least 60 time units, at most 5% of the tim

(iii) Formalise **controller design ideas**:

- Des-1 := $\Box([L] \implies \ell \leq 1)$
- "make leakage phases last for at most one time unit"
- Des-2 := $\Box([L] : [\neg L] : [L] \implies \ell > 30)$



Gas Burner Revisited



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- "make leakage phases last for at most one time unit"
- Des-2 := $\Box([L] : [\neg L] : [L] \implies \ell > 30)$

"ensure: non-leakage phases between two leakage phases last at lea

(iv) Prove **correctness**, i.e. prove $\models (\text{Des-1} \wedge \text{Des-2} \implies \text{Req})$.

(Or do we want " $\models_0 \dots$ "?)

A Correct Gas Burner Controller Design

$$\text{Req} := \square(\ell \geq 60 \implies 20 \cdot \int L \leq \ell)$$

$$\text{Des-1} := \square([L] \implies \ell \leq 1), \quad \text{Des-2} := \square([L]; [\neg L]; [L] \implies$$

- A controller for the gas burner which guarantees Des-1 and Des-1 is **correct**

$$\models (\text{Des-1} \wedge \text{Des-2} \implies \text{Req})$$

(shown in **Theorem 2.16**)

- We do prove (in **Lemma 2.19**)

$$\models (\text{Des-1} \wedge \text{Des-2}) \implies \text{Req-1.}$$

for the the **simplified requirement**

$$\text{Req-1} := \square(\ell \leq 30 \implies \int L \leq 1).$$

("intervals of length at most 30 time units have at most 1 time unit of accu

- Showing

$$\models \text{Req-1} \implies \text{Req}$$

(in **Lemma 2.17**) completes the overall proof.

Lemma 2.17

Claim:

$$\models \underbrace{\square(\ell \leq 30 \implies \int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\square(\ell \geq 60 \implies 20 \cdot \int L)}_{\text{Req}}$$

Proof:

- Assume that 'Req-1' holds.
- Let $I_{\mathcal{I}}$ be any interpretation of L , and $[b, e]$ an interval with $e - b \geq$
- We need to show that

$$20 \cdot \int L \leq \ell$$

evaluates to 'tt' on **interval** $[b, e]$ under **interpretation** \mathcal{I} (and any \mathbf{v} :

- We have

$$\mathcal{I} \llbracket 20 \cdot \int L \leq \ell \rrbracket (\mathbf{v}, [b, e]) = \text{tt}$$

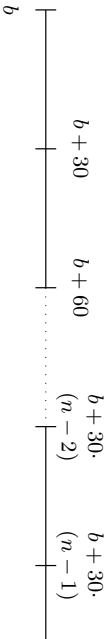
\iff (by DC semantics)

$$20 \cdot \int_b^e L_{\mathcal{I}}(t) dt \leq (e - b)$$

Lemma 2.17 Cont'd

$$\begin{aligned} \mathbb{R} &= \underbrace{\mathbb{R}}_{\ell \leq 30} \cup \underbrace{\mathbb{R}}_{\ell \geq 60} \\ &\Rightarrow \mathbb{R} \end{aligned}$$

- Set $n := \lceil \frac{e-b}{30} \rceil$, i.e. $n \in \mathbb{N}$ with $n - 1 < \frac{e-b}{30} \leq n$, and split the interval as follows:

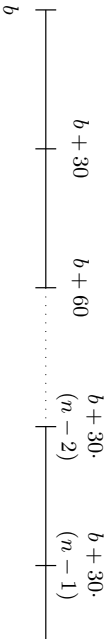


$$\begin{aligned} & 20 \cdot \int_b^e L_X(t) dt \\ &= 20 \left(\underbrace{\sum_{i=0}^{n-2} \int_{b+30i}^{b+30(i+1)} L_X(t) dt}_{\leq 1} + \underbrace{\int_{b+30(n-1)}^e L_X(t) dt}_{\leq 1} \right) \\ \{\text{Req-1}\} &\leq 20 \cdot \sum_{i=0}^{n-2} 1 + (20 \cdot 1) \end{aligned}$$

Lemma 2.17 Cont'd

$$\begin{aligned} \mathbb{R} &= \underbrace{\mathbb{R}}_{\ell \leq 30} \cup \underbrace{\mathbb{R}}_{\ell \geq 60} \\ &\Rightarrow \mathbb{R} \end{aligned}$$

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$$\begin{aligned} & 20 \cdot \int_b^e L_X(t) dt \\ &= 20 \left(\sum_{i=0}^{n-2} \int_{b+30i}^{b+30(i+1)} L_X(t) dt + \int_{b+30(n-1)}^e L_X(t) dt \right) \\ \{\text{Req-1}\} &\leq 20 \cdot \sum_{i=0}^{n-2} 1 + 20 \cdot 1 = 20 \cdot n \\ \{(*)\} &< 20 \cdot \left(\frac{e-b}{30} + 1 \right) = \frac{2}{3}(e-b) + 20 \\ \{e-b \geq 60\} &\leq e-b \end{aligned}$$

Some Laws of the DC Integral Operator

Theorem 2.18.

For all state assertions P and all real numbers $r_1, r_2 \in \mathbb{R}$,

- (i) $\models \int P \leq \ell$,
- (ii) $\models (\int P = r_1) ; (\int P = r_2) \implies (\int P = (r_1 + r_2))$
- (iii) $\models [\neg P] \implies \int P = 0$,
- (iv) $\models \square \implies \int P = 0$.

Lemma 2.19

- (i) $\models \int P \leq \ell$, (iii) $\models [\neg P$
- (ii) $\models (\int P = r_1) ; (\int P = r_2)$
- (iv) $\models \square \implies \int P = 0$.

Claim:

$$\models \underbrace{(\square([L]) \implies \ell \leq 1)}_{\text{Des-1}} \wedge \underbrace{(\square([L] ; [\neg L] ; [L]) \implies \ell > 30)}_{\text{Des-2}} \implies \square(\ell \leq ;$$

Proof:

$$\{ \text{Des-2} \} \implies \square$$

$$\ell \leq 30$$

$$\vee [L] ; (\square \vee [\neg L])$$

$$\vee [\neg L] ; (\square \vee [L])$$

$$\vee [\neg L] ; [L] ; [\neg L] \quad \square$$

$$\ell = 29$$

$$\vdash \square \vdash \square$$

Lemma 2.19

$$\begin{aligned} (i) & \models fP \leq \ell, & (iii) & \models \lceil \neg P \\ (ii) & \models (fP = r_1); (fP = r_2) \\ (iv) & \models \top \implies fP = 0. \end{aligned}$$

Claim: $\models \underbrace{\Box([L])}_{\text{Des-1}} \implies \ell \leq 1 \wedge \underbrace{\Box([L]; \lceil \neg L \rceil; [L])}_{\text{Des-2}} \implies \ell > 30 \implies \underbrace{\Box(\ell \leq ;)}_{\text{Des-1}}$

Proof:

$$\ell \leq 30$$

$$\{\text{Des-2}\} \implies \top$$

$$\vee [L]; (\top \vee \lceil \neg L \rceil)$$

$$\vee \lceil \neg L \rceil; (\top \vee [L])$$

$$\vee \lceil \neg L \rceil; [L]; \lceil \neg L \rceil$$

$$\{\text{Des-1}\} \implies \top$$

$$\vee (\ell \leq 1); (\top \vee \lceil \neg L \rceil)$$

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$$\{\text{Des-1}\} \implies \top$$

$$\vee (\ell \leq 1); (\top \vee \lceil \neg L \rceil)$$

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$$\vee \lceil \neg L \rceil; (\ell \leq 1); \lceil \neg L \rceil$$

$$\{\emptyset\} \implies \top$$

$$\vee (fL \leq 1); (\top \vee \lceil \neg L \rceil)$$

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Claim: $\models \underbrace{\Box([L])}_{\text{Des-1}} \implies \underbrace{\ell \leq 1 \wedge \Box([L]; \lceil \neg L \rceil; [L])}_{\text{Des-2}} \implies \ell > 30 \implies \underbrace{\Box(\ell \leq 1)}_{\text{Des-1}}$

Proof:

$$\begin{aligned} \ell \leq 30 & \\ \{\text{Des-2}\} & \implies \top \\ & \vee [L]; (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee [L]) \\ & \vee \lceil \neg L \rceil; [L]; \lceil \neg L \rceil \\ \{\text{Des-1}\} & \implies \top \\ & \vee (\ell \leq 1); (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee (\ell \leq 1)) \\ & \vee \lceil \neg L \rceil; (\ell \leq 1); \lceil \neg L \rceil \\ \{\emptyset\} & \implies \top \\ & \vee (fL \leq 1); (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee (fL \leq 1)) \\ & \vee \lceil \neg L \rceil; (fL \leq 1); \lceil \neg L \rceil \end{aligned}$$

$$\{(iv), (iii)\} \implies fL = 0$$

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Proof:

$$\begin{aligned} \ell \leq 30 & \\ \{\text{Des-2}\} & \implies \top \\ & \vee [L]; (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee [L]) \\ & \vee \lceil \neg L \rceil; [L]; \lceil \neg L \rceil \\ \{\text{Des-1}\} & \implies \top \\ & \vee (\ell \leq 1); (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee (\ell \leq 1)) \\ & \vee \lceil \neg L \rceil; (\ell \leq 1); \lceil \neg L \rceil \\ \{\emptyset\} & \implies \top \\ & \vee (fL \leq 1); (\top \vee \lceil \neg L \rceil) \\ & \vee \lceil \neg L \rceil; (\top \vee (fL \leq 1)) \\ & \vee \lceil \neg L \rceil; (fL \leq 1); \lceil \neg L \rceil \end{aligned}$$

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$$\text{Claim: } \underbrace{\models (\Box([L]) \implies \ell \leq 1)}_{\text{Des-1}} \wedge \underbrace{\models (\Box([L]; [\neg L]; [L]) \implies \ell > 30)}_{\text{Des-2}} \implies \underbrace{\models \Box(\ell \leq 1)}_{\text{Des-1}}$$

Proof:

$$\begin{aligned} \{\text{Des-2}\} & \implies \square \implies \ell \leq 30 & \{\text{(iv), (iii)}\} & \implies fL = 0 \\ & \vee [L]; (\square \vee [\neg L]) & & \vee (fL \leq 1); (fL \\ & \vee [\neg L]; (\square \vee [L]) & & \vee fL = 0; (fL = \\ & \vee [\neg L]; [L]; [\neg L] & & \vee fL = 0; (fL \\ \{\text{Des-1}\} & \implies \square & \{\text{(ii)}\} & \implies fL = 0 \\ & \vee (\ell \leq 1); (\square \vee [\neg L]) & & \vee fL \leq 1 + 0 \\ & \vee [\neg L]; (\square \vee (\ell \leq 1)) & & \vee fL = 0 + 1 \\ & \vee [\neg L]; (\ell \leq 1); [\neg L] & & \vee fL \leq 0 + 1 + \\ \{\text{(i)}\} & \implies \square & & \implies fL \leq 1 \\ & \vee (fL \leq 1); (\square \vee [\neg L]) & & \\ & \vee [\neg L]; (\square \vee (fL \leq 1)) & & \\ & \vee [\neg L]; (fL \leq 1); [\neg L] & & \end{aligned}$$

Content

- **Semantics-based Correctness Proofs**
 - Example: **Gas Burner Controller**
 - **Theorem 2.16:** Des-1 and Des-2 is a correct design wrt. Req ✓
 - **Lemma 2.19:** Des-1 and Des-2 imply a simplified requirement (Req-1)
 - **Some laws of the DC Integral Operator**
 - **Lemma 2.17:** Req-1 implies Req
- **Obstacles (in a Non-Ideal World)**
 - requirements may be **unrealisable** without considering plant assumptions
 - **intermediate design levels**
 - **different observables**
 - **proving correctness** may be difficult
- If time permits: **A Calculus for DC**

Obstacles in Non-Ideal World

Methodology: The World is Not Ideal...

- (i) Choose a collection of **observables** 'Obs';
- (ii) Provide **specification** 'Req' (conjunction of DC formulae over 'C
- (iii) Provide a description 'Ctrl' of the **controller** (DC formula over 'C
- (iv) Prove 'Ctrl' **correct** (wrt. 'Req'), i.e. prove $\models \text{Ctrl} \implies \text{Req}$.

That looks **too simple to be practical**.

Typical obstacles:

- (i) It may be **impossible** to realise 'Req' if it doesn't consider properties of the plant.
- (ii) There are typically intermediate **design levels** between 'Req' and
- (iii) 'Req' and 'Ctrl' may use **different observables**.
- (iv) Proving validity of the implication is **not trivial**.

(i) Assumptions As A Form of Plant Model

- Often the controller will (or can) operate correctly only under some
- For instance, with a **level crossing**
- we may assume an **upper bound** on the **speed of approaching t** (otherwise wed need to close the gates arbitrarily fast)
- we may assume that trains are **not arbitrarily slow** in the crossin (otherwise we can't make promises to the road traffic)
- We shall **specify such assumptions** as a DC formula 'Asm' on the **input observables** and verify correctness of 'Ctrl' wrt. 'Req' by proving validity (from 0

$$\text{Ctrl} \wedge \text{Asm} \implies \text{Req}$$
- Shall we **care** whether 'Asm' is satisfiable?

(ii) Intermediate Design Levels

- A **top-down development approach** may involve
 - Req – **specification/requirements**
 - Des – **design**
 - Ctrl – **implementation**
- Then **correctness** is established by proving validity of

$$\text{Ctrl} \implies \text{Des}$$
 and

$$\text{Des} \implies \text{Req}$$
 (and then concluding 'Ctrl \implies Req' by transitivity).
- Any preference on the order (of (1) and (2))?

(iii): Different Observables

- Assume, 'Req' uses **more abstract observables** Obs_A and 'Ctrl' **more concrete observables** Obs_C .
- **For instance:**
 - in Obs_A : only consider **one gas valve, open or closed** – $(G : \{0, 1$
 - in Obs_C : may consider **two valves and intermediate positions**, for instance, to react to different heating requests – $G_i : \{0, 1, 2,$
- To **prove correctness**,
- we need information **how the observables are related**,
- an **invariant** which **links** the data values of Obs_A and Obs_C .
- If we're given the linking invariant as a DC formula, say 'Link $_{C,A}$ ', the correctness of 'Ctrl' wrt. 'Req' amounts to proving

$$\models_0 \text{Ctrl} \wedge \text{Link}_{C,A} \implies \text{Req}.$$
- For instance, $\text{Link}_{C,A} := \prod \vee [G \iff (G_1 > 0 \vee G_2 > 0)]$.

Obstacle (iv): How to Prove Correctness?

Main options:

- **by hand** on the basis of DC semantics (as demonstrated before),
- using **proof rules** from a calculus (\rightarrow later),
- sometimes a **general theorem** may fit (e.g. cycle times of PLC auto algorithms as in Uppaal (\rightarrow later)).

Tell Them What You've Told Them...

- **Design ideas** for the behaviour of real-time system controllers can also be described using DC formulae.
- The **correctness** of a design idea wrt. requirements can principally be proven "on foot" (using the DC semantics and analysis results).
- This approach is not limited to over-simplified (?) **gas burner** controllers:
- Consider **plant assumptions**.
- Use **intermediate designs** in a step-by-step development
- Link **different observables** by invariants.
- Consider other **proof techniques**.

References

References

Olderog, E.-R. and Dietke, H. (2008). *Real-Time Systems - Formal Specification and Autor*
Cambridge University Press.