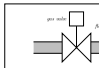
Duration Calculus: Preview



- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an (**implicitly given**) interval.

- $G, F, I, H : \{0,$
- Define $L : \{0, 1\}$

Strangest operators:

- **almost everywhere** – Example: $\lceil G \rceil$
  (Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere.)
- **chop** – Example: $(\lceil \neg I \rceil \,;\, \lceil I \rceil \,;\, \lceil \neg I \rceil) \implies \ell \geq 1$
  (Ignition phases last at least one time unit.)
- **integral** – Example: $\ell \geq 60 \implies \int L \leq \frac{\ell}{20}$
  (At most 5% leakage time within intervals of at least 60 time units.)

Content

Methodology *(in an ideal world)*

In order to **prove** a controller design **correct** wrt. a **specification**:

(i) Choose **observables** 'Obs'.

(ii) Formalise the **requirements** 'Req' as a conjunction of DC formulae (over 'Obs').

(iii) Formalise a **controller design** 'Ctrl' as a conjunction of DC formulae (over 'Obs').

(iv) We say 'Ctrl' is **correct** (wrt. 'Req') iff

$$\models_0 \text{Ctrl} \implies \text{Req},$$

so "just" prove $\models_0 \text{Ctrl} \implies \text{Req}$.

Real-Time Systems

Lecture 5: Duration Calculus

2017-11-09

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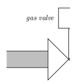
Content

Specification and Semantics-based Correctness I of Real-Time Systems with DC

Gas Burner Revisited



(i) Choose **observables**:
- $F : \{0,1\}$: value 1 models "flame sensed now" **(input)**
- $G : \{0,1\}$: value 1 models "gas valve is open now" **(output)**
- define $L := G \wedge \neg F$ to model **leakage**

(ii) Formalise the **requirement**:

$$\text{Req} := \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

"in each interval of length at least 60 time units, at most 5% of the tim

(iii) Formalise **controller design ideas**:
- Des-1 $:= \Box(\lceil L \rceil \implies \ell \leq 1)$
  "**make** leakage phases last for at most one time unit"
- Des-2 $:= \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$
  "**ensure**: non-leakage phases between two leakage phases last at lea

(iv) Prove **correctness**, i.e. prove $\models$ (Des-1 $\wedge$ Des-2 $\implies$ Req).
(Or do we want "$\models_0$"...?)

Lemma 2.17

Claim:

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L}_{\text{Req}}$$

Proof:
- Assume that 'Req-1' holds.
- Let $L_\mathcal{I}$ be any interpretation of L, and $[b,e]$ an interval with $e - b \geq$
- We need to show that
  $$20 \cdot \textstyle\int L \leq \ell$$
  evaluates to 'tt' on **interval** $[b,e]$ under **interpretation** $\mathcal{I}$ (and any va
- We have
  $$\mathcal{I}[\![20 \cdot \textstyle\int L \leq \ell]\!](\mathcal{V}, [b,e]) = \text{tt}$$
  $\iff$ *(by DC semantics)*
  $$\hat{20} \,\hat{\cdot} \int_b^e L_\mathcal{I}(t)\,dt \;\hat{\leq}\; (e-b)$$

Lemma 2.17 Cont'd

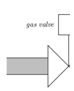
$\models \Box(\ell \leq 30$
R
$\implies \Box(\ell \geq 6$

- Set $n := \lceil \frac{e-b}{30} \rceil$, i.e. $n \in \mathbb{N}$ with $n - 1 < \frac{e-b}{30} \leq n$, $(*)$
  and split the interval as follows:

b | $b+30$ | $b+60$ | ⋯ | $b+30\cdot(n-2)$ | $b+30\cdot(n-1)$

$$20 \cdot \int_b^e L_\mathcal{I}(t)\,dt$$
$$= 20\left(\sum_{i=0}^{n-2} \int_{b+30i}^{b+30(i+1)} L_\mathcal{I}(t)\,dt + \int_{b+30(n-1)}^{e} L_\mathcal{I}(t\right.$$
$$\{\text{Req-1}\} \leq 20 \cdot \sum_{i=0}^{n-2} 1 + 20 \cdot 1 = 20 \cdot n$$
$$\{(*)\} < 20 \cdot \left(\frac{e-b}{30} + 1\right) = \frac{2}{3}(e-b) + 20$$
$$\{e - b \geq 60\} \leq e - b$$

Gas Burner Revisited



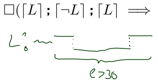
(i) Choose **observables**:
- $F : \{0,1\}$: value 1 models "flame sensed now" **(input)**
- $G : \{0,1\}$: value 1 models "gas valve is open now" **(output)**
- define $L := G \wedge \neg F$ to model **leakage**

(ii) Formalise the **requirement**:

$$\text{Req} := \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

"in each interval of length at least 60 time units, at most 5% of the tim

(iii) Formalise **controller design ideas**:
- Des-1 $:= \Box(\lceil L \rceil \implies \ell \leq 1)$
  "**make** leakage phases last for at most one time unit"
- Des-2 $:= \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$



A Correct Gas Burner Controller Design

Req $:= \Box(\ell \geq 60 \implies 20 \cdot \int L \leq \ell)$

Des-1 $:= \Box(\lceil L \rceil \implies \ell \leq 1)$, Des-2 $:= \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies$

- A **controller for the gas burner** which guarantees Des-1 and Des-1 is **corre**
  $$\models (\text{Des-1} \wedge \text{Des-2} \implies \text{Req})$$
  (shown in Theorem 2.16)
- We do prove (in Lemma 2.19)
  $$\models (\text{Des-1} \wedge \text{Des-2}) \implies \text{Req-1}.$$
  for the the **simplified requirement**
  $$\text{Req-1} := \Box(\ell \leq 30 \implies \textstyle\int L \leq 1).$$
  ("intervals of length at most 30 time units have at most 1 time unit of acc
- Showing
  $$\models \text{Req-1} \implies \text{Req}$$
  (in Lemma 2.17) completes the overall proof.

Lemma 2.17 Cont'd

$\models \Box(\ell \leq 30$
R
$\implies \Box(\ell \geq 6$

- Set $n := \lceil \frac{e-b}{30} \rceil$, i.e. $n \in \mathbb{N}$ with $n - 1 < \frac{e-b}{30} \leq n$,
  and split the interval as follows:

b | $b+30$ | $b+60$ | ⋯ | $b+30\cdot(n-2)$ | $b+30\cdot(n-1)$

$$20 \cdot \int_b^e L_\mathcal{I}(t)\,dt$$
$$= 20\left(\underbrace{\sum_{i=0}^{n-2} \int_{b+30i}^{b+30(i+1)} L_\mathcal{I}(t)\,dt}_{\leq 1} + \underbrace{\int_{b+30(n-1)}^{e} L_\mathcal{I}(t}_{\leq 1}\right.$$
$$\{\text{Req-1}\} \leq \left(20 \cdot \sum_{i=0}^{n-2} 1\right) + \left(20 \cdot 1\right)$$

Some Laws of the DC Integral Operator

Theorem 2.18.
For all state assertions P and all real numbers $r_1, r_2 \in \mathbb{R}$,

(i) $\models \int P \le \ell$,

(ii) $\models \big((\int P = r_1)\,;(\int P = r_2)\big) \Longrightarrow \big(\int P = (r_1 + r_2)\big)$

(iii) $\models \lceil \neg P \rceil \implies \int P = 0$,

(iv) $\models \lceil\!\lceil \implies \int P = 0$.

Lemma 2.19

(i) $\models \int P \le \ell$, (iii) $\models \lceil \neg P$
(ii) $\models (\int P = r_1)\,;(\int P = r_2)$
(iv) $\models \lceil\!\lceil \implies \int P = 0$.

Claim:

$$\models (\overbrace{\Box(\lceil L \rceil \implies \ell \le 1)}^{\text{Des-1}} \wedge \overbrace{\Box(\lceil L \rceil\,;\lceil \neg L \rceil\,;\lceil L \rceil \implies \ell > 30)}^{\text{Des-2}}) \implies \Box(\ell \le$$

Proof:

$$\begin{array}{rll}
 & \ell \le 30 & \\
\{\text{Des-2}\} \implies & \lceil\!\lceil & \\
 & \vee\, \lceil L \rceil\,;(\lceil\!\lceil \vee \lceil \neg L \rceil) & \\
 & \vee\, \lceil \neg L \rceil\,;(\lceil\!\lceil \vee \lceil L \rceil) & \\
 & \vee\, \lceil \neg L \rceil\,;\lceil L \rceil\,;\lceil \neg L \rceil & \\
\{\text{Des-1}\} \implies & \lceil\!\lceil & \\
 & \vee\, (\ell \le 1)\,;(\lceil\!\lceil \vee \lceil \neg L \rceil) & \\
 & \vee\, \lceil \neg L \rceil\,;(\lceil\!\lceil \vee (\ell \le 1)) & \\
 & \vee\, \lceil \neg L \rceil\,;(\ell \le 1)\,;\lceil \neg L \rceil & \\
\{(i)\} \implies & \lceil\!\lceil & \\
 & \vee\, (\int L \le 1)\,;(\lceil\!\lceil \vee \lceil \neg L \rceil) & \\
 & \vee\, \lceil \neg L \rceil\,;(\lceil\!\lceil \vee (\int L \le 1)) & \\
 & \vee\, \lceil \neg L \rceil\,;(\int L \le 1)\,;\lceil \neg L \rceil &
\end{array}$$

$$\begin{array}{rll}
\{(iv),\,(iii)\} \implies & \int L = 0 & \\
 & \vee\, (\int L \le 1)\,;(\int L & \\
 & \vee\, \int L = 0\,;(\int L = & \\
 & \vee\, \int L = 0\,;(\int L & \\
\{(ii)\} \implies & \int L = 0 & \\
 & \vee\, \int L \le 1 + 0 & \\
 & \vee\, \int L \not< 0 + 1 & \\
 & \vee\, \int L \le 0 + 1 + &
\end{array}$$

Content

- **Semantics-based Correctness Proofs**
  - Example: **Gas Burner Controller**
  - Theorem 2.16: Des-1 and Des-2 is a correct design wrt. Req
  - Lemma 2.19: Des-1 and Des-2 imply a simplified requirement (Req-1)
  - **Some Laws of the DC Integral Operator**
  - Lemma 2.17: Req-1 implies Req
- **Obstacles (in a Non-Ideal World)**
  - requirements may be **unrealisable** without considering plant assumptions
  - **intermediate** design levels
  - **different observables**
  - **proving correctness** may be difficult
- If time permits:
  A Calculus for DC

Methodology: The World is Not Ideal...

(i) Choose a collection of **observables** 'Obs'.

(ii) Provide **specification** 'Req' (conjunction of DC formulae over 'C

(iii) Provide a description 'Ctrl' of the **controller** (DC formula over 'C

(iv) Prove 'Ctrl' **correct** (wrt. 'Req'), i.e. prove $\models \text{Ctrl} \implies \text{Req}$.

That looks **too simple to be practical**.

Typical **obstacles**:

(i) It may be **impossible** to realise 'Req' if it doesn't consider properties of the plant.

(ii) There are typically intermediate **design levels** between 'Req' and

(iii) 'Req' and 'Ctrl' may use **different observables**.

(iv) Proving validity of the implication is **not trivial**.

(ii) Intermediate Design Levels

- A **top-down development approach** may involve
  - Req – **specification/requirements**
  - Des – **design**
  - Ctrl – **implementation**
- Then **correctness** is established by proving validity of

$$\text{Ctrl} \implies \text{Des}$$

and

$$\text{Des} \implies \text{Req}$$

(and then concluding 'Ctrl $\implies$ Req' by transitivity).

- Any preference on the order (of (1) and (2))?

Lemma 2.19

(i) $\models \int P \leq \ell$, (iii) $\models \lceil \neg P$
(ii) $\models (\int P = r_1); (\int P = r_2)$
(iv) $\models \lceil\rceil \implies \int P = 0$.

Claim:

$$\models (\overbrace{\Box(\lceil L \rceil \implies \ell \leq 1)}^{\text{Des-1}} \wedge \overbrace{\Box(\lceil L \rceil ; \lceil \neg L \rceil ; \lceil L \rceil \implies \ell > 30)}^{\text{Des-2}}) \implies \Box(\ell \leq$$

Proof:

$$\begin{array}{rl}
 & \ell \leq 30 \\
\{\text{Des-2}\} \implies & \lceil\rceil \\
 & \vee \lceil L \rceil ; (\lceil\rceil \vee \lceil \neg L \rceil) \\
 & \vee \lceil \neg L \rceil ; (\lceil\rceil \vee \lceil L \rceil) \\
 & \vee \lceil \neg L \rceil ; \lceil L \rceil ; \lceil \neg L \rceil \\
\{\text{Des-1}\} \implies & \lceil\rceil \\
 & \vee (\ell \leq 1) ; (\lceil\rceil \vee \lceil \neg L \rceil) \\
 & \vee \lceil \neg L \rceil ; (\lceil\rceil \vee (\ell \leq 1)) \\
 & \vee \lceil \neg L \rceil ; (\ell \leq 1) ; \lceil \neg L \rceil \\
\{\text{i}\} \implies & \lceil\rceil \\
 & \vee (\int L \leq 1) ; (\lceil\rceil \vee \lceil \neg L \rceil) \\
 & \vee \lceil \neg L \rceil ; (\lceil\rceil \vee (\int L \leq 1)) \\
 & \vee \lceil \neg L \rceil ; (\int L \leq 1) ; \lceil \neg L \rceil
\end{array}$$

$$\begin{array}{rl}
\{\text{(iv), (iii)}\} \implies & \int L = 0 \\
 & \vee (\int L \leq 1) ; (\int L \\
 & \vee \int L = 0 ; (\int L = \\
 & \vee \int L = 0 ; (\int L \\
\{\text{ii}\} \implies & \int L = 0 \\
 & \vee \int L \leq 1 + 0 \\
 & \vee \int L = 0 + 1 \\
 & \vee \int L \leq 0 + 1 + \\
\implies & \int L \leq 1
\end{array}$$

Obstacles in Non-Ideal World

(i) Assumptions As A Form of Plant Model

- Often the controller will (or can) operate correctly only under some
- For instance, with a **level crossing**
  - we may assume an **upper bound** on the **speed of approaching t** (otherwise we'd need to close the gates arbitrarily fast)
  - we may assume that trains are **not arbitrarily slow** in the crossin (otherwise we can't make promises to the road traffic)
- We shall **specify such assumptions** as a DC formula 'Asm' on the **input observables** and verify correctness of 'Ctrl' wrt. 'Req' by proving validity (from O

$$\text{Ctrl} \wedge \text{Asm} \implies \text{Req}$$

- Shall we **care** whether 'Asm' is satisfiable?

Obstacle (iv): How to Prove Correctness?

Main options:

- **by hand** on the basis of DC semantics (as demonstrated before),
- using **proof rules** from a calculus (→ later),
- sometimes a **general theorem** may fit (e.g. cycle times of PLC auto
- **algorithms** as in Uppaal (→ later).

(iii): Different Observables

- Assume, 'Req' uses **more abstract observables** Obs_A and 'Ctrl' **more concrete observables** Obs_C.
- **For instance**:
  - in Obs_A: only consider **one gas valve**, **open or closed** – ($G : \{0, 1$
  - in Obs_C: may consider **two valves** and **intermediate positions**, for instance, to react to different heating requests – $G_i : \{0, 1, 2,$
- To **prove correctness**,
  - we need information **how the observables are related**,
  - an **invariant** which **links** the data values of Obs_A and Obs_C.
- **If** we're given the linking invariant as a DC formula, say '$\mathrm{Link}_{C,A}$', **th** correctness of 'Ctrl' wrt. 'Req' amounts to proving

$$\models_0 \mathrm{Ctrl} \wedge \mathrm{Link}_{C,A} \implies \mathrm{Req}.$$

- For instance, $\mathrm{Link}_{C,A} := \lceil\!\lceil \vee \lceil G \iff (G_1 > 0 \vee G_2 > 0)\rceil\!\rceil$.

Tell Them What You've Told Them...

- **Design ideas** for the behaviour of real-time system controlle can also be described using DC formulae.
- The **correctness** of a design idea wrt. requirements can principally be proven "on foot" (using the DC semantics and analysis results).
- This approach is not limited to over-simplified (?) **gas burner** controllers:
  - Consider **plant assumptions**.
  - Use **intermediate designs** in a step-by-step development
  - Link **different observables** by invariants.
  - Consider other **proof techniques**.

References

References

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Autor* Cambridge University Press.