Real-Time Systems

Lecture 8: DC Implementables I

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Content

Introduction

- **Observables and Evolutions** ✓
- **Duration Calculus** (DC) ✓
- Semantical Correctness Proofs ✓
- DC Decidability ✓
- DC Implementables 8–9
- **PLC-Automata** 10

$obs : \mathsf{Time} \to \mathscr{D}(obs)$

- **Timed Automata** (TA), Uppaal 11–
- Networks of Timed Automata
- Region/Zone-Abstraction
- TA model-checking
- Extended Timed Automata
- Undecidability Results

$\langle obs_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_0} \langle obs_1, \nu_1 \rangle, t_1 \ldots$

- **Automatic Verification...**
  ...whether a TA satisfies a DC formula, observer-based
- **Recent Results:**
  - **Timed Sequence Diagrams**, or **Quasi-equal Clocks**, or **Automatic Code Generation**, or ...

Content

- **Motivation:** Why DC Implementables?
  - What can we assume of controller platforms?
- **DC Standard Forms**
  - Followed-by, Followed-by-initially
  - (Timed) Leads-to
  - (Timed) Up-to, (Timed) Up-to-initially
- **Control Automata**
  - phases, basic phases
- **DC Implementables**
  - Initialisation, Sequencing, Progress
  - Synchronisation, (Un)Bounded Stability
  - (Un)Bounded Initial Stability
- **Example:**
  A correct controller for the **Gas Burner** specified by **DC Implementables**

DC Implementables: Motivation

Requirements vs. Implementations

- **Problem:** in general, a DC requirement doesn't tell **how** to achieve it, how to build a controller/write a program which ensures it.

$$\Box((\lceil \neg B \rceil \wedge \ell = 5 ; \lceil B \rceil) \implies (\lceil L = \mathsf{yellow} \rceil ; true))$$

"whenever a pedestrian presses the button **5 time units from now**, then **now** the traffic lights should **already be yellow**"

Plus: road traffic should not see 'yellow' all the time.

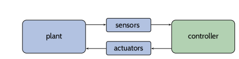
$$\Box((\lceil B \wedge L = \mathsf{green} \rceil ; \ell = 5) \implies (true ; \lceil L = \mathsf{red} \rceil))$$

"whenever a pedestrian presses the button **now** while road traffic sees 'green', then **5 time units later** (the latest) road traffic **should see 'red'**"

Requirements vs. Implementations

- **Problem:** in general, a DC requirement doesn't tell **how** to achieve it, how to build a controller/write a program which ensures it.
- What **a controller** (clearly) **can do** is:
  - consider **inputs now**,
  - **change (local) state**, or
  - **wait**,
  - set **outputs now**.

  (But not, e.g., consider future inputs now.)



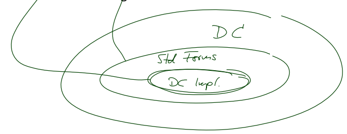
- So, if we have
  - a DC requirement 'Req',
  - a description 'Impl' in DC of the controller behaviour, which "uses" **just these four** operations,

  then
  - proving correctness (still) amounts to proving $\models_0$ Impl $\implies$ Req (**in DC**)
  - and we (more or less) **know how to program** (the correct) 'Impl' in a PLC language, or in C on a real-time OS, or or or...

Approach: Control Automata and DC Implementables

Plan:

- Introduce **DC Standard Forms** (a sub-language of DC)
- Introduce **Control Automata**
- Introduce **DC Implementables** as a subset of **DC Standard Forms**
- **Example**: a correct controller design for the notorious Gas Burner



DC Standard Forms

DC Standard Forms: Followed-by

In the following: F is a DC **formula**, P a **state assertion**, θ a **rigid term**.

- **Followed-by:**

$$F \longrightarrow \lceil P \rceil :\iff \neg\Diamond(F ; \lceil \neg P \rceil) \iff \Box\neg(F ; \lceil \neg P \rceil)$$

in other symbols

$$\forall x \bullet \Box((F \wedge \ell = x) ; \ell > 0 \implies (F \wedge \ell = x) ; \lceil P \rceil ; true)$$

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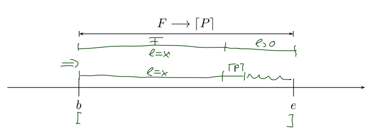

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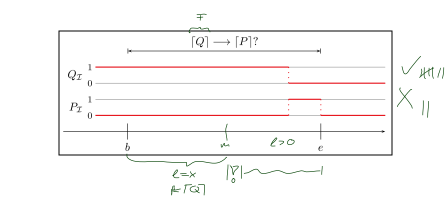

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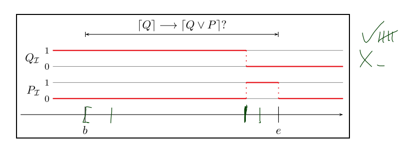

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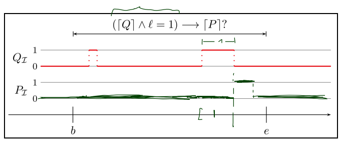
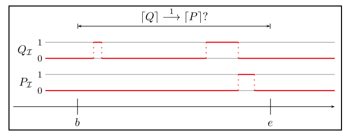


DC Standard Forms: (Timed) leads-to

- **(Timed) leads-to:**

$$F \xrightarrow{\theta} \lceil P \rceil :\iff (F \wedge \ell = \theta) \longrightarrow \lceil P \rceil$$



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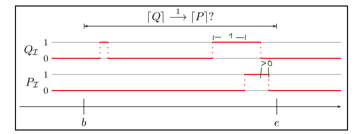


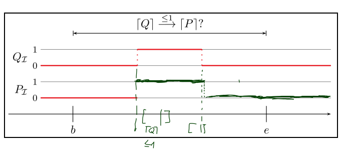
"if F persists for (at least) θ time units from time t,
then there is $\lceil P \rceil$ after $\theta + t$"

DC Standard Forms: (Timed) up-to

$\forall x \bullet \Box((F \wedge \ell = x) ; \ell > 0 \implies (F \wedge \ell = x) ; \lceil P \rceil ; true)$

- **(Timed) up-to:**

$$F \xrightarrow{\leq\theta} \lceil P \rceil :\iff (F \wedge \ell \leq \theta) \longrightarrow \lceil P \rceil$$



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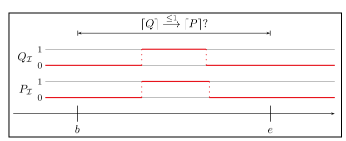

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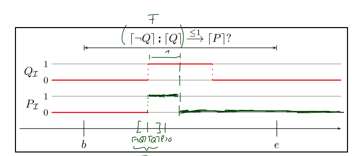
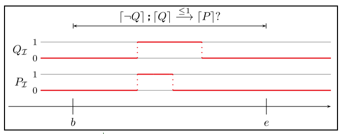


DC Standard Forms: (Timed) up-to

$$\forall x \bullet \Box((F \land \ell = x) ; \ell > 0 \implies (F \land \ell = x) ; \lceil P \rceil ; true)$$

- **(Timed) up-to:**

$$F \xrightarrow{\leq \theta} \lceil P \rceil :\iff (F \land \ell \leq \theta) \longrightarrow \lceil P \rceil$$

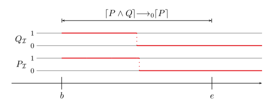


"during all Q-phases of at most θ time units, there needs to be $\lceil P \rceil$ as well"

DC Standard Forms: Initialisation

- **Followed-by-initially:**

$$F \longrightarrow_0 \lceil P \rceil :\iff \neg(F ; \lceil \neg P \rceil)$$



"after an initial phase with $\lceil P \land Q \rceil$, $\lceil P \rceil$ persists for some non-point interval"

- **(Timed) up-to-initially:**

$$F \xrightarrow{\leq \theta}_0 \lceil P \rceil :\iff (F \land \ell \leq \theta) \longrightarrow_0 \lceil P \rceil$$

- **Initialisation:**

$$\sqcap \lor \lceil P \rceil ; true$$

Control Automata

Control Automata

- Let $X_1, \dots, X_k$ be state variables with **finite** domains $\mathcal{D}(X_1), \dots, \mathcal{D}(X_k)$.
- $X_1, \dots, X_k$ together with a DC formula 'Impl' (over $X_1, \dots, X_k$) is called **system of k control automata**.
- 'Impl' is typically a conjunction of **DC implementables**. ($\to$ in a minute)

Example: (Simplified) **traffic lights**: $X : \{\text{red}, \text{green}, \text{yellow}\}$,

Impl $:= (\lceil \text{red} \rceil \longrightarrow \lceil \text{red} \lor \text{green} \rceil) \quad \land \quad (\lceil \text{green} \rceil \longrightarrow \lceil \text{green} \lor \text{yellow} \rceil)$
$\land \quad (\lceil \text{yellow} \rceil \longrightarrow \lceil \text{yellow} \lor \text{red} \rceil) \quad \land \quad (\sqcap \lor \lceil \text{red} \rceil ; true)$

state var. — $\mathcal{D}(X)$ — system of 1 control automaton

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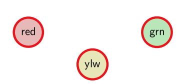
- Where's the **automaton**? Here, look:



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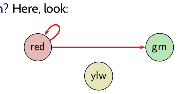

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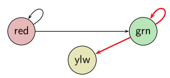

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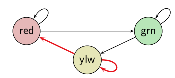

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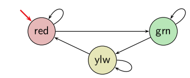

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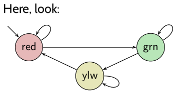


Phases

- A state assertion of the form

$$X_i = d_i, \quad d_i \in \mathcal{D}(X_i),$$

which constrains the values of X_i, is called **basic phase** of X_i.

- A **phase** of X_i is a Boolean combination of basic phases of X_i.

- **Abbreviations:**
  - Write X_i instead of $X_i = 1$, if X_i is Boolean.
  - Write d_i instead of $X_i = d_i$, if $\mathcal{D}(X_i)$ is disjoint from $\mathcal{D}(X_j)$, $i \neq j$.

- **Examples**
  - **Basic phases** of X: $(X = \text{green})$ (green) (red) (yellow)
  - **Phases** of X: $(X = \text{green} \vee X = \text{yellow})$ (green $\vee$ yellow) ($\neg$red) ...
  - Not a phase: $(X = \text{green} \wedge B = \text{pressed})$ [two different observables]

DC Implementables

DC Implementables

- …are special **patterns** of **DC Standard Forms** (due to A.P. Ravn).
- Within one pattern,
  - $\pi, \pi_1, \dots, \pi_n$, $n \geq 0$, denote **phases** of **the same** state variable X_i,
  - φ denotes a state assertion **not depending** on X_i.
  - θ denotes a **rigid** term.

- **Initialisation**: $\lceil\rceil \vee \lceil \pi \rceil \,;\, true$

  "initially, the control automaton is in phase π"

- **Sequencing**: $\lceil \pi \rceil \longrightarrow \lceil \pi \vee \pi_1 \vee \dots \vee \pi_n \rceil$

  "when the control automaton is in π, it subsequently stays in π or moves to one of $\pi_1, \dots \pi_n$"

- **Progress**: $\lceil \pi \rceil \xrightarrow{\theta} \lceil \neg\pi \rceil$

  "after the control automaton stayed in phase π for θ time units, is subsequently leaves this phase, thus progresses"

DC Implementables Cont'd

- **Synchronisation**: $\lceil \pi \wedge \varphi \rceil \xrightarrow{\theta} \lceil \neg\pi \rceil$

  "after the control automaton stayed for θ time units in phase π with the condition φ being true, it subsequently leaves this phase"

- **Bounded Stability**:

  $$\lceil \neg\pi \rceil \,;\, \lceil \pi \wedge \varphi \rceil \xrightarrow{\leq\theta} \lceil \pi \vee \pi_1 \vee \dots \vee \pi_n \rceil$$

  "if the control automaton changed its phase to π with the condition φ being true and the time since this change does not exceed θ time units, it subsequently stays in π or moves to one of $\pi_1, \dots, \pi_n$"

- **Unbounded Stability**:

  $$\lceil \neg\pi \rceil \,;\, \lceil \pi \wedge \varphi \rceil \longrightarrow \lceil \pi \vee \pi_1 \vee \dots \vee \pi_n \rceil$$

  "if the control automaton changed its phase to π with the condition φ being true, it subsequently stays in π or moves to one of $\pi_1, \dots, \pi_n$"

DC Implementables Cont'd

- **Bounded initial stability**:

  $$\lceil \pi \wedge \varphi \rceil \xrightarrow{\leq\theta}_0 \lceil \pi \vee \pi_1 \vee \dots \vee \pi_n \rceil$$

  "when the control automaton initially is in phase π with condition φ being true and the current time does not exceed θ time units, the control automaton subsequently stays in π or moves to one of $\pi_1, \dots, \pi_n$"

- **Unbounded initial stability**:

  $$\lceil \pi \wedge \varphi \rceil \longrightarrow_0 \lceil \pi \vee \pi_1 \vee \dots \vee \pi_n \rceil$$

  "when the control automaton initially is in phase π with condition φ being true, the control automaton subsequently stays in π or moves to one of $\pi_1, \dots, \pi_n$"

Using DC Implementables for (Controller) Specifications

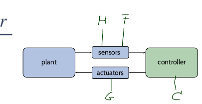
- Let $X_1, \dots, X_k$ be a **system of k control automata.**
- Let 'Impl' be a conjunction of **DC implementables**.
- Then 'Impl' **specifies** / **denotes** all interpretations $\mathcal{I}$ of $X_1, \dots, X_k$ and all valuations $\mathcal{V}$ such that $\mathcal{I}, \mathcal{V} \models_0$ Impl
- In other words: 'Impl' denotes the set $\{(\mathcal{I}, \mathcal{V}) \mid \mathcal{I}, \mathcal{V} \models_0 \text{Impl}\}$ of **interpretations** and **valuations** which **realise** 'Impl' **from** 0.
- **Controller Verification**:
  If 'Impl' describes (exactly or over-approximating) the behaviour of a controller, then proving the controller correct wrt. requirements 'Req' amounts to showing

  $$\models_0 \text{Impl} \implies \text{Req}$$

- **Controller Specification**: Dear programmers,
  'Impl' describes my design idea (and I have shown $\models_0$ Impl $\implies$ Req),
  please provide a controller program whose behaviour is a subset of 'Impl';
  that is: a correct implementation of my design.

Example: Gas Burner

Control Automata for the Gas Burner



A **gas burner controller** can be modelled as a **system of four control automata**:

- **inputs** / sensors:
  - $H : \{0, 1\}$ – heating request
  - $F : \{0, 1\}$ – flame sensor

  implementables constraining phases of H, F express **environment assumptions**;
  H, F in controller implementables correspond to **reading sensor values**,

- **outputs** / actuators:
  - $G : \{0, 1\}$ – gas valve

  implementables constraining phases of G
  describe the connection between **controller states and actuators**.

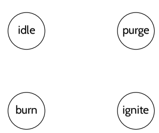
- **local state** / controller:
  - $C : \{\text{idle}, \text{purge}, \text{ignite}, \text{burn}\}$,

  to produce the desired behaviour, the controller makes use of four **local states**.

Gas Burner Controller: Control State Changes

C : {idle, purge, ignite, burn}

$\lceil\rceil \vee \lceil \text{idle} \rceil \, ; \, true$ (Init-1)
$\lceil \text{idle} \rceil \longrightarrow \lceil \text{idle} \vee \text{purge} \rceil$ (Seq-1)
$\lceil \text{purge} \rceil \longrightarrow \lceil \text{purge} \vee \text{ignite} \rceil$ (Seq-2)
$\lceil \text{ignite} \rceil \longrightarrow \lceil \text{ignite} \vee \text{burn} \rceil$ (Seq-3)
$\lceil \text{burn} \rceil \longrightarrow \lceil \text{burn} \vee \text{idle} \rceil$ (Seq-4)



Gas Burner Controller: Control State Changes

C : {idle, purge, ignite, burn}

$\lceil\rceil \vee \lceil \text{idle} \rceil \, ; \, true$ (Init-1)
$\lceil \text{idle} \rceil \longrightarrow \lceil \text{idle} \vee \text{purge} \rceil$ (Seq-1)
$\lceil \text{purge} \rceil \longrightarrow \lceil \text{purge} \vee \text{ignite} \rceil$ (Seq-2)
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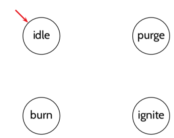

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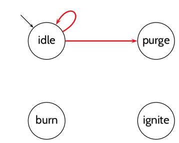

Gas Burner Controller: Control State Changes

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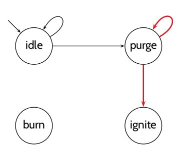

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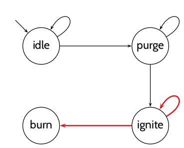

Gas Burner Controller: Control State Changes

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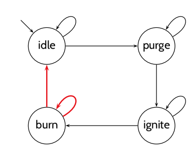

Gas Burner Controller: Control State Changes

C : {idle, purge, ignite, burn}

$\lceil\rceil \vee \lceil \text{idle} \rceil ; true$	(Init-1)
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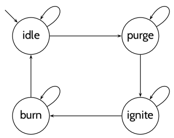
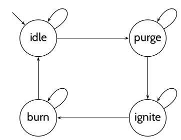


Gas Burner Controller: Timing Constraints

$\lceil \neg\text{purge} \rceil ; \lceil \text{purge} \rceil \xrightarrow{\leq 30} \lceil \text{purge} \rceil$	(Stab-2)
$\lceil \text{purge} \rceil \xrightarrow{30+\varepsilon} \lceil \neg\text{purge} \rceil$	(Prog-1)

"after changing to 'purge', **stay there for at least** 30 time units (or: leave after 30 the earliest);
you may **stay** in 'purge' **for at most** $30+\varepsilon$ time units"



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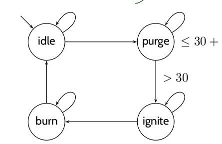

Gas Burner Controller: Timing Constraints

$\lceil \neg\text{purge} \rceil ; \lceil \text{purge} \rceil \xrightarrow{\leq 30} \lceil \text{purge} \rceil$	(Stab-2)
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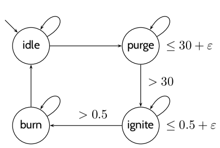
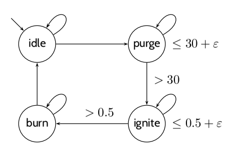
"after changing to 'purge', **stay there for at least** 30 time units (or: leave after 30 the earliest);
you may **stay** in 'purge' **for at most** $30+\varepsilon$ time units"

$\lceil \neg\text{ignite} \rceil ; \lceil \text{ignite} \rceil \xrightarrow{\leq 0.5} \lceil \text{ignite} \rceil$	(Stab-3)
$\lceil \text{ignite} \rceil \xrightarrow{0.5+\varepsilon} \lceil \neg\text{ignite} \rceil$	(Prog-2)



Gas Burner Controller: Inputs

$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg\text{idle} \rceil$	(Syn-1)
$\lceil \text{burn} \wedge (\neg H \vee \neg F) \rceil \xrightarrow{\varepsilon} \lceil \neg\text{burn} \rceil$	(Syn-2)
$\lceil \neg\text{idle} \rceil ; \lceil \text{idle} \wedge \neg H \rceil \longrightarrow \lceil \text{idle} \rceil$	(Stab-1)
$\lceil \text{idle} \wedge \neg H \rceil \longrightarrow_0 \lceil \text{idle} \rceil$	(Stab-1-init)
$\lceil \neg\text{burn} \rceil ; \lceil \text{burn} \wedge H \wedge F \rceil \longrightarrow \lceil \text{burn} \rceil$	(Stab-4)



Gas Burner Controller: Inputs

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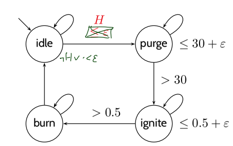

Gas Burner Controller: Inputs

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$\lceil \neg\text{idle} \rceil \,;\, \lceil \text{idle} \wedge \neg H \rceil \longrightarrow \lceil \text{idle} \rceil$	(Stab-1)
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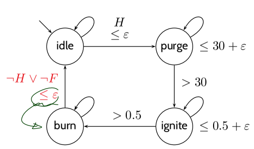

Gas Burner Controller: Inputs

$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg\text{idle} \rceil$	(Syn-1)
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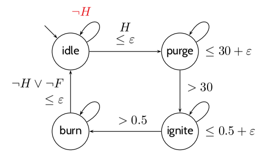

Gas Burner Controller: Inputs

$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg\text{idle} \rceil$	(Syn-1)
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$\lceil \neg\text{idle} \rceil \,;\, \lceil \text{idle} \wedge \neg H \rceil \longrightarrow \lceil \text{idle} \rceil$	(Stab-1)
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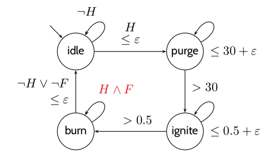
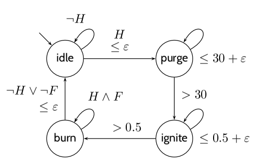
$\lceil \neg\text{burn} \rceil \,;\, \lceil \text{burn} \wedge H \wedge F \rceil \longrightarrow \lceil \text{burn} \rceil$	(Stab-4)



Gas Burner Controller: Inputs

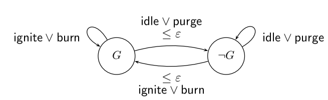
$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg\text{idle} \rceil$	(Syn-1)
$\lceil \text{burn} \wedge (\neg H \vee \neg F) \rceil \xrightarrow{\varepsilon} \lceil \neg\text{burn} \rceil$	(Syn-2)
$\lceil \neg\text{idle} \rceil \,;\, \lceil \text{idle} \wedge \neg H \rceil \longrightarrow \lceil \text{idle} \rceil$	(Stab-1)
$\lceil \text{idle} \wedge \neg H \rceil \longrightarrow_0 \lceil \text{idle} \rceil$	(Stab-1-init)
$\lceil \neg\text{burn} \rceil \,;\, \lceil \text{burn} \wedge H \wedge F \rceil \longrightarrow \lceil \text{burn} \rceil$	(Stab-4)



Gas Burner Controller: Outputs

$G : \{0, 1\}$

$\lceil G \wedge (\text{idle} \vee \text{purge}) \rceil \xrightarrow{\varepsilon} \lceil \neg G \rceil$	(Syn-3)
$\lceil \neg G \wedge (\text{ignite} \vee \text{burn}) \rceil \xrightarrow{\varepsilon} \lceil G \rceil$	(Syn-4)
$\lceil G \rceil \,;\, \lceil \neg G \wedge (\text{idle} \vee \text{purge}) \rceil \longrightarrow \lceil \neg G \rceil$	(Stab-6)
$\lceil \neg G \wedge (\text{idle} \vee \text{purge}) \rceil \longrightarrow_0 \lceil \neg G \rceil$	(Stab-6-init)
$\lceil \neg G \rceil \,;\, \lceil G \wedge (\text{ignite} \vee \text{burn}) \rceil \longrightarrow \lceil G \rceil$	(Stab-7)



Gas Burner Controller: Environment Assumptions

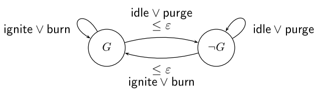
$G : \{0, 1\}$

$\lceil\rceil \vee \lceil \neg G \rceil \,;\, true$	(Init-4)

Gas Burner Controller: Environment Assumptions

$G : \{0, 1\}$

$\lceil\rceil \vee \lceil \neg G \rceil \,;\, true$ **(Init-4)**



Gas Burner Controller: Environment Assumptions

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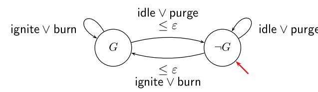

Gas Burner Controller: Environment Assumptions

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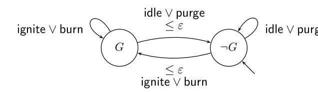


Gas Burner Controller: Environment Assumptions

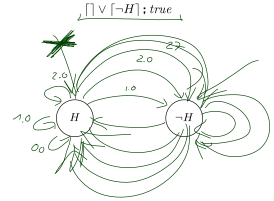
$H : \{0, 1\}$

$\lceil\rceil \vee \lceil \neg H \rceil \,;\, true$ **(Init-2)**

Gas Burner Controller: Environment Assumptions

$H : \{0, 1\}$

$\lceil\rceil \vee \lceil \neg H \rceil \,;\, true$ **(Init-2)**



Gas Burner Controller: Environment Assumptions

$H : \{0, 1\}$

$\lceil\rceil \vee \lceil \neg H \rceil \,;\, true$ **(Init-2)**

Gas Burner Controller: Environment Assumptions

$F : \{0,1\}$

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$ **(Init-3)**

$\lceil F \rceil$; $\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow \lceil \neg F \rceil$ **(Stab-5)**

$\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow_0 \lceil \neg F \rceil$ **(Stab-5-init)**

Gas Burner Controller: Environment Assumptions

$F : \{0,1\}$

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$ **(Init-3)**

$\lceil F \rceil$; $\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow \lceil \neg F \rceil$ **(Stab-5)**

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Gas Burner Controller: Environment Assumptions

$F : \{0,1\}$

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$ **(Init-3)**

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Gas Burner Controller: Environment Assumptions

$F : \{0,1\}$

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$ **(Init-3)**

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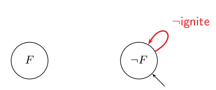

Gas Burner Controller: Environment Assumptions

$F : \{0,1\}$

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$ **(Init-3)**

$\lceil F \rceil$; $\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow \lceil \neg F \rceil$ **(Stab-5)**

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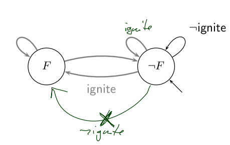

Gas Burner Controller: The Complete Specification

Controller: (local)

$\lceil\,\rceil \vee \lceil \text{idle} \rceil$; $true$, **(Init-1)**

$\lceil \text{idle} \rceil \longrightarrow \lceil \text{idle} \vee \text{purge} \rceil$ **(Seq-1)**

$\lceil \text{purge} \rceil \longrightarrow \lceil \text{purge} \vee \text{ignite} \rceil$ **(Seq-2)**

$\lceil \text{ignite} \rceil \longrightarrow \lceil \text{ignite} \vee \text{burn} \rceil$ **(Seq-3)**

$\lceil \text{burn} \rceil \longrightarrow \lceil \text{burn} \vee \text{idle} \rceil$ **(Seq-4)**

$\lceil \text{purge} \rceil \xrightarrow{30+\varepsilon} \lceil \neg \text{purge} \rceil$ **(Prog-1)**

$\lceil \text{ignite} \rceil \xrightarrow{0.5+\varepsilon} \lceil \neg \text{ignite} \rceil$ **(Prog-2)**

$\lceil \neg \text{purge} \rceil$; $\lceil \text{purge} \rceil \xrightarrow{\leq 30} \lceil \text{purge} \rceil$ **(Stab-2)**

$\lceil \neg \text{ignite} \rceil$; $\lceil \text{ignite} \rceil \xrightarrow{\leq 0.5} \lceil \text{ignite} \rceil$ **(Stab-3)**

$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg \text{idle} \rceil$ **(Syn-1)**

$\lceil \text{burn} \wedge (\neg H \vee \neg F) \rceil \xrightarrow{\varepsilon} \lceil \neg \text{burn} \rceil$ **(Syn-2)**

$\lceil \neg \text{idle} \rceil$; $\lceil \text{idle} \wedge \neg H \rceil \longrightarrow \lceil \text{idle} \rceil$ **(Stab-1)**

$\lceil \text{idle} \wedge \neg H \rceil \longrightarrow_0 \lceil \text{idle} \rceil$ **(Stab-1-init)**

$\lceil \neg \text{burn} \rceil$; $\lceil \text{burn} \wedge H \wedge F \rceil \longrightarrow \lceil \text{burn} \rceil$ **(Stab-4)**

Gas Valve: (output)

$\lceil\,\rceil \vee \lceil \neg G \rceil$; $true$ **(Init-4)**

$\lceil G \wedge (\text{idle} \vee \text{purge}) \rceil \xrightarrow{\varepsilon} \lceil \neg G \rceil$ **(Syn-3)**

$\lceil \neg G \wedge (\text{ignite} \vee \text{burn}) \rceil \xrightarrow{\varepsilon} \lceil G \rceil$ **(Syn-4)**

$\lceil G \rceil$; $\lceil \neg G \wedge (\text{idle} \vee \text{purge}) \rceil \longrightarrow \lceil \neg G \rceil$ **(Stab-6)**

$\lceil \neg G \wedge (\text{idle} \vee \text{purge}) \rceil \longrightarrow_0 \lceil \neg G \rceil$ **(Stab-6-init)**

$\lceil \neg G \rceil$; $\lceil G \wedge (\text{ignite} \vee \text{burn}) \rceil \longrightarrow \lceil G \rceil$ **(Stab-7)**

Heating Request: (input)

$\lceil\,\rceil \vee \lceil \neg H \rceil$; $true$, **(Init-2)**

Flame: (input)

$\lceil\,\rceil \vee \lceil \neg F \rceil$; $true$, **(Init-3)**

$\lceil F \rceil$; $\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow \lceil \neg F \rceil$ **(Stab-5)**

$\lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow_0 \lceil \neg F \rceil$ **(Stab-5-init)**

Tell Them What You've Told Them...

- Controller hardware platforms can
  - **read inputs**, **change local state**,
  - **wait**, **write outputs**.
- If we limit **controller behaviour descriptions** to these "operations", there's (at least) no principle **obstacle** to **implement** the design.
- One such **limited specification language**:
  - **DC Implementables**,
  - a set of patterns of **DC Standard Forms**.
- **DC Implementables** basically constrain:
  - local state changes, synchronisation with inputs
  - and outputs, timed stability and progress
- This is sufficient to formalise a correct (safe) **Gas Burner** controller design specification.

References

References

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.