Parallel Composition

Definition 4.12. The parallel composition \( A_1 \parallel A_2 \) of two timed automata \( A_i = (L_i, B_i, X_i, I_i, E_i, \ell_{ini}) \), \( i = 1, 2 \), with disjoint sets of clocks \( X_1 \) and \( X_2 \) yields the timed automaton

\[
A = (L_1 \times L_2, B_1 \cup B_2, X_1 \cup X_2, I, E, (\ell_{ini}, 1, \ell_{ini}, 2))
\]

where

- \( I(\ell_1, \ell_2) := I(\ell_1) \land I(\ell_2) \), and
- \( E \) consists of handshake (or rendezvous) and asynchronous communication edges. (→ next slide)

Helper: Action Complementation

The complementation function \( \cdot : \text{Act} \rightarrow \text{Act} \) is defined pointwise as follows:

- \( a! = a? \)
- \( a? = a! \)
- \( \tau = \tau \)

Note: \( \alpha = \alpha \) for all \( \alpha \in \text{Act} \).

Parallel Composition: Handshake and Asynchrony

- **Handshake Edges**: If there is \( a \in B_1 \cup B_2 \) such that \((\ell_1, \alpha, \phi_1, Y_1, \ell'_1) \in E_1\), \((\ell_2, \bar{\alpha}, \phi_2, Y_2, \ell'_2) \in E_2\), and \(\{\alpha, \bar{\alpha}\} = \{a!, a?\}\), then \((\ell_1, \ell_2), \tau, \phi_1 \land \phi_2, Y_1 \cup Y_2, (\ell'_1, \ell'_2)) \in E.\)

- **Asynchronous Edges**: If \((\ell_1, \alpha, \phi_1, Y_1, \ell'_1) \in E_1\) then for all \(\ell_2 \in L_2\), \((\ell_1, \ell_2), \alpha, \phi_1, Y_1, (\ell'_1, \ell_2)) \in E.\)

If \((\ell_2, \alpha, \phi_2, Y_2, \ell'_2) \in E_2\) then for all \(\ell_1 \in L_1\), \((\ell_1, \ell_2), \alpha, \phi_2, Y_2, (\ell_1, \ell'_2)) \in E.\)
• Parallel Composition of TA
• handshake edges
• asynchronous edges
• Restriction / Channel Hiding
• Networks of Timed Automata
• closed networks
• Operational Semantics of Networks of Timed Automata
• a semantical approach
• The Uppaal tool
• Demo I: Model Editor
  • Simulator

Operational Semantics of Networks of TA: The Plan

\[ (N^\parallel)^T = T(N^\parallel) \]

Def. 4.12 + 4.13
Def. 4.4
Lemma 4.16
Operational Semantics of Networks

(iii) $\langle \vec{\ell}, \nu \rangle^t \rightarrow \langle \vec{\ell}, \nu \rangle^n$.
The parallel composition of two timed automata is again a timed automaton.

IOW: the set of timed automata is closed under parallel composition.

Channel restriction introduces local channels.

Hiding all channels yields a closed network.

Uppaal always interprets a network as closed.

Behaviour of a network can alternatively be characterised semantically.

The Uppaal tool is one way to model and analyse (networks of) timed automata.

(And to verify → next lecture(s).)

References

