Real-Time Systems

Lecture 4: Duration Calculus II

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Content

Duration Calculus: Formulae

Duration Calculus: Overview

We will introduce **four syntactical categories** (and **abbreviations**):

(i) **Symbols:**

$$\overbrace{true, false, =, <, >, \leq, \geq}^{p,q}, \quad f, g, \quad X, Y, Z, \quad d, \quad x, y, z,$$

(ii) **State Assertions:**

$$P ::= 0 \mid 1 \mid X = d \mid \neg P_1 \mid P_1 \wedge P_2$$

(iii) **Terms:**

$$\theta ::= x \mid \ell \mid \int P \mid f(\theta_1, \dots, \theta_n)$$

(iv) **Formulae:**

chop operator

$$F ::= p(\theta_1, \dots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

(v) **Abbreviations:**

$$\lceil\rceil, \quad \lceil P \rceil, \quad \lceil P \rceil^t, \quad \lceil P \rceil^{\leq t}, \quad \Diamond F, \quad \Box F$$

Formulae: Syntax

- The set of **DC formulae** is defined by the following grammar:

$$F ::= p(\theta_1, \dots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

where p is a predicate symbol, θ_i are terms, and x is a global variable.

- **chop operator**: ';'
- **atomic formula**: $p(\theta_1, \dots, \theta_n)$
- **rigid formula**: all terms are rigid (no ℓ, no $\int P$)
- **chop free**: ';' doesn't occur
- usual notion of **free** and **bound** (global) variables

$\forall x \bullet \int P > x \wedge y < 10$

bound

free

- Note: quantification only over (**first-order**) global variables, not over (**second-order**) state variables.

↳ $\exists X \bullet \int X > 3$ NOT

Formulae: Priority Groups

- To avoid parentheses, we define the following five **priority groups** from highest to lowest priority (or precedence):
  - $\neg$ (negation)
  - $;$ (chop)
  - $\wedge, \vee$ (and/or)
  - $\implies, \iff$ (implication/equivalence)
  - $\exists, \forall$ (quantifiers)

Examples:

- $\neg F \,;\, F \vee G$

$(\neg(F\,;F)) \vee G$ —

$((\neg F)\,;F) \vee G$ ✓

$(\neg F)\,;(F \vee G)$

- $\forall x \bullet \quad F \quad \wedge \quad G$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

- We use
$$F[x := \theta]$$
to denote the formula that results from performing the following steps:
  (i) transform F into $\tilde{F}$ by (consistently) **renaming bound variables** such that **no free occurrence** of x in $\tilde{F}$ appears within a **quantified subformula** $\exists\, z \bullet G$ or $\forall\, z \bullet G$ for some z **occurring in term** θ,
  (ii) textually replace all free occurrences of x in $\tilde{F}$ by θ.

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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$

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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$
- $\theta_2 := \ell + z, \quad F \qquad = (x \quad \geq y \implies \exists\, z \bullet z \geq 0 \wedge x \quad = y + z)$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

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suddenly bound

- $\theta_2 := \ell + z, \quad F[x := \theta_2] = (\ell + z \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell + z = y + z)$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

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$$F[x := \theta]$$
to denote the formula that results from performing the following steps:
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Example:

- $\theta_1 := \ell, \quad F[x := \theta_1] = (\ell \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell = y + z)$ ✓
- $\theta_2 := \ell + z, \quad F[x := \theta_2] = (\ell + z \geq y \implies \exists\, z \bullet z \geq 0 \wedge \ell + z = y + z)$ ✗
- $F[x := \theta_2] = \ell + z \geq y \implies \exists\, \tilde{z} \bullet \tilde{z} \geq 0 \wedge \ell + z = y + \tilde{z})$ ✓

Formulae: Semantics

- The **semantics** of a **formula** is a function
$$\mathcal{I}[\![F]\!] : \mathsf{Val} \times \mathsf{Intv} \to \{\mathsf{tt}, \mathsf{ff}\}$$
$\mathcal{I}[\![F]\!](\mathcal{V}, [b,e])$: truth value of F under interpretation $\mathcal{I}$ and valuation $\mathcal{V}$ in the interval $[b,e]$.

- $\mathcal{I}[\![F]\!](\mathcal{V}, [b,e])$ is defined **inductively** over the structure of F:

$$\mathcal{I}[\![p(\theta_1, \dots, \theta_n)]\!](\mathcal{V}, [b,e]) = \hat{p}(\,\mathcal{I}[\![\theta_1]\!](\mathcal{V}, [b,e]),\ \dots,\ \mathcal{I}[\![\theta_n]\!](\mathcal{V}, [b,e])\,),$$

base step

$$\mathcal{I}[\![\neg F_1]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b,e]) = \mathsf{ff},$$

$$\mathcal{I}[\![F_1 \wedge F_2]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_i]\!](\mathcal{V}, [b,e]) = \mathsf{tt}, i \in \{1,2\},$$

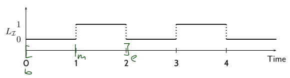
function symbol

$$\mathcal{I}[\![\forall\, x \bullet F_1]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff for all } a \in \mathbb{R},\ \mathcal{I}[\![F_1[x := a]]\!](\mathcal{V}, [b,e]) = \mathsf{tt}$$

$$\mathcal{I}[\![F_1 \,;\, F_2]\!](\mathcal{V}, [b,e]) = \text{ iff there is an } m \in [b,e] \text{ such that } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b,m]) = \mathsf{tt} \text{ and } \mathcal{I}[\![F_2]\!](\mathcal{V}, [m,e]) = \mathsf{tt}.$$

Formulae: Example

$F := \int L = 0 \,;\, \int L = 1$



- $\mathcal{I}[\![F]\!](\mathcal{V},[0,2]) = \mathrm{tt}$

  Proof:

  - Choose $m = 1$ as **chop point**.

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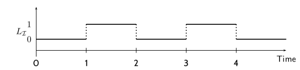

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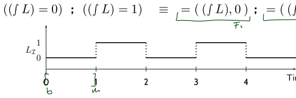

- $\mathcal{I}[\![F]\!](\mathcal{V},[0,2]) = \mathrm{tt}$

  Proof:

  - Choose $m = 1$ as **chop point**. Then
  - $\mathcal{I}[\![= ((\int L), 0)]\!](\mathcal{V},[0,1]) = \hat{=}(\,\mathcal{I}[\![\int L]\!](\mathcal{V},[0,1]),\, \mathcal{I}[\![0]\!](\mathcal{V},[0,1])\,)$
    $= \hat{=}\left(\int_0^1 L_\mathcal{I}(t)\,dt, \quad \hat{0}\right) = \hat{=}(0,0) = \mathrm{tt},$

Formulae: Example

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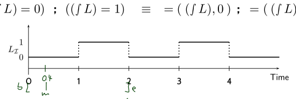

- $\mathcal{I}[\![F]\!](\mathcal{V},[0,2]) = \mathrm{tt}$ $\quad \int_{0.4}^{2} L_\mathcal{I}(t)\,dt = 1$

  Proof:

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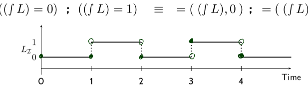

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- Is the **chop point** m **unique**?

Formulae: Example

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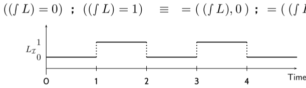

- $\mathcal{I}[\![F]\!](\mathcal{V},[0,2]) = \mathrm{tt}$

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  No, all $m \in [0,1]$ are proper chop points (and only those)

- Is the **chop point** m **unique**?

  $\mathcal{I}[\![\int \neg L < 1 \,;\, \int L < 1]\!](\mathcal{V},[0,?]) = \mathrm{ff}$

- Would the **chop point** for formula $\int \neg L = 1 \,;\, \int L = 1$ be **unique**?

Formulae: Remarks

$x+y>3$; $x+y>5$

- **rigid formula**: all terms are rigid
- **rigid term**: no length or integral operators
- **chop free**: ';' doesn't occur

Remark 2.10. [*Rigid and chop-free*] Let F be a duration formula, $\mathcal{I}$ an interpretation, $\mathcal{V}$ a valuation, and $[b,e] \in$ Intv.

- If F is **rigid**, then

$$\forall\, [b',e'] \in \mathsf{Intv} : \mathcal{I}[\![F]\!](\mathcal{V}, [b,e]) = \mathcal{I}[\![F]\!](\mathcal{V}, [b',e']).$$

- If F is **chop-free** or θ is **rigid**,
  then in the calculation of the semantics of F,
  every occurrence of θ denotes the same value.

Substitution Lemma

Lemma 2.11. [*Substitution*]
Consider a formula F, a global variable x, and a term θ such that F is **chop-free** or θ is **rigid**.
Then for all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and intervals $[b,e]$,

$$\mathcal{I}[\![F[x := \theta]]\!](\mathcal{V}, [b,e]) = \mathcal{I}[\![F]\!](\mathcal{V}[x := a], [b,e])$$

where $a = \mathcal{I}[\![\theta]\!](\mathcal{V}, [b,e])$.

- **Negative Example**: $F := (\ell = x) ; (\ell = x) \implies (\ell = 2 \cdot x)$, $\theta := \ell$
  - $\mathcal{I}[\![F[x := \ell]]\!](\mathcal{V}, [b,e]) = \mathcal{I}[\![(\ell = \ell);(\ell = \ell) \Rightarrow (\ell = 2 \cdot \ell)]\!](\mathcal{V}, [b,e])$
    $\hookrightarrow$ yields ff for $b<e$
  - $\mathcal{I}[\![F]\!](\mathcal{V}[x := a], [b,e]) = tt$ (even valid)

Duration Calculus: Overview

We will introduce **four syntactical categories** (and **abbreviations**):

(i) **Symbols:**

$$\overbrace{true, false, =, <, >, \leq, \geq}^{p,q}, \quad f, g, \quad X, Y, Z, \quad d, \quad x, y, z,$$

(ii) **State Assertions:**

$$P ::= 0 \mid 1 \mid X = d \mid \neg P_1 \mid P_1 \wedge P_2$$

(iii) **Terms:**

$$\theta ::= x \mid \ell \mid \textstyle\int P \mid f(\theta_1, \ldots, \theta_n)$$

(iv) **Formulae:**

$$F ::= p(\theta_1, \ldots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall x \bullet F_1 \mid F_1 \,;\, F_2$$

(v) **Abbreviations:**

$$\lceil\,\rceil, \quad \lceil P \rceil, \quad \lceil P \rceil^t, \quad \lceil P \rceil^{\leq t}, \quad \Diamond F, \quad \Box F$$

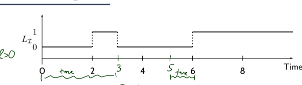
Duration Calculus Abbreviations

Abbreviations

- $\lceil\,\rceil := \ell = 0$ **(point interval)**
- $\lceil P \rceil := (\int P = \ell) \wedge (\ell > 0)$ **(almost everywhere)** — state assertion
- $\lceil P \rceil^t := \lceil P \rceil \wedge \ell = t$ **(for time t)**
- $\lceil P \rceil^{\leq t} := \lceil P \rceil \wedge \ell \leq t$ **(up to time t)**
- $\Diamond F := true \,;\, F \,;\, true$ **(for some subinterval)** — diamond
- $\Box F := \neg\Diamond\neg F$ **(for all subintervals)** — box

- $\Diamond \lceil P \rceil$ not satisfied on any point interval

Abbreviations: Examples

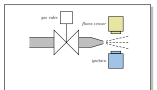


$(\int(\neg L)) = \ell \wedge \ell > 0$

true ; $\lceil L \rceil$; true

$\mathcal{I}[\![\ (\int L) = 0$	$]\!](\mathcal{V}, [0,2])$	$= tt$	
$\mathcal{I}[\![\ \int L = 1$	$]\!](\mathcal{V}, [2,6])$	$= tt$	
$\mathcal{I}[\![\ \int L = 0 \,;\, \int L = 1$	$]\!](\mathcal{V}, [0,6])$	$= tt$	, $m = 2$
$\mathcal{I}[\![\ \lceil \neg L \rceil$	$]\!](\mathcal{V}, [0,2])$	$= tt$	
$\mathcal{I}[\![\ \lceil L \rceil$	$]\!](\mathcal{V}, [2,3])$	$= tt$	
$\mathcal{I}[\![\ \lceil \neg L \rceil \,;\, \lceil L \rceil$	$]\!](\mathcal{V}, [0,3])$	$= tt$	, $m = 2$
$\mathcal{I}[\![\ \lceil \neg L \rceil \,;\, \lceil L \rceil \,;\, \lceil \neg L \rceil$	$]\!](\mathcal{V}, [0,6])$	$= tt$	, $m_1 = 2$, $m_2 = 3$
$\mathcal{I}[\![\ \Diamond \lceil L \rceil$	$]\!](\mathcal{V}, [0,6])$	$= tt$	$m_1 = 2$ $m_2 = 3$
$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil$	$]\!](\mathcal{V}, [0,6])$	$=$	
$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil^2$	$]\!](\mathcal{V}, [0,6])$	$= tt$	$m_1 = 3$ or 0, $m_2 = 5$ or 2
$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil^2 \,;\, \lceil L \rceil^1 \,;\, \lceil \neg L \rceil^3$	$]\!](\mathcal{V}, [0,6])$	$= tt$	$m_1 = 2$ $m_2 = 3$

Duration Calculus: Preview



- $G, F, I, H : \{0, 1\}$
- Define $L : \{0, 1\}$ as $G \wedge \neg F$.

- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an **(implicitly given)** interval.

Strangest operators:

- **almost everywhere** – Example: $\lceil G \rceil$
  (Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere.)
- **chop** – Example: $(\lceil \neg I \rceil \,;\, \lceil I \rceil \,;\, \lceil \neg I \rceil) \implies \ell \geq 1$
  (Ignition phases last at least one time unit.)
- **integral** – Example: $\ell \geq 60 \implies \int L \leq \frac{\ell}{20}$
  (At most 5% leakage time within intervals of at least 60 time units.)

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Content

- **Formulae**
  - syntax, priority groups
  - syntactic substitution
  - semantics
  - well-definedness
  - remarks, substitution lemma
- **DC Abbreviations**
  - point interval, almost everywhere
  - for some subinterval / for all subintervals
- **Validity, Satisfiability, Realisability**
  - realisability / validity from 0
- Proving design ideas correct: **Method**
  - Example: gas burner

DC Validity, Satisfiability, Realisability

Validity, Satisfiability, Realisability

Let $\mathcal{I}$ be an interpretation, $\mathcal{V}$ a valuation, $[b, e]$ an interval, and F a DC formula.

- $\mathcal{I}, \mathcal{V}, [b, e] \models F$ (read: F **holds** in $\mathcal{I}, \mathcal{V}, [b, e]$) iff $\mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \mathrm{tt}$.
- F is called **satisfiable** iff it **holds** in some $\mathcal{I}, \mathcal{V}, [b, e]$.

- $\mathcal{I}, \mathcal{V} \models F$ (read: $\mathcal{I}$ and $\mathcal{V}$ **realise** F) iff $\forall [b, e] \in \mathsf{Intv} : \mathcal{I}, \mathcal{V}, [b, e] \models F$.
- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ **realise** F.

- $\mathcal{I} \models F$ (read: $\mathcal{I}$ **realises** F) iff $\forall \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models F$.

- $\models F$ (read: F is **valid**) iff $\forall \mathcal{I} : \mathcal{I} \models F$.

Validity vs. Satisfiability vs. Realisability

Remark 2.13. For all DC formulae F,

- F is satisfiable if and only if $\neg F$ is not valid,
  F is valid if and only if $\neg F$ is not satisfiable.
- If F is valid then F is realisable, but not vice versa.
- If F is realisable then F is satisfiable, but not vice versa.

Examples: Valid? Realisable? Satisfiable?

- $\ell \geq 0$
- $\ell = \int 1$
- $\ell = 30 \iff \ell = 10 \,;\, \ell = 20$
- $((F \,;\, G) \,;\, H) \iff (F \,;\, (G \,;\, H))$
- $\int L \leq x$
- $\ell = 2$

Initial Values

- $\mathcal{I}, \mathcal{V} \models_0 F$ (read: $\mathcal{I}$ and $\mathcal{V}$ **realise F from 0**) iff

$$\forall\, t \in \mathsf{Time} : \mathcal{I}, \mathcal{V}, [0,t] \models F.$$

- F is called **realisable from 0** iff some $\mathcal{I}$ and $\mathcal{V}$ **realise** F from 0.
- **Intervals** of the form $[0,t]$ are called **initial intervals**.
- $\mathcal{I} \models_0 F$ (read: $\mathcal{I}$ **realises F from 0**) iff $\forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models_0 F$.
- $\models_0 F$ (read: F is **valid from 0**) iff $\forall\, \mathcal{I} : \mathcal{I} \models_0 F$.

Initial or not Initial...

Remark. For all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and DC formulae F,

(i) $\mathcal{I}, \mathcal{V} \models F$ implies $\mathcal{I}, \mathcal{V} \models_0 F$,

(ii) if F is realisable then F is realisable from 0, but not vice versa,

(iii) F is valid iff F is valid from 0.

Content

Specification and Semantics-based Correctness Proofs of Real-Time Systems with DC

Methodology *(in an ideal world)*

In order to **prove** a controller design **correct** wrt. a **specification**:

(i) Choose **observables** 'Obs'.

(ii) Formalise the **requirements** 'Spec' as a conjunction of DC formulae (over 'Obs').

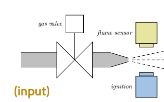
(iii) Formalise a **controller design** 'Ctrl' as a conjunction of DC formulae (over 'Obs').

(iv) We say 'Ctrl' is **correct** (wrt. 'Spec') iff

$$\models_0 \mathsf{Ctrl} \implies \mathsf{Spec},$$

so "just" prove $\models_0 \mathsf{Ctrl} \implies \mathsf{Spec}$.

Gas Burner Revisited



(i) Choose **observables**:
- $F : \{0,1\}$: value 1 models "flame sensed now" **(input)**
- $G : \{0,1\}$: value 1 models "gas valve is open now" **(output)**
- define $L := G \wedge \neg F$ to model **leakage**

(ii) Formalise the **requirement**:

$$\mathsf{Req} := \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

"in each interval of length at least 60 time units, at most 5% of the time leakage"

(iii) Formalise **controller design ideas**:
- Des-1 $:= \Box(\lceil L \rceil \implies \ell \leq 1)$
  "leakage phases last for at most one time unit"
- Des-2 $:= \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$
  "non-leakage phases between two leakage-phases last at least 30 time units"

(iv) Prove **correctness**, i.e. prove $\models$ (Des-1 $\wedge$ Des-2 $\implies$ Req).
(Or do we want "$\models_0$"...?)

Content

- **Formulae**
  - **syntax, priority groups**
  - **syntactic substitution**
  - **semantics**
  - **well-definedness**
  - **remarks, substitution lemma**
- **DC Abbreviations**
  - **point interval, almost everywhere**
  - **for some subinterval** / **for all subintervals**
- **Validity, Satisfiability, Realisability**
  - **realisability** / **validity** from 0
- Proving design ideas correct: **Method**
  - Example: **gas burner**

Tell Them What You've Told Them. . .

- **Duration Calculus Formulae**
  - using, e.g., the **chop operator**

  are **evaluated** for **intervals** and **valuations**.

  The **semantics** of a **DC formula** is a **truth value**.
- The following **abbreviations** are sometimes useful
  - **point interval** ($\lceil\rceil$), **almost everywhere** ($\lceil P \rceil$),
  - **for some subinterval** ($\Diamond F$), **for all subintervals** ($\Box F$)
- **DC Formulae** have notions of
  - **satisfiability** and **validity** (as usual),
  - **realisability** ("for all subintervals")
  - also: from 0
- Outlook on next lecture:
  proving design ideas correct wrt. requirements.

References

References

Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.

EXAM

– oral / written

– DATE
(mid / late March)

↳ Tue fix on