The goal of this sheet is to become familiar with the notion of a linear-time property as such (and, for the most part, not in relation to an already given transition system, as in the previous exercise sheet).

**Exercise 1: Linear-Time Properties**

Assume $AP = \{a, b\}$. For each of the following properties $P$,

(a) formalize $P$ as a set of traces using set comprehension
(for example: ”always $a$“ can be formalized as $\{A_1A_2A_3\cdots | \forall i. a \in A_i\}$),

(b) formalize $P$ as a set of traces using $\omega$-regular expressions
(for example, $(\{a\} + \{a, b\})^\omega$),

(c) give an example of a trace that satisfies $P$,

(d) give an example of a trace that does not satisfy $P$,

(e) give all states of the transition system below that satisfy $P$, and

(f) state whether or not the transition system below satisfies $P$.

\[
\begin{array}{c}
\{a\} \\
\text{s}_1 \\
\{a\} \\
\text{s}_3 \\
\{a, b\} \\
\text{s}_4 \\
\{a\} \\
\text{s}_2 \\
\end{array}
\]

$(P_1)$ Always (at any point of time) $a$ or $b$ holds.

$(P_2)$ Always (at any point of time) $a$ and $b$ holds.

$(P_3)$ Never $b$ holds before $a$ holds.

$(P_4)$ Every time $a$ holds there will be eventually a point of time where $b$ holds.

$(P_5)$ At exactly three points of time, $a$ holds.

$(P_6)$ If there are infinitely many points of time where $a$ holds, then there are infinitely many points of time where $b$ holds.

$(P_7)$ There are only finitely many points of time where $a$ holds.