The goal of this exercise sheet is to understand transition systems, and how they can be used to model cyber-physical systems. The exercises are meant to train you in translating between different formalisms, both graphical and mathematical.

Exercise 1: Hardware Circuit and Transition System 4 Points
The goal of this exercise is to go from a pictorial representation of a hardware system to a formal model.

Consider the following sequential hardware circuit.

Draw the transition system of the hardware circuit. That is, the states are the valuations of the input $x$ and the register $r$. The transitions represent the stepwise behavior where the value of the input bit $x$ may or may not change in each step.

You may assume that initially the register $r$ has the value false.

For your reference: $\bigcirc$ = AND gate, $\bigtriangledown$ = OR gate, $\overline{\bigcirc}$ = NOT gate

Exercise 2: Transition Systems 6 Points
The goal of this exercise is to understand the connection between mathematical notation and graphical representation of transition systems.

(a) Let $T = (S, Act, \rightarrow, S_0, AP, L)$ be a transition system with

- the set of states $S = \{\text{locked}, \text{checking}, \text{opened}\}$,
- the set of action $Act = \{\text{insert ticket}, \text{unlock}, \text{enter}, \text{error}\}$,
- the transition relation
  \[ \rightarrow = \{(\text{locked}, \text{insert ticket}, \text{checking}), (\text{checking}, \text{unlock}, \text{opened}), (\text{opened}, \text{enter}, \text{locked}), (\text{checking}, \text{error}, \text{locked})\}, \]
- the initial states $S_0 = \{\text{locked}\}$,
• the set of atomic propositions \( AP = \{ \text{light\_red}, \text{light\_green} \} \),
• and the labeling function \( L \) with \( L(\text{locked}) = \{ \text{light\_red} \} \), \( L(\text{checking}) = \emptyset \) and \( L(\text{opened}) = \{ \text{light\_green} \} \).

Draw this transition system. Can you see what cyber-physical system it models?

(b) The transition system shown below models an elevator. Give the corresponding mathematical definition, i.e., define the tuple \( T = (S, \text{Act}, \to, S_0, AP, L) \) that is described by the picture, in the style of (a).

In which states is the elevator door closed?