ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Dr. Stefan Schwoon
Sergiy Bogomolov

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Tutorials for Model Checking
Exercise sheet 2

Exercise 1: Closure of Büchi-Automata

A DBA is total if for each state s and each input symbol a there is a transition of the form (s, a, s'). Given two total DBAs $\mathcal{A}_1$ and $\mathcal{A}_2$, construct a total DBA that recognizes $\mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$.

Exercise 2: Product automaton

Let $A^1 = (Q^1, \Sigma, \delta^1, q_0^1, F^1)$ and $A^2 = (Q^2, \Sigma, \delta^2, q_0^2, F^2)$ be Büchi automata. Consider the following versions of a "product":

$$A = (Q^1 \times Q^2, \Sigma, \delta, \langle q_0^1, q_0^2 \rangle, F)$$

where $\delta((q^1, q^2), a) = \delta^1(q^1, a) \times \delta^2(q^2, a)$ $(a \in \Sigma,\ q^1 \in Q^1,\ q^2 \in Q^2)$ and

(a) $F = F^1 \times F^2$

(b) $F = F^1 \times Q^2 \cup Q^1 \times F^2$

Provide examples where neither version recognizes the intersection $\mathcal{L}(A^1) \cap \mathcal{L}(A^2)$. Is there an example where no choice of $F \subseteq Q^1 \times Q^2$ recognizes the intersection?

Exercise 3: Bakery mutual exclusion

Consider the following Promela model implementing the so-called bakery mutual exclusion protocol.

```
byte turn_A; /* turn_A = 0 iff A does not want to enter CS */
byte turn_B; /* turn_A = 0 iff A does not want to enter CS */

proctype A() {
  think: skip;
  req:   turn_A = 1;
         turn_A = turn_B + 1;
  wait:  turn_B == 0 || turn_A < turn_B;
  crit:  skip; /* critical section */
  leave: turn_A = 0;
  goto think
}
```

```
proctype B() {
  think: skip;
  req:   turn_B = 1;
         turn_B = turn_A + 1;
  wait:  turn_A == 0 || turn_B < turn_A;
  crit:  skip; /* critical section */
  leave: turn_B = 0;
         goto think
}
```

```
init { atomic { run A(); run B(); } }
```

(a) Express in LTL the properties mutual exclusion (never both processes at label use) and response (if a process is at wait then it will proceed to use eventually).

(b) Verify the above properties using SPIN.

(c) Why does mutual exclusion not hold?

(d) Explain why mutual exclusion holds if the protocol does not run too long.