



Tutorials for Model Checking Exercise sheet 3

Exercise 1: LTL Formulae Inclusion

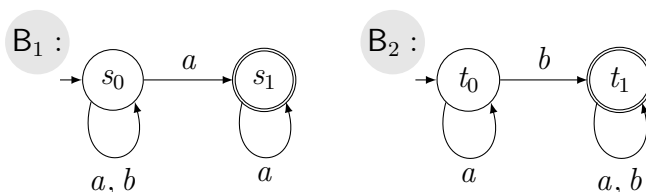
Given atomic propositions p and q , for each LTL formulae ϕ and ψ below, determine which of these inclusions hold:

- i $\mathcal{L}(\phi) \subseteq \mathcal{L}(\psi)$ ii $\mathcal{L}(\psi) \subseteq \mathcal{L}(\phi)$

Justify your answer. If an inclusion does not hold, produce a Kripke structure demonstrating it.

- (a) $\phi = \mathbf{GF} p$, $\psi = \mathbf{FG} p$
(b) $\phi = (\mathbf{G} p) \mathbf{U} q$, $\psi = (p \mathbf{U} q) \vee \mathbf{G} p$
(c) $\phi = p \mathbf{U} (\mathbf{F} q)$, $\psi = \mathbf{F} q$

Exercise 2: LTL and Büchi automata

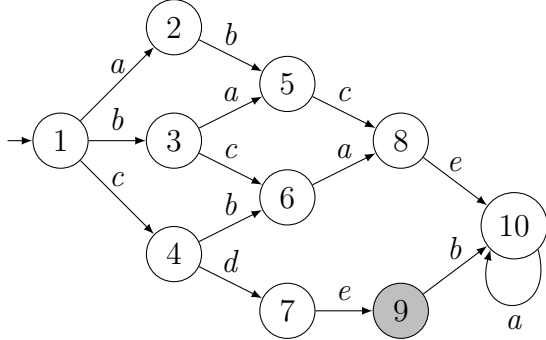


- Given the Büchi automata \mathbf{B}_1 and \mathbf{B}_2 over the alphabet $\Sigma = \{a, b\}$ as shown above, construct an automaton that accepts the intersection of $\mathcal{L}(\mathbf{B}_1)$ and $\mathcal{L}(\mathbf{B}_2)$.
- In the following LTL formulas, the atomic proposition *send* means that a message is sent, and *recv* means that a message is received.
 - (a) Which of the following LTL formulas expresses the following: “Only finitely many messages are sent.”
 - i. $\mathbf{GF} \neg \text{send}$
 - ii. $\mathbf{G} \neg \text{send}$
 - iii. $\mathbf{F} \text{send}$
 - iv. $\mathbf{FG} \neg \text{send}$
 - (b) Which of the following LTL formulas expresses the following: “Whenever a message is received, a message has been sent in the previous step.”

- (i) $G ((X \text{ send}) \rightarrow \text{recv})$
- (ii) $G ((\neg \text{send}) \rightarrow X \neg \text{recv})$
- (iii) $G (\text{send} \rightarrow X \text{recv})$

Exercise 3: Partial-order Reduction

Consider a Kripke structure $\mathcal{K} = (\{1, \dots, 10\}, \rightarrow, 1, AP, \nu)$, where $AP = \{p\}$, $\nu(p) = \{9\}$ and $\nu(s) = \emptyset$ for $s \neq 9$, and whose transitions, labelled with actions from the set $A = \{a, b, c, d, e\}$, are shown below.



- (a) Determine the maximal independence relation.
- (b) Indicate which actions from A are visible and which are not.
- (c) Which of the following sets can be chosen for $red(1)$?
 - i. $\{b\}$
 - ii. \emptyset
 - iii. $\{a, c\}$
 - iv. $\{b, c\}$

Check whether the given sets conform with conditions C0–C3. If a set violates one of the conditions, please explain why.

Exercise 4: Stuttering Invariant

Prove the statement: All LTL formulae without temporal operator X are stuttering-invariant.