Formal Methods for Java

Lecture 21: Verification of Data Structures in Jahob

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The Jahob system

Focus of Jahob: verifying properties of data structures.

Developed at

- EPFL, Lausanne, Switzerland (Viktor Kuncak)
- MIT, Cambridge, USA (Martin Rinard)
- Freiburg, Germany (Thomas Wies)

References

- Jahob webpage: `http://lara.epfl.ch/w/jahob_system`
- Viktor Kuncak's PhD thesis

Core syntax of HOL

Jahob's assertion language is a subset of the interactive theorem prover Isabelle/HOL which is built on the simply typed lambda calculus.

		Terms and Formulas:	
f	$::=$	$\lambda x :: t.\, f$	lambda abstraction (λ is also written %)
	$\mid$	$f_1\ f_2$	function application
	$\mid$	x	variable or constant
	$\mid$	$f :: t$	typed formula
		Types:	
t	$::=$	bool	truth values
	$\mid$	int	integers
	$\mid$	obj	uninterpreted objects
	$\mid$	$t_1 \Rightarrow t_2$	total functions
	$\mid$	t set	sets
	$\mid$	$t_1 * t_2$	*pairs*

Function with Several Arguments

A function with two arguments $g(x, y)$ has the type

$$g : (t_1 * t_2) \Rightarrow t_3$$

In HOL, usually one defines a function with two arguments as

$$f : t_1 \Rightarrow t_2 \Rightarrow t_3,$$

and the application as

$$f\ x\ y = g(x, y)$$

Note that $\Rightarrow$ is right-associative and function application is left-associative:

$$(t_1 \Rightarrow t_2 \Rightarrow t_3) = (t_1 \Rightarrow (t_2 \Rightarrow t_3)) \quad \text{and} \quad f\ x\ y = (f\ x)y.$$

Lambda Abstraction

Suppose, you want to define a function or relation:

$$inc\ x = x + 1 \quad \text{or} \quad succ\ x\ y \equiv (y = x + 1).$$

With lambda abstraction these can be written as

$$inc = (\lambda\ x.\ x + 1) \quad \text{resp.} \quad succ = (\lambda\ x\ y.\ y = x + 1).$$

This is especially useful if you need a function argument:

$$\text{rtrancl\_pt}\ succ\ 0\ z$$

can be written as

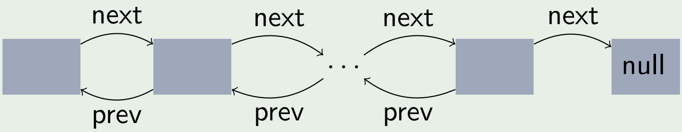
$$\text{rtrancl\_pt}\ (\lambda\ x\ y.\ y = x + 1)\ 0\ z$$

Data Structure Consistency

Statically verify data structure consistency properties.

Example

Internal Data Structure Consistency

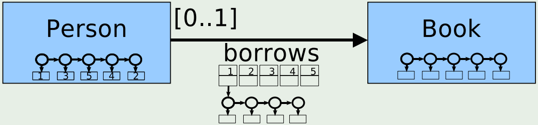


- field prev is inverse of field next
- field next is acyclic

➜ inconsistency can cause program crashes.

External Consistency Properties

Example (Library)



- if a book is loaned to a person, then
  - the person is registered with the library, and
  - the book is in the catalog
- Can loan a book to at most one person at a time

- correlate multiple data structures
- depend on internal consistency
- capture design constraints (object models)

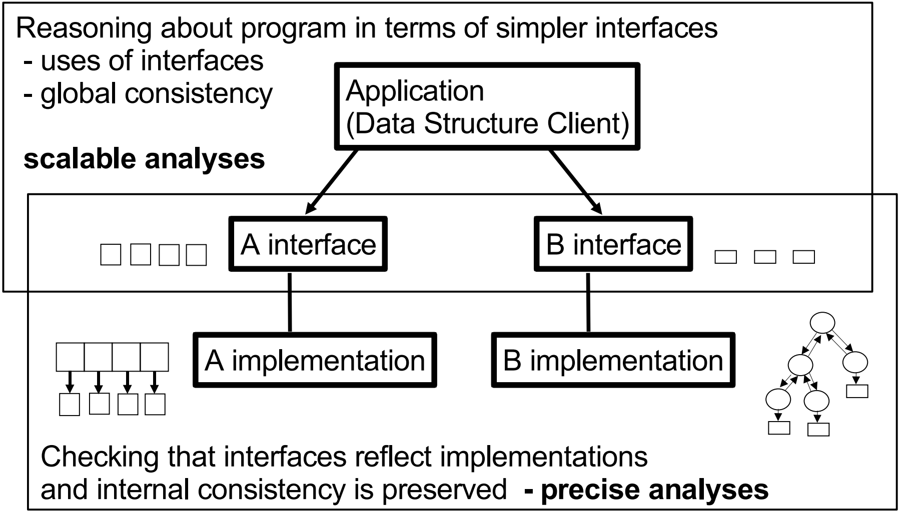
➜ inconsistency can cause policy violations.

Goal

Proof data structure consistency properties

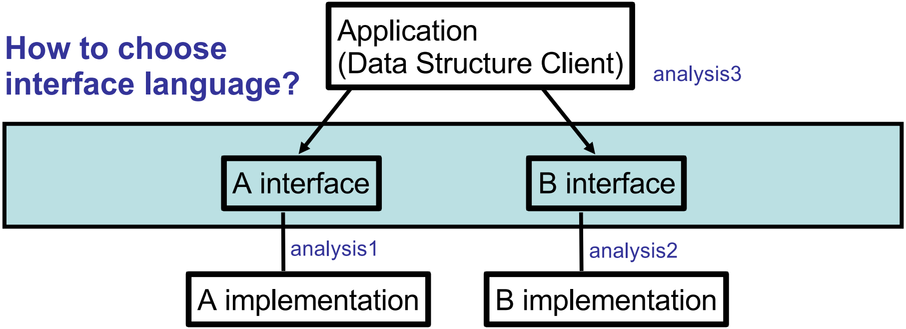
- for all program executions (sound)
- with high level of automation
- both internal and external consistency properties
- both implementation and use of data structures.

Overview of the Jahob Approach

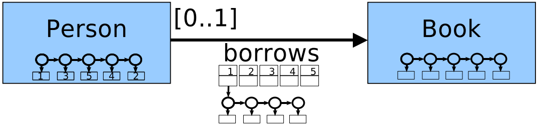


Overview of the Jahob Approach

Key question in automating approach (while keeping it useful)



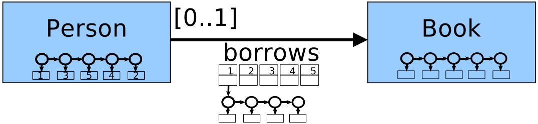
The Jahob Approach through an Example



Data structures to record who borrowed which book. These consist of

- a set of persons, implemented by a linked list.
  Each person has a unique id.
- a set of books, implemented by a linked list.
  Each book has a unique id.
- a relation borrows, implemented by an array indexed by the person unique id.
  Array contains a linked list of books borrowed by that person.

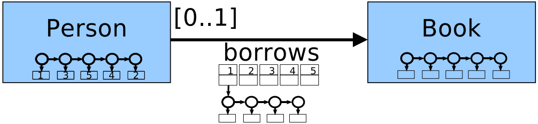
The Jahob Approach through an Example




```
class Library {
    public static Set persons;
    public static Set books;
    public static Relation borrows;
    ...
}
class Relation {
    private Set[] a;
    private int size;
    ...
    public void add(int i, Object o1){
      ...
    }
}
```

```
class Set {
    private Node first;
    ...
    public void add(Object o1){
      Node n = new Node();
      n.data = o1;
      n.next = first;
      first = n;
    }
}
```

Factoring Out Complexity



if a person has borrowed a book, then

- the person is registered with the library, and
- the book is in the catalog

$$\forall\, p\ b\, .\, (p, b) \in \text{borrows.content} \rightarrow$$
$$p \in \text{persons.content}$$
$$\wedge\ b \in \text{books.content}$$

Specification Variables

$$\text{Set.content} = \{\, x \mid \exists\, n\, .\, n \in \text{first.next}^* \wedge n.\text{data} = b \,\}$$

$$\text{Relation.content} = \{\, (x, y) \mid a[x] \neq \text{null} \wedge y \in a[x].\text{content} \,\}$$

Defining Interfaces using Specification Variables

```
class Node {
    Object data;
    Node next;
}
class Set {
    public Node first;
    /*: public specvar content :: objset;
    ...
```

How can we define the set of data values in the linked list?

```
content == first.next*.data
```

Jahob supports reflexive transitive closure but with a different syntax:

Definition (rtrancl_pt)

Let $R : \alpha \Rightarrow \alpha \Rightarrow$ bool be a relation on some type α, then rtrancl_pt R is the reflexive transitive closure of R:
rtrancl_pt $R\ x\ y$ holds if there is a sequence $x = x_0, \dots, x_n = y$, $n \geq 0$ such that $R\ x_i\ x_{i+1}$ holds for $0 \leq i < n$.

Using the rtrancl_pt predicate

Definition (rtrancl_pt)

Let $R : \alpha \Rightarrow \alpha \Rightarrow$ bool be a relation on some type α, then rtrancl_pt R is the reflexive transitive closure of R:
rtrancl_pt R x y holds if there is a sequence $x = x_0, \ldots, x_n = y$, $n \geq 0$ such that R x_i x_{i+1} holds for $0 \leq i < n$.

Define the successor relation using the field `Node.next`:

```
R == (% x y. x..Node.next = y)
```

Note: % is λ-abstraction.

The set of all nodes on the list is:

```
nodes == {n. rtrancl_pt (% x y. x..Node.next = y) first n}
```

and the set of all values on the list is:

```
contents == {x. EX n. n..Node.data = x
                    & rtrancl_pt (% v1 v2. v1..Node.next = v2) first n}
```

Jahob Code

```
class Set {
    private Node first;
    ...
    /*: public specvar content :: objset;
    vardefs "content == {x. EX n. n..Node.data = x &
          rtrancl_pt (% v1 v2. v1..Node.next = v2) first n}";
    ...
    invariant "tree [Node.next]";
    */
    public void add(Object o1)
     /*: requires "o1 ~: content"
       modifies "content"
       ensures "content = old content Un {o1}"
    */
    { ... }
}
```

Use Interfaces to Verify Data Structure Clients

```
class Library {
  public static Set persons;
  ...
  /*: invariant "ALL p b. (p,b) : borrows..Relation.content -->
      p : persons..Set.content & b : books..Set.content" */

  public static void checkOutBook(Person p, Book b)
  /*:
    requires "p ~= null & b ~= null &
       b : books..Set.content & p : persons..Set.content"
    modifies "borrows..Relation.content"
    ensures "((ALL p1. (p1,b) ~: old borrows..Relation.content) -->
       borrows..Relation.content =
           old (borrows..Relation.content) Un {(p,b)})
       & (EX p1. (p1,b) : old borrows..Relation.content -->
       borrows..Relation.content = old borrows..Relation.content)"
    */
    { ... }
}
```

Demo

Example: Doubly Linked List

```
public /*: claimedby DoublyLinkedList */ class Node {
    public Node next;
    public Node prev;
    public Object data;
}

class DoublyLinkedList
{
    private static Node first;
    private static Node last;
```