Software Design, Modelling and Analysis in UML

Lecture 02: Semantical Model

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Contents & Goals

Last Lecture:

- Motivation: model-based development of things (houses, software) to cope with complexity, detect errors early
- Model-based (or -driven) Software Engineering
- UML Mode of the Lecture: Blueprint.

This Lecture:

- **Educational Objectives:** Capabilities for these tasks/questions:
  - Why is UML of the form it is?
  - Shall one feel bad if not using all diagrams during software development?
  - What is a signature, an object, a system state, etc.?
    What's the purpose of signature, object, etc. in the course?
  - How do Basic Object System Signatures relate to UML class diagrams?
- **Content:**
  - Brief history of UML
  - Course map revisited
  - Basic Object System Signature, Structure, and System State

Why (of all things) UML?

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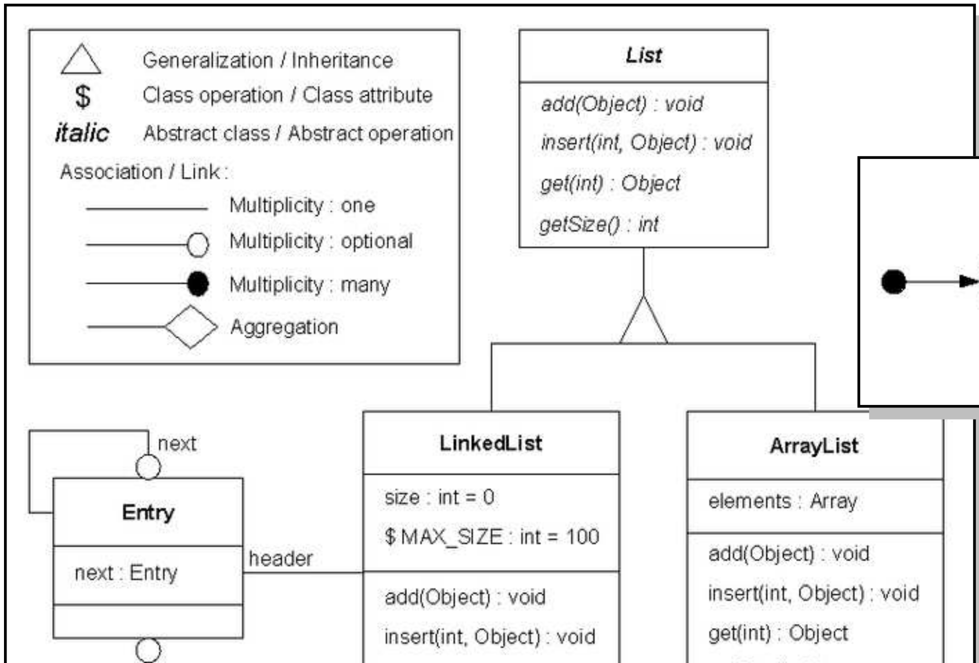
- Note: being a **modelling** languages doesn't mean being graphical (or: being a visual formalism [Harel]).
- For instance, [Kastens and Büning, 2008] also name:
  - Sets, Relations, Functions
  - Terms and Algebras
  - Propositional and Predicate Logic
  - Graphs
  - XML Schema, Entity Relation Diagrams, UML Class Diagrams
  - Finite Automata, Petri Nets, UML State Machines
- **Pro**: visual formalisms are found appealing and easier to **grasp**. Yet they are not necessarily easier to **write**!
- **Beware**: you may meet people who dislike visual formalisms just for being graphical — maybe because it is easier to "trick" people with a meaningless picture than with a meaningless formula.

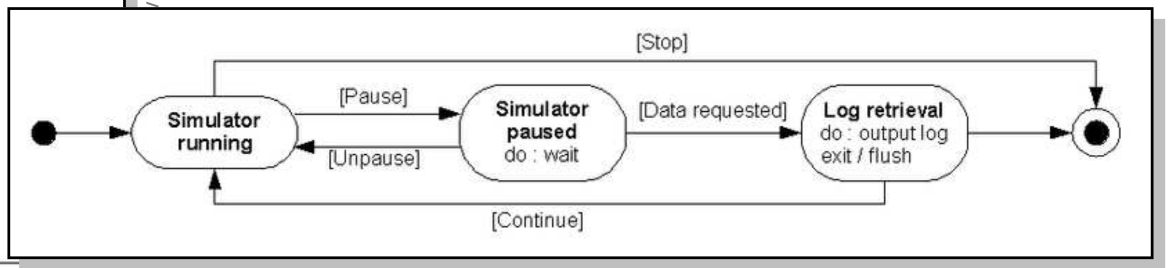
  More serious: it's maybe easier to misunderstand a picture than a formula.

A Brief History of UML

- Boxes/lines and finite automata are used to visualise software **for ages**.
- **1970's**, **Software Crisis**™
  — Idea: learn from engineering disciplines to handle growing complexity.

  Languages: **Flowcharts, Nassi-Shneiderman, Entity-Relation Diagrams**
- Mid **1980**'s: **Statecharts** [Harel, 1987], **StateMate**™ [Harel et al., 1990]
- Early **1990's**, advent of **Object-Oriented**-Analysis/Design/Programming
  — Inflation of notations and methods, most prominent:
  - **Object-Modeling Technique** (OMT) [Rumbaugh et al., 1990]





A Brief Histo

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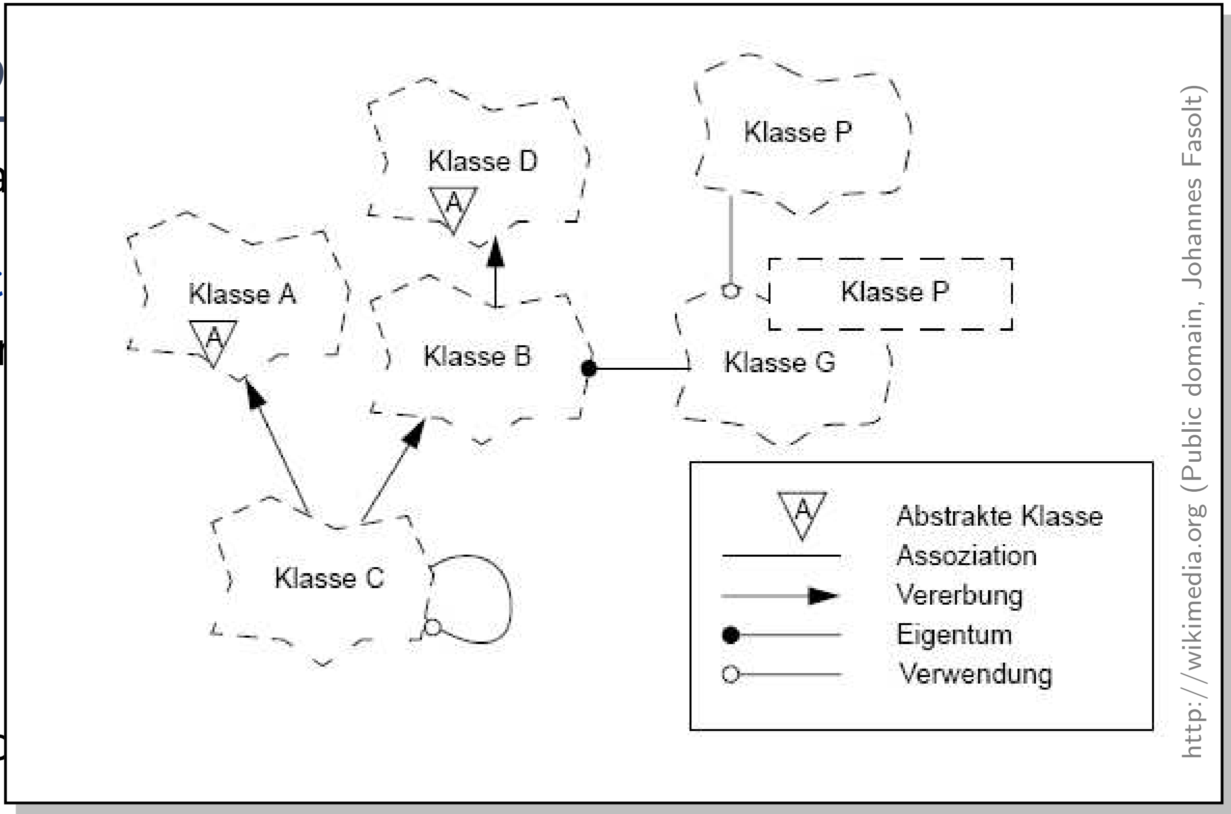

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  - **Object-Oriented Software Engineering** (OOSE) [Jacobson et al., 1992]

  Each "persuasion" selling books, tools, seminars...
- Late **1990's**: joint effort **UML 0.x, 1.x**

  Standards published by **Object Management Group** (OMG), "*international, open membership, not-for-profit* **computer industry** *consortium*".
- Since **2005**: **UML 2.x**

UML Overview [OMG, 2007b, 684]

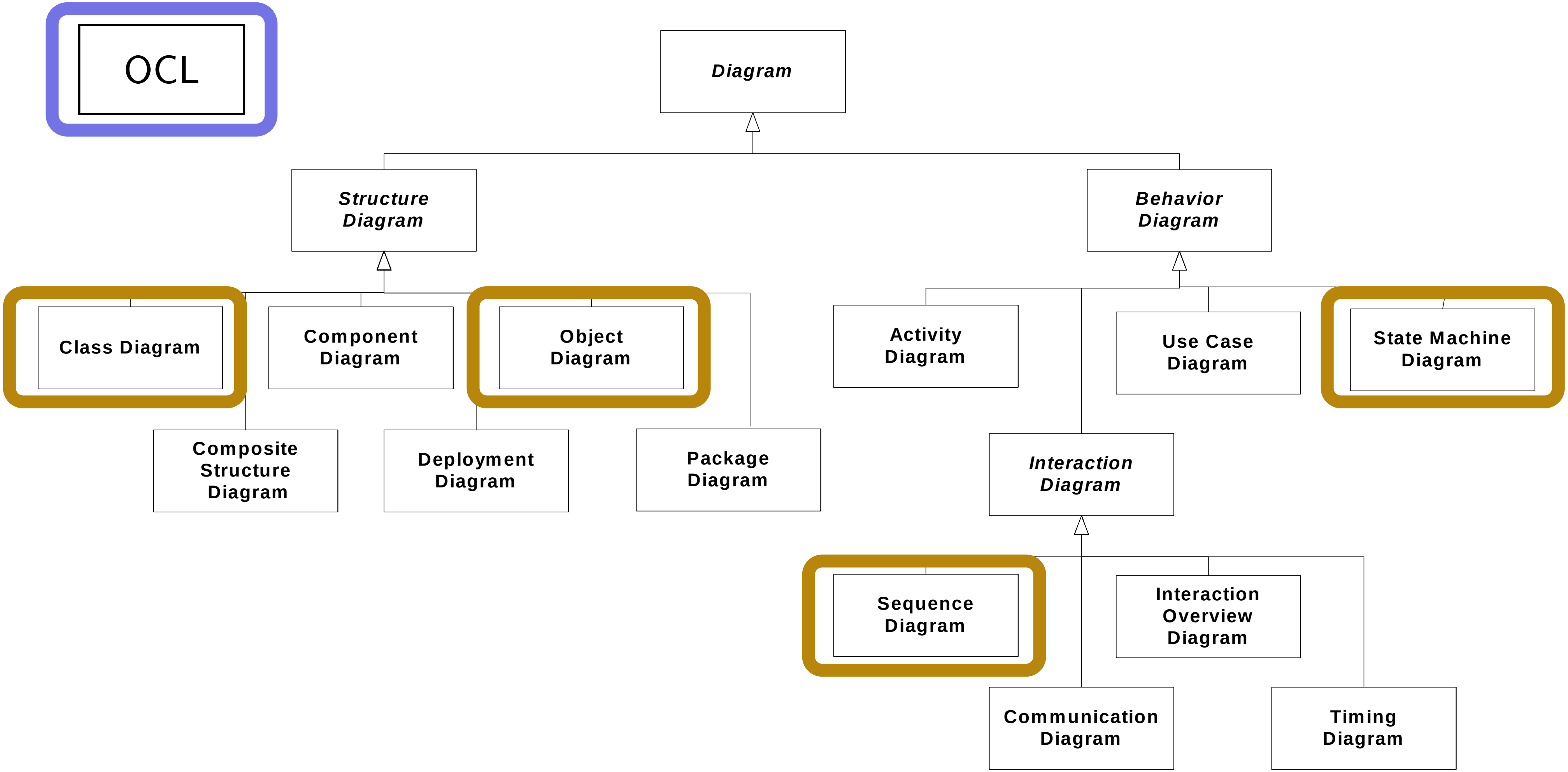


Figure A.5 - The taxonomy of structure and behavior diagram

[Dobing and Parsons, 2006]

Common Expectations on UML

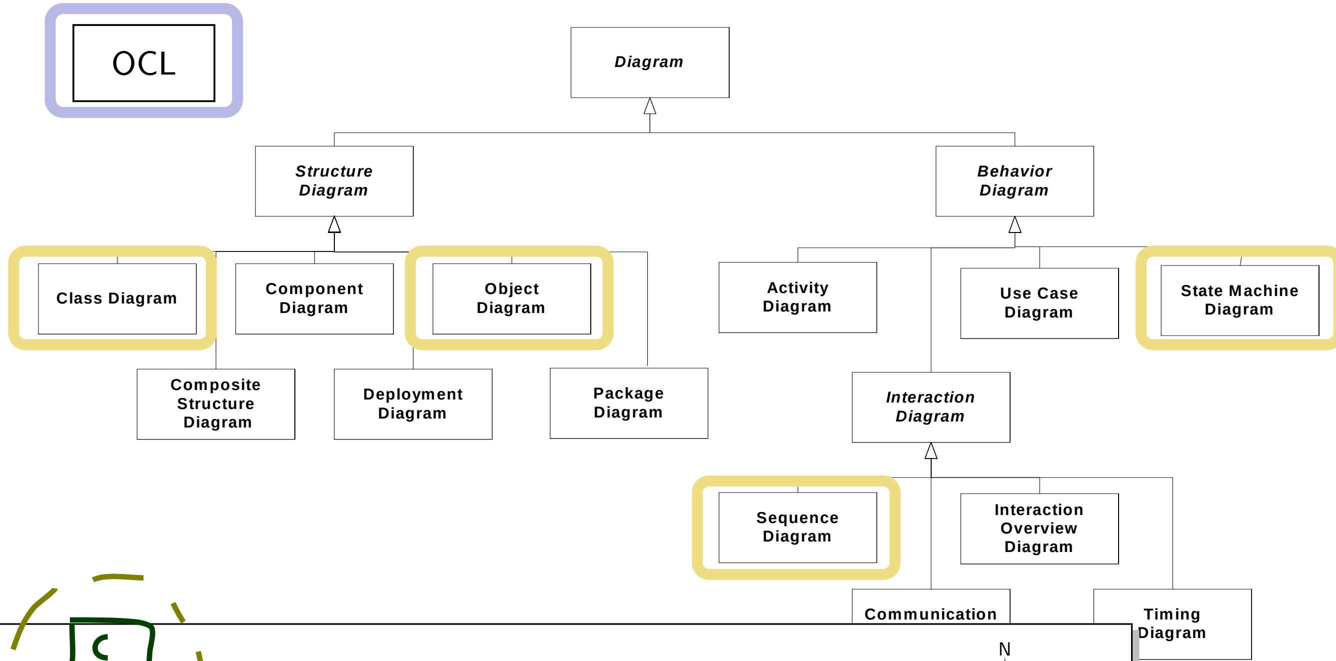
- Easily writeable, readable even by customers
- Powerful enough to bridge the gap between idea and implementation
- Means to tame complexity by separation of concerns ("views")
- Unambiguous
- Standardised, exchangeable between modelling tools
- UML standard says how to develop software
- Using UML leads to better software
- ...

We will see...

Seriously: After the course, you should have an own opinion on each of these claims. In how far/in what sense does it hold? Why? Why not? How can it be achieved? Which ones are really only hopes and expectations? ...?

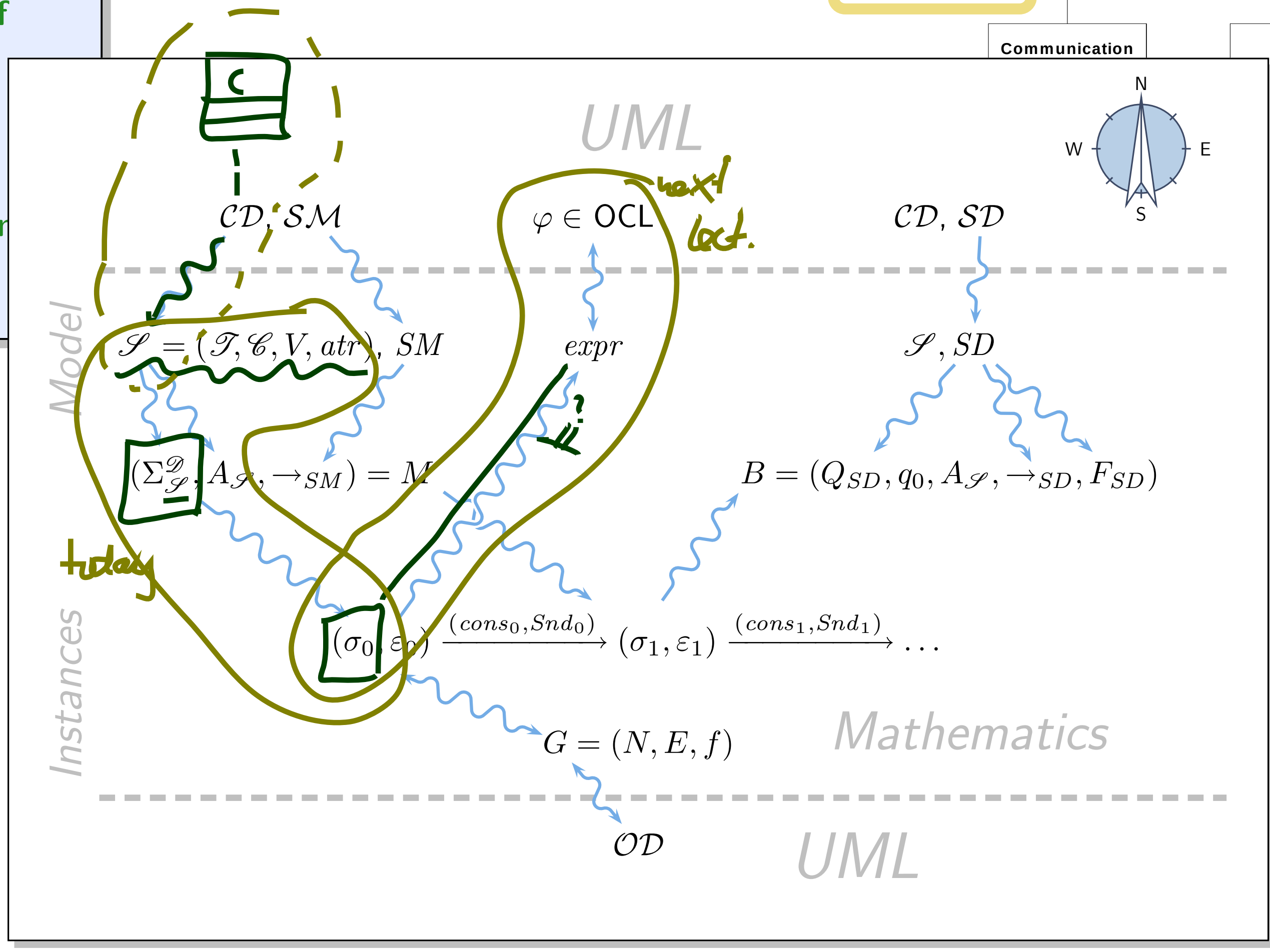
Course Map Revisited

The Plan



Recall:

- **Overall aim**: a formal language for software blueprints.
- **Approach**:
  - (i) Common semantical domain.
  - (ii) UML fragments as **syntax**.
  - (iii) Abstract **representation of diagrams**.
  - (iv) **Informal semantics**: UML standard
  - (v) **assign meaning to diagrar**
  - (vi) Define, e.g., **consistency**.



UML: Semantic Areas

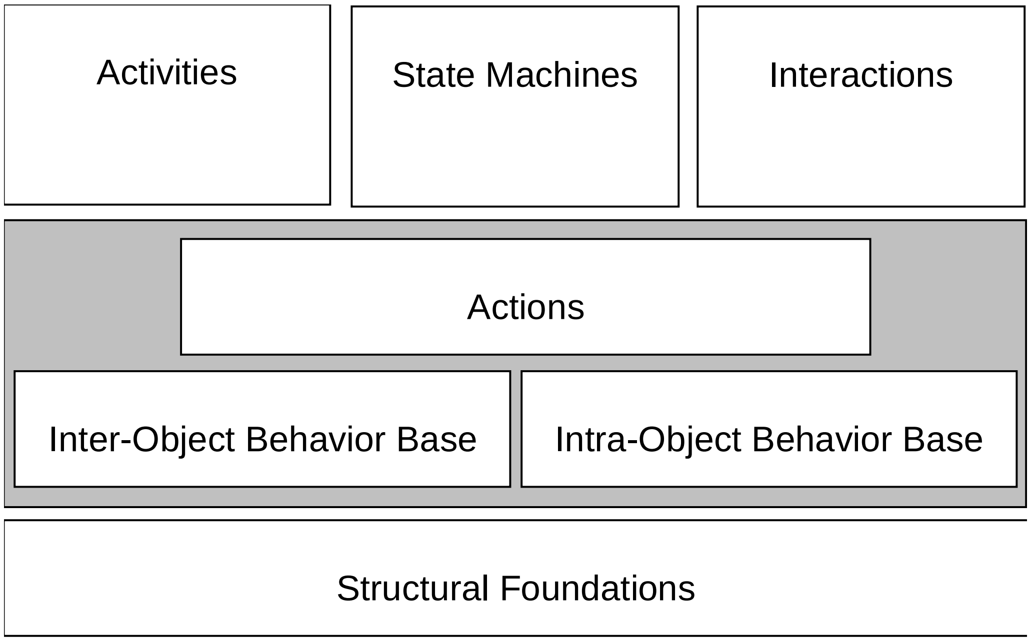


Figure 6.1 - A schematic of the UML semantic areas and their dependencies

[OMG, 2007b, 11]

Common Semantical Domain

Basic Object System Signature

Definition. A (Basic) Object System Signature is a quadruple

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$$

where

- $\mathscr{T}$ is a set of (basic) types,
- $\mathscr{C}$ is a finite set of classes,
- V is a finite set of typed attributes, i.e., each $v \in V$ has type
  - $\tau \in \mathscr{T}$ or
  - $C_{0,1}$ or C_*, where $C \in \mathscr{C}$

  (written $v : \tau$ or $v : C_{0,1}$ or $v : C_*$),
- $atr : \mathscr{C} \to 2^V$ maps each class to its set of attributes.

Note: Inspired by OCL 2.0 standard [OMG, 2006], Annex A.

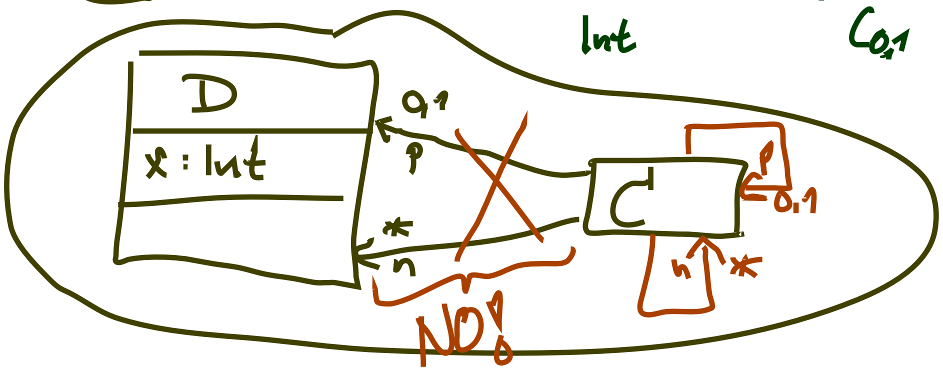
Basic Object System Signature Example

$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$, (both finite),
- typed attributes V, τ from $\mathscr{T}$ or $C_{0,1}$ or C_*, $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

Example:

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$



Basic Object System Signature Another Example

$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ where

- (basic) types $\mathscr{T}$ and classes $\mathscr{C}$, (both finite),
- typed attributes V, τ from $\mathscr{T}$ or $C_{0,1}$ or C_*, $C \in \mathscr{C}$,
- $atr : \mathscr{C} \to 2^V$ mapping classes to attributes.

Example:

$\mathscr{S} = (\{Int, Compl\},$
$\{C, D\},$
$\{C::x : Int, D::x : Compl\},$
$\{C \mapsto \{C::x\}, D \mapsto \{D::x\}\})$

one name! could also be x vs. x♡

What about
- class C with attribute x : Int, and
- class D with attribute x : Compl ?

Rename!

Basic Object System Structure

Definition. A Basic Object System Structure of

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$$

is a domain function $\mathscr{D}$ which assigns to each type a domain, i.e.

- $\tau \in \mathscr{T}$ is mapped to $\mathscr{D}(\tau)$,
- $C \in \mathscr{C}$ is mapped to an infinite set $\mathscr{D}(C)$ of (object) identities.

  Note: Object identities only have the "=" operation;
  object identities of different classes are disjoint, i.e.
  $\forall\, C, D \in \mathscr{C} : C \neq D \rightarrow \mathscr{D}(C) \cap \mathscr{D}(D) = \emptyset$.
- C_* **and** $C_{0,1}$ for $C \in \mathscr{C}$ are mapped to $2^{\mathscr{D}(C)}$.

We use $\mathscr{D}(\mathscr{C})$ to denote $\bigcup_{C \in \mathscr{C}} \mathscr{D}(C)$; analogously $\mathscr{D}(\mathscr{C}_*)$.

Note: We identify objects and object identities, because both uniquely determine each other (cf. OCL 2.0 standard).

Basic Object System Structure Example

Wanted: a structure for signature

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

Recall: by definition, seek a $\mathscr{D}$ which maps

- $\tau \in \mathscr{T}$ to **some** $\mathscr{D}(\tau)$,
- $c \in \mathscr{C}$ to **some** identities $\mathscr{D}(C)$ (infinite, disjoint for different classes),
- C_* and $C_{0,1}$ for $C \in \mathscr{C}$ to $\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) = 2^{\mathscr{D}(C)}$.

$$\mathscr{D}(Int) = \mathbb{Z}$$

(could also be $\{-100, \dots, 99, 100\}$)

$$\mathscr{D}(C) = \mathbb{N}^+ \times \{C\} \cong \{1_C, 2_C, 3_C, \dots\}$$

$$\mathscr{D}(D) = \mathbb{N}^+ \times \{D\} \cong \{1_D, 2_D, 3_D, \dots\}$$

$$\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) = 2^{\mathscr{D}(C)}$$

$$\mathscr{D}(D_{0,1}) = \mathscr{D}(D_*) = 2^{\mathscr{D}(D)}$$

System State

all object identities — partial function — attributes — domain of non-class types

Definition. Let $\mathscr{D}$ be a structure of $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$.
A system state of $\mathscr{S}$ wrt. $\mathscr{D}$ is a **type-consistent** mapping

$$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*))).$$

That is, for each $u \in \mathscr{D}(C)$, $C \in \mathscr{C}$, if $u \in \mathrm{dom}(\sigma)$

- $\mathrm{dom}(\sigma(u)) = atr(C)$
  $: V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*))$
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$
- $\sigma(u)(v) \in \mathscr{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathscr{C}$

We call $u \in \mathscr{D}(\mathscr{C})$ alive in σ if and only if $u \in \mathrm{dom}(\sigma)$.

We use $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ to denote the set of all system states of $\mathscr{S}$ wrt. $\mathscr{D}$.

System State Example

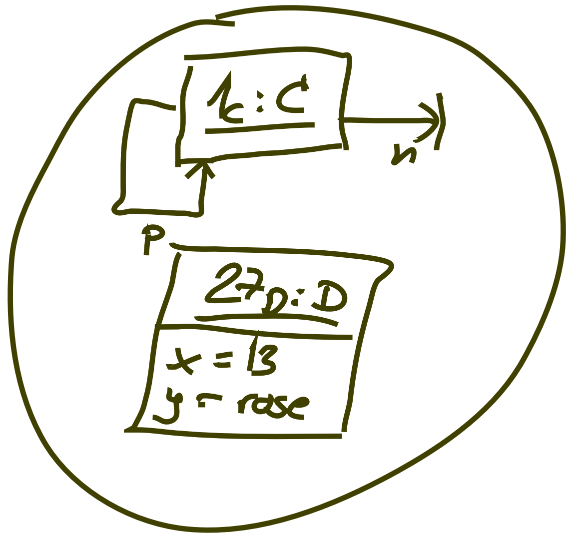
Signature, Structure:

$$\mathscr{S}_0 = (\{Int\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{x\}\})$$

$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, ...\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, ...\}$$

Wanted: $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that

- $\mathrm{dom}(\sigma(u)) = atr(C)$,
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$,
- $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ with $D \in \mathscr{C}$.



System State Example

Signature, **Structure**:

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- **Concrete, explicit**:

$$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}.$$

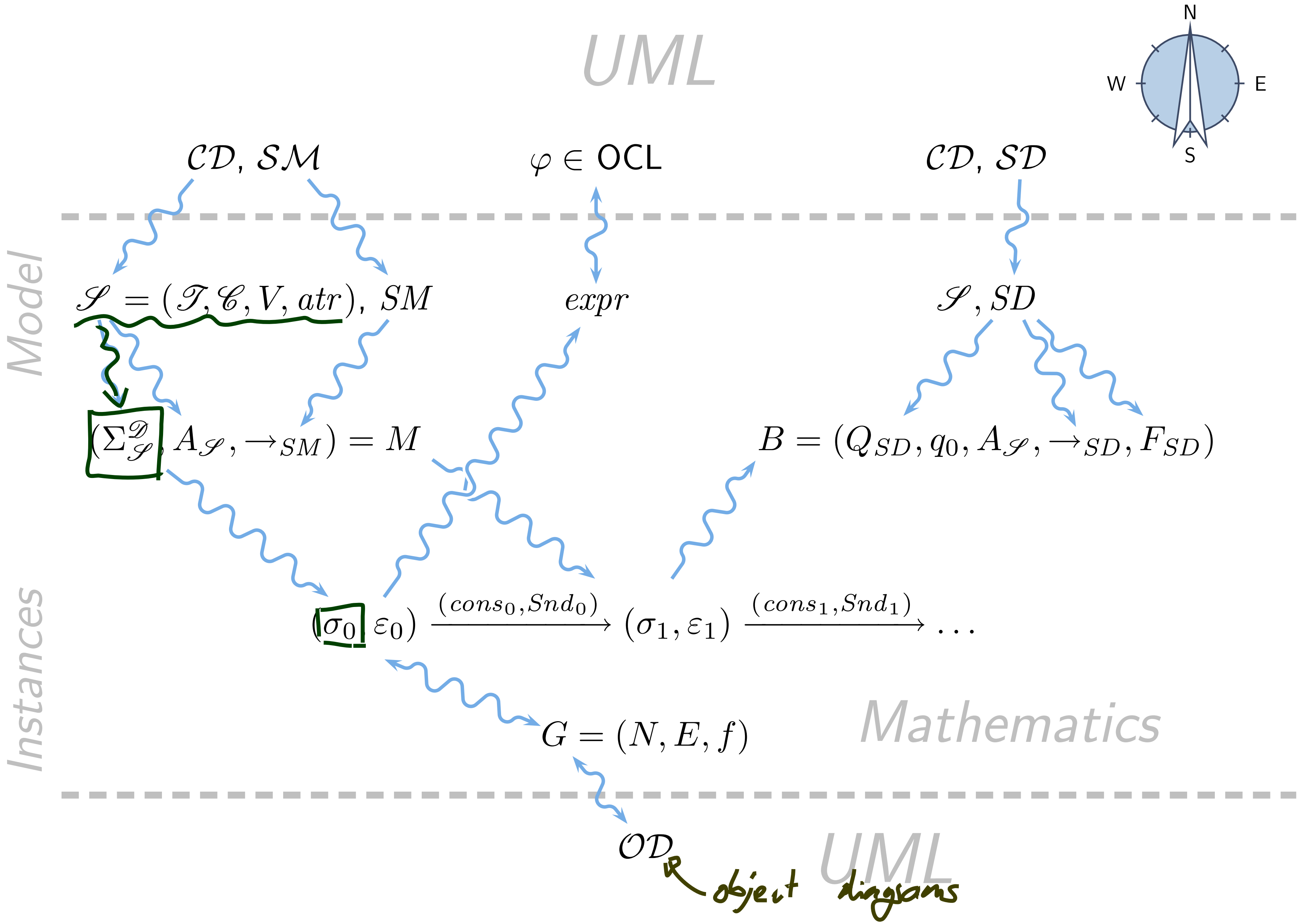
- **Alternative**: **symbolic** system state

$$\sigma = \{c_1 \mapsto \{p \mapsto \emptyset, n \mapsto \{c_2\}\}, c_2 \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, d \mapsto \{x \mapsto 23\}\}$$

assuming $c_1, c_2 \in \mathscr{D}(C)$, $d \in \mathscr{D}(D)$, $c_1 \neq c_2$.

You Are Here.

Course Map



UML
N
W
E
S
Model
Instances
$\mathcal{CD}$, $\mathcal{SM}$
$\varphi \in$ OCL
$\mathcal{CD}$, $\mathcal{SD}$
$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$, SM
$expr$
$\mathscr{S}$, SD
$(\Sigma_{\mathscr{S}}^{\mathscr{D}}, A_{\mathscr{S}}, \rightarrow_{SM}) = M$
$B = (Q_{SD}, q_0, A_{\mathscr{S}}, \rightarrow_{SD}, F_{SD})$
$(\sigma_0, \varepsilon_0) \xrightarrow{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \xrightarrow{(cons_1, Snd_1)} \ldots$
$G = (N, E, f)$
Mathematics
$\mathcal{OD}$
UML
object diagrams

References

References

[Booch, 1993] Booch, G. (1993). *Object-oriented Analysis and Design with Applications*. Prentice-Hall.

[Dobing and Parsons, 2006] Dobing, B. and Parsons, J. (2006). How UML is used. *Communications of the ACM*, 49(5):109–114.

[Harel, 1987] Harel, D. (1987). Statecharts: A visual formalism for complex systems. *Science of Computer Programming*, 8(3):231–274.

[Harel et al., 1990] Harel, D., Lachover, H., et al. (1990). Statemate: A working environment for the development of complex reactive systems. *IEEE Transactions on Software Engineering*, 16(4):403–414.

[Jacobson et al., 1992] Jacobson, I., Christerson, M., and Jonsson, P. (1992). *Object-Oriented Software Engineering - A Use Case Driven Approach*. Addison-Wesley.

[Kastens and Büning, 2008] Kastens, U. and Büning, H. K. (2008). *Modellierung, Grundlagen und Formale Methoden*. Carl Hanser Verlag München, 2nd edition.

[OMG, 2006] OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.

[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.

[Rumbaugh et al., 1990] Rumbaugh, J., Blaha, M., Premerlani, W., Eddy, F., and Lorensen, W. (1990). *Object-Oriented Modeling and Design*. Prentice Hall.