Software Design, Modelling and Analysis in UML

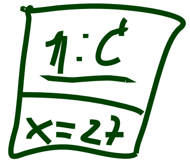
Lecture 04: Object Diagrams

2011-11-09

Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals



Last Lecture:

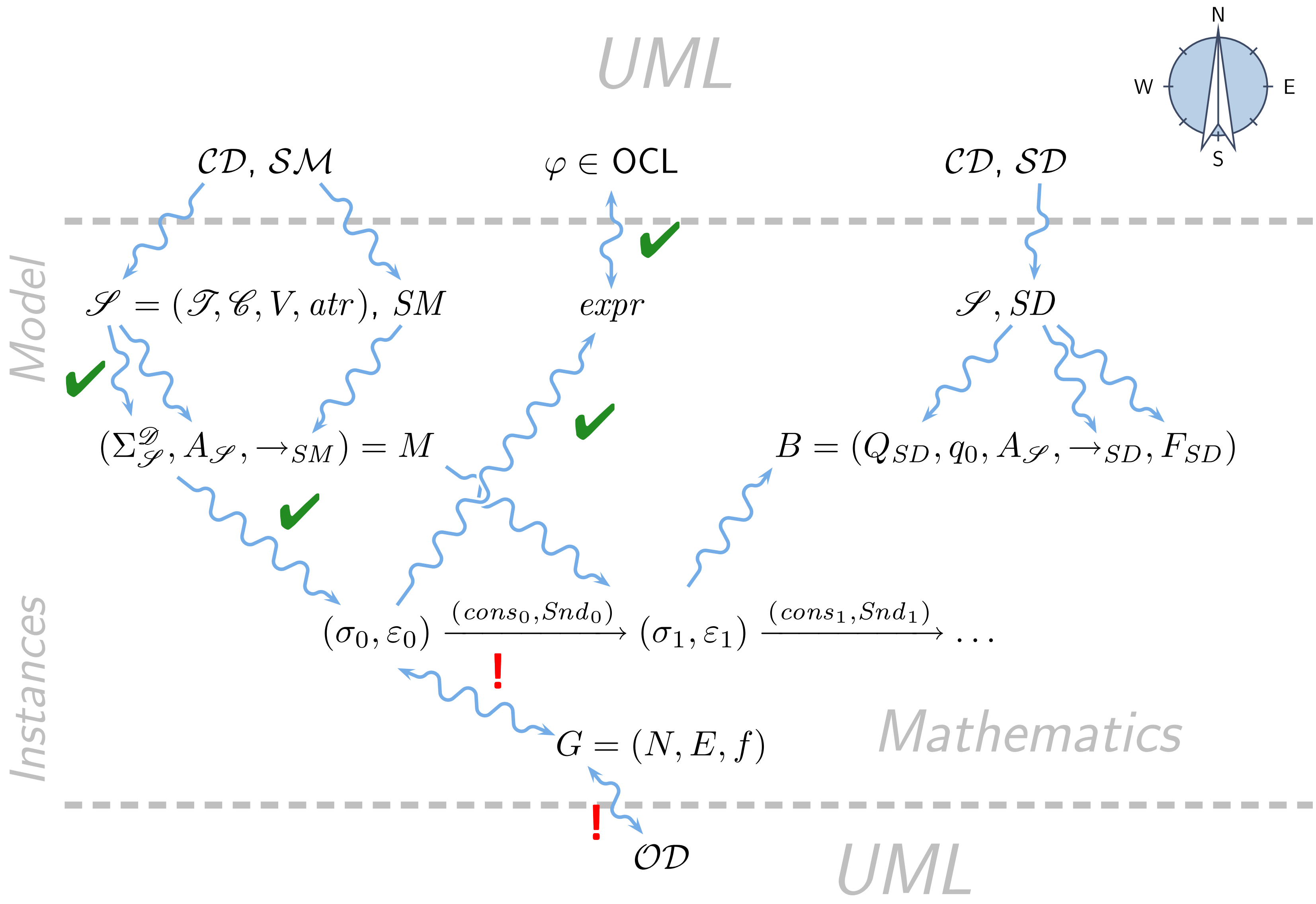
- OCL Syntax and Semantics

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is an object diagram? What are object diagrams good for?
  - When is an object diagram called partial? What are partial ones good for?
  - When is an object diagram an object diagram (wrt. what)?
  - Is this an object diagram wrt. to that other thing?
  - How are system states and object diagrams related?
  - What does it mean that an OCL expression is satisfiable?
  - When is a set of OCL constraints said to be consistent?
  - Can you think of an object diagram which violates this OCL constraint?
- **Content:**
  - Object Diagrams
  - Example: Object Diagrams for Documentation
  - OCL: consistency, satisfiability

Where Are We?

You Are Here.



UML
N
W
E
S
$\mathcal{CD}$, $\mathcal{SM}$
$\varphi \in$ OCL
$\mathcal{CD}$, $\mathcal{SD}$
Model
$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$, SM
$expr$
$\mathscr{S}$, SD
$(\Sigma_{\mathscr{S}}^{\mathscr{D}}, A_{\mathscr{S}}, \to_{SM}) = M$
$B = (Q_{SD}, q_0, A_{\mathscr{S}}, \to_{SD}, F_{SD})$
Instances
$(\sigma_0, \varepsilon_0) \xrightarrow{(cons_0, Snd_0)} (\sigma_1, \varepsilon_1) \xrightarrow{(cons_1, Snd_1)} \dots$
!
$G = (N, E, f)$
Mathematics
!
$\mathcal{OD}$
UML

Object Diagrams

Graph

Definition. A node labelled graph is a triple

$$G = (N, E, f)$$

consisting of

- vertexes N,
- edges E,
- node labeling $f : N \to X$, where X is some label domain,

note: we are allowed to have edges, we choose which to put

Definition. Let $\mathscr{D}$ be a structure of signature $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr)$ and $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$ a system state.

note: we may have values of $V_{0,1,*}$ attributes in the labelling (may be redundant)

Then any graph $G = (N, E, f)$ with

- nodes are identities (not necessarily alive), i.e.

  or vertices

$$N \subset \mathscr{D}(\mathscr{C}) \text{ finite},$$

- edges correspond to "links" of objects, i.e.

  source attribute source object alive destination

$$E \subseteq N \times \underbrace{\{v : \tau \in V \mid \tau \in \{C_{0,1}, C_* \mid C \in \mathscr{C}\}\}}_{V_{0,1,*} :=} \times N,$$

$$\forall\,(u_1, r, u_2) \in E : u_1 \in \operatorname{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r),$$

set of objects referred to via r by u_1

- objects are labelled with attribute valuations and non-alive identities marked with "X", i.e.

$$X = \{\mathsf{X}\} \mathbin{\dot\cup} (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$$

label of $u \in N$ gives values acc. to $\sigma(u)$, not necessarily all

$$\forall\, u \in N \cap \operatorname{dom}(\sigma) : f(u) \subseteq \sigma(u)$$

$: V \nrightarrow \mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)$

$$\forall\, u \in N \setminus \operatorname{dom}(\sigma) : f(u) = \{\mathsf{X}\}$$

is called object diagram of σ.

Graphical Representation of Object Diagrams

$$N \subset \mathscr{D}(\mathscr{C}) \text{ finite}, \quad E \subset N \times V_{0,1,*} \times N, \quad X = \{\mathsf{X}\} \,\dot\cup\, (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$$
$$u_1 \in \mathrm{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r), \qquad f(u) \subseteq \sigma(u) \text{ or } f(u) = \{\mathsf{X}\}$$

- Assume $\mathscr{S} = (\{Int\}, \{C\}, \{v_1 : Int, v_2 : Int, r : C_*\}, \{C \mapsto \{v_1, v_2, r\}\})$.
- Consider
  $\sigma = \{\underbrace{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{u_2\}\}}, \underbrace{u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset\}}\}$ $\quad u_1 \neq u_2$
- Then $G = (N, E, f)$

$$= (\underbrace{\{u_1, u_2\}}, \underbrace{\{(u_1, r, u_2)\}}, \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4\}\},$$

is an object diagram of σ wrt. $\mathscr{S}$ and any $\mathscr{D}$ with $\mathscr{D}(Int) \supseteq \{1, 2, 3, 4\}$.

- $G = (\emptyset, \emptyset, \emptyset)$ $\qquad$ $G = (\{u_1\}, \emptyset, u_1 \mapsto \emptyset)$
  $G = (\{u_1\}, \emptyset, \{u_1 \mapsto \{v_2 \mapsto 2\}\})$ $\qquad$ $G = (\{u_1\}, \emptyset, u_1 \mapsto \{r \mapsto \{u_2\}\})$
- $\sigma_2 = \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{u_2\}\}\}$
  $G = (\{u_1, u_2\}, \{(u_1, r, u_2)\}, \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2\}, u_2 \mapsto \cancel{\emptyset}\ \{\mathsf{X}\}\})$ ← u_2 is not alive!

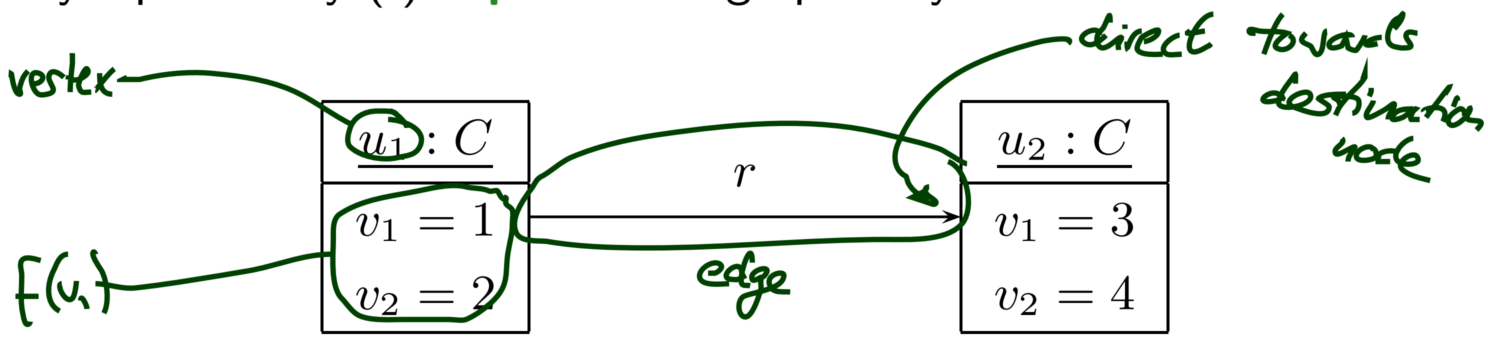
Graphical Representation of Object Diagrams

$N \subset \mathscr{D}(\mathscr{C})$ finite, $\quad E \subset N \times V_{0,1,*} \times N, \quad X = \{\mathsf{X}\} \,\dot{\cup}\, (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$
$u_1 \in \mathrm{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r), \qquad f(u) \subseteq \sigma(u)$ or $f(u) = \{\mathsf{X}\}$

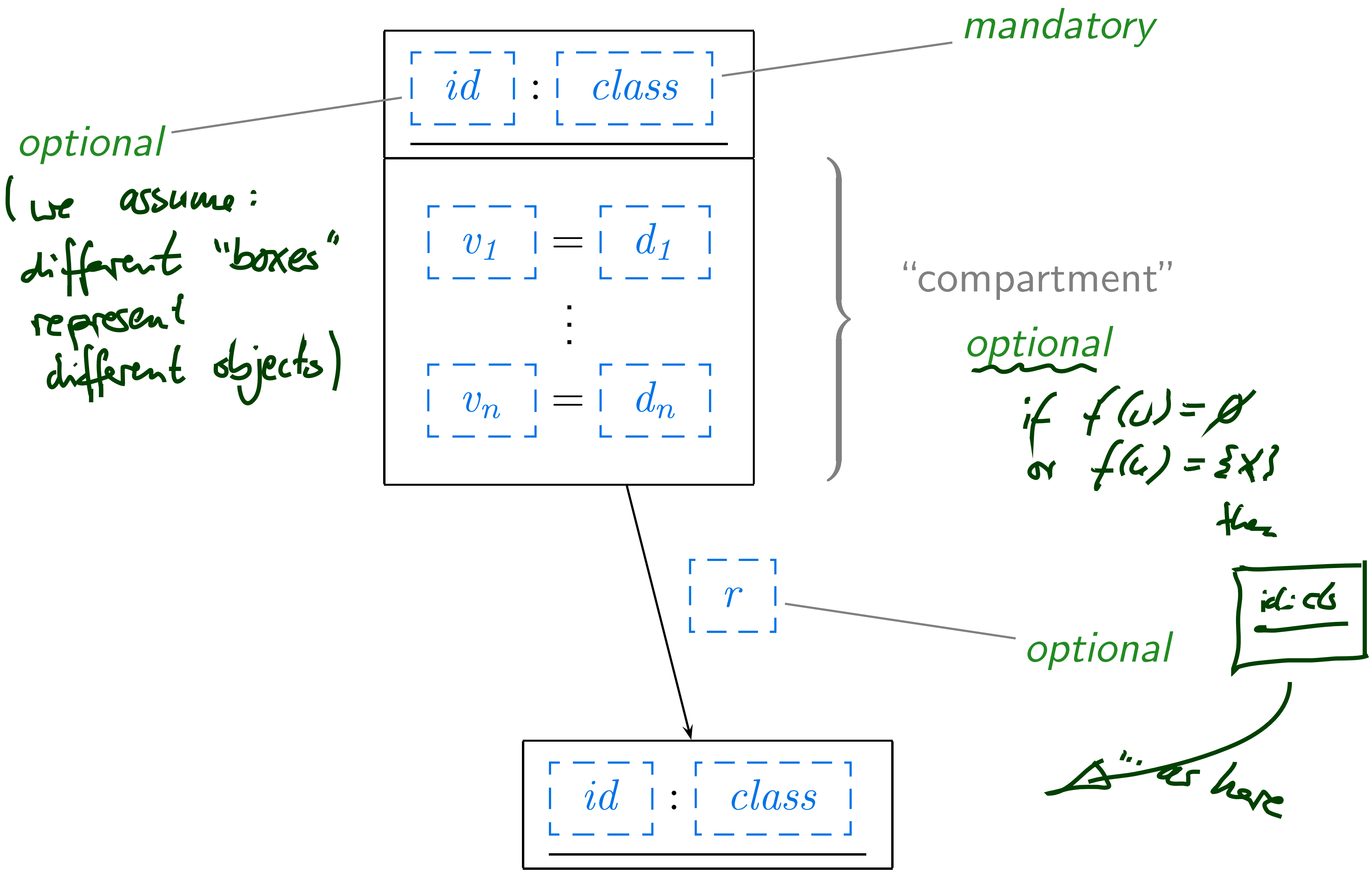
- Assume $\mathscr{S} = (\{Int\}, \{C\}, \{v_1 : Int, v_2 : Int, r : C_*\}, \{C \mapsto \{v_1, v_2, r\}\})$.
- Consider
  $\sigma = \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{u_2\}\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset\}\}$
- Then $G = (N, E, f)$

  $= (\{u_1, u_2\}, \{(u_1, r, u_2)\}, \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4\}\},$

  is an object diagram of σ wrt. $\mathscr{S}$ and any $\mathscr{D}$ with $\mathscr{D}(Int) \supseteq \{1, 2, 3, 4\}$.
- We may equivalently (!) **represent** G graphically as follows:



UML Notation for Object Diagrams



Object Diagrams: More Examples

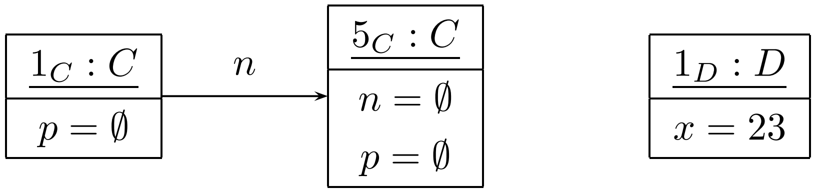
$$N \subset \mathscr{D}(\mathscr{C}) \text{ finite}, \quad E \subset N \times V_{0,1,*} \times N, \quad X = \{\mathsf{X}\} \,\dot\cup\, (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$$
$$u_1 \in \mathrm{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r), \qquad f(u) \subseteq \sigma(u) \text{ or } f(u) = \{\mathsf{X}\}$$

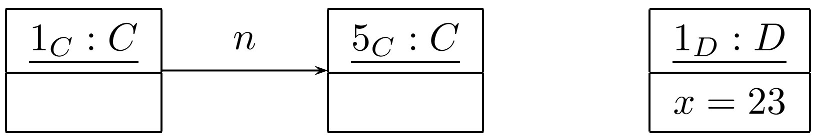
$$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$$

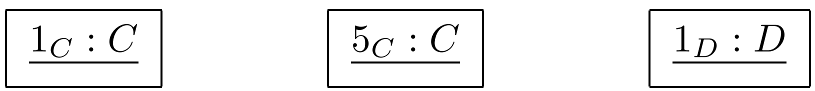
vs.

$\mathscr{S} = (\{Int\}, \{C, D\}, \{x : Int, p : D_{0,1}, n : C_{0,1}\}, C \mapsto \{n, p\}, D \mapsto \{x\}))$

the empty picture

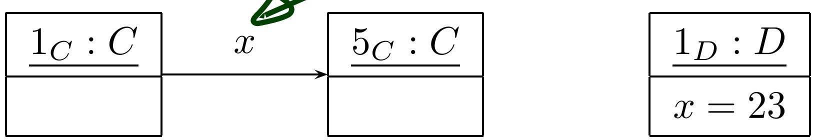
- $(\emptyset, \emptyset, \emptyset)$ ✓
- ✓
- 

✓
- 

✓
- 

✓

NO, $x \notin V_{0,1,*}$

- 

Complete vs. Partial Object Diagram

Definition. Let $G = (N, E, f)$ be an object diagram of system state $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$.

We call G complete wrt. σ if and only if

- G is object complete, i.e.
  - G ~~comprises~~ consists of all alive objects, i.e. $N \overset{=}{\cancel{\subseteq}} \mathrm{dom}(\sigma)$,
- G is attribute complete, i.e.
  - G comprises all "links" between alive objects, i.e. if $u_2 \in \sigma(u_1)(r)$ for some $u_1, u_2 \in \mathrm{dom}(\sigma)$ and $r \in V$, then $(u_1, r, u_2) \in E$, and
  - each node is labelled with the values of all $\mathscr{T}$-typed attributes, i.e. for each $u \in \mathrm{dom}(\sigma)$,

$$f(u) = \underbrace{\sigma(u)|_{V_{\mathscr{T}}}}_{\text{function restriction}} \cup \{r \mapsto \overbrace{(\sigma(u)(r) \setminus N)}^{\text{the non-alive objects referred to via } r \text{ by } u \ldots} \mid \underbrace{r \in V : \sigma(u)(r) \setminus N \neq \emptyset}_{\ldots \text{ if } u \text{ does refer to some non-alive objects}}\}$$

where $V_{\mathscr{T}} := \{v : \tau \in V \mid \tau \in \mathscr{T}\}$.

Otherwise we call G partial.

Complete vs. Partial Examples

- $N = \mathrm{dom}(\sigma)$, if $u_2 \in \sigma(u_1)(r)$, then $(u_1, r, u_2) \in E$,
- $f(u) = \sigma(u)|_{V_{\mathscr{T}}} \cup \{r \mapsto (\sigma(u)(r) \setminus N) \mid \sigma(u)(r) \setminus N\}$

Complete or partial? (wrt. system state σ)

$$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$$

complete acc. to def. if we remove these

complete: /

- $\underline{1_C : C}$ | $(p = \emptyset)$ $\xrightarrow{n}$ $\underline{5_C : C}$ | $(n = \emptyset)$, $(p = \emptyset)$; $\underline{1_D : D}$ | $x = 23$

$\sigma' = \sigma$
$\cup \{27_D \mapsto \{x \mapsto 70\}\}$

- $\underline{1_C : C}$ $\xrightarrow{n}$ $\underline{5_C : C}$; $\underline{1_D : D}$ | $x = 23$ complete

- $\underline{1_C : C}$; $\underline{5_C : C}$; $\underline{1_D : D}$
  - object-complete
  - not attribute complete

Complete/Partial is Relative

- Claim:
  - Each finite system state has **exactly one complete** object diagram.
  - A finite system state can have **many partial** object diagrams.

- Each object diagram G represents a set of system states, namely

$$G^{-1} := \{\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \mid G \text{ is an object diagram of } \sigma\}$$

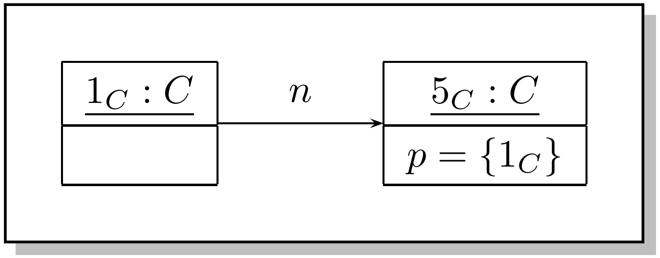
- **Observation**: If somebody **tells us**, that a given (consistent) object diagram G is **complete**, we can uniquely reconstruct the corresponding system state.

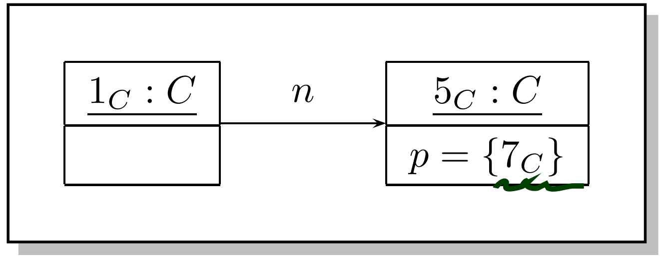
  In other words: G^{-1} is then a singleton.

Corner Cases

Closed Object Diagrams vs. Dangling References

Find the 10 differences! (Both diagrams shall be complete.)





Definition. Let σ be a system state. We say attribute $v \in V_{0,1,*}$ has a dangling reference in object $u \in \mathrm{dom}(\sigma)$ if and only if the attribute's value comprises an object which is not alive in σ, i.e. if

$$\sigma(u)(v) \not\subseteq \mathrm{dom}(\sigma).$$

alive objects

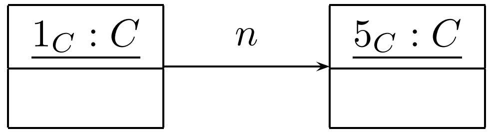
We call σ closed if and only if no attribute has a dangling reference in any object alive in σ.

Observation: Let G be the (!) complete object diagram of a **closed** system state σ. Then the nodes in G are labelled with $\mathscr{T}$-typed attribute/value pairs only.

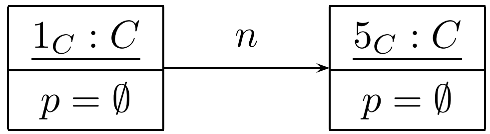
Special Notation

- $\mathscr{S} = (\{Int\}, \{C\}, \{n, p : C_*\}, \{C \mapsto \{n, p\}\})$.

- Instead of

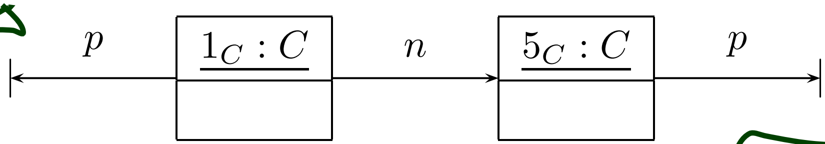


we ~~want to~~ write
can



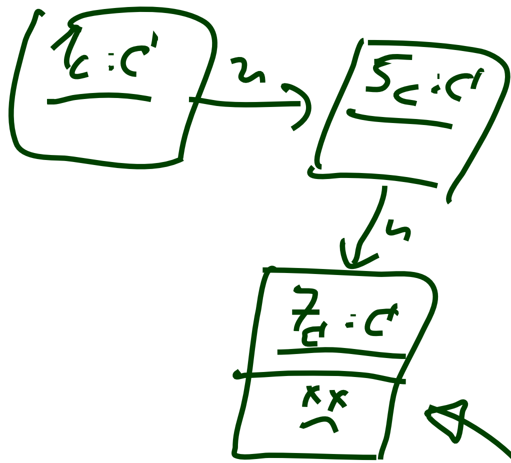
or

not
OMG
UML



to **explicitly** indicate that attribute $p : C_*$ has value $\emptyset$ (also for $p : C_{0,1}$).

possible extension:

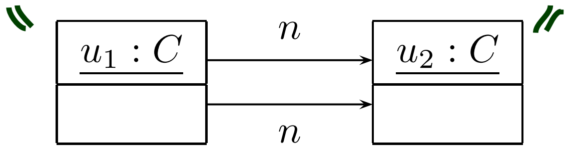


notation for non-alive object

Aftermath

We slightly deviate from the standard (for reasons):

- In the course, $C_{0,1}$ and C_*-typed attributes **only** have **sets as values**.
  UML also considers multisets, that is, they can have

  (This is not an object diagram in the sense of our definition because of the requirement on the edges E. Extension is straightforward but tedious.)

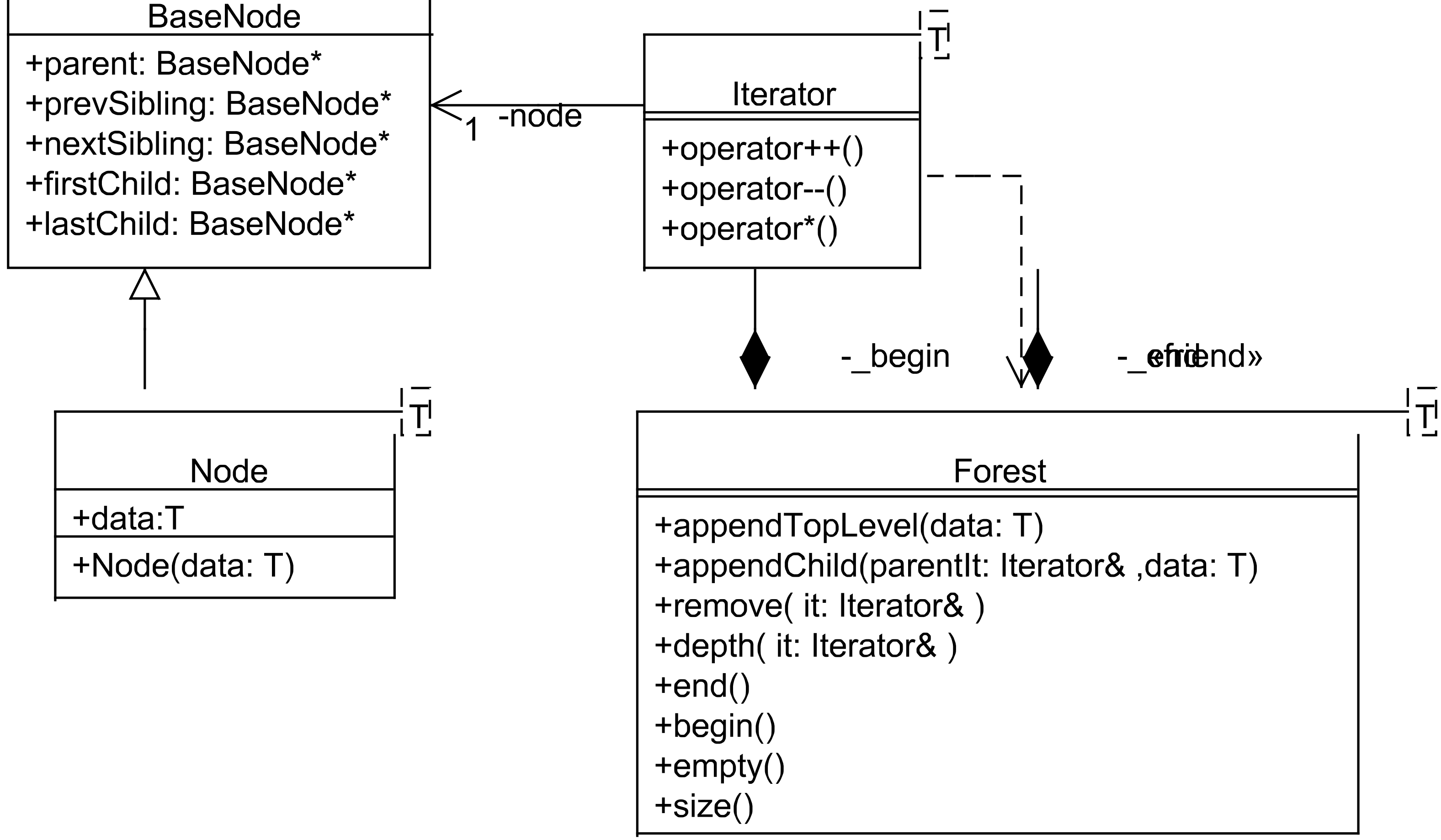
- We **allow** to give the valuation of $C_{0,1}$- or C_*-typed attributes in the **values compartment**.
  - Allows us to indicate that a certain r is not referring to another object.
  - Allows us to represent "dangling references", i.e. references to objects which are not alive in the current system state.

- We introduce a graphical representation of $\emptyset$ values.

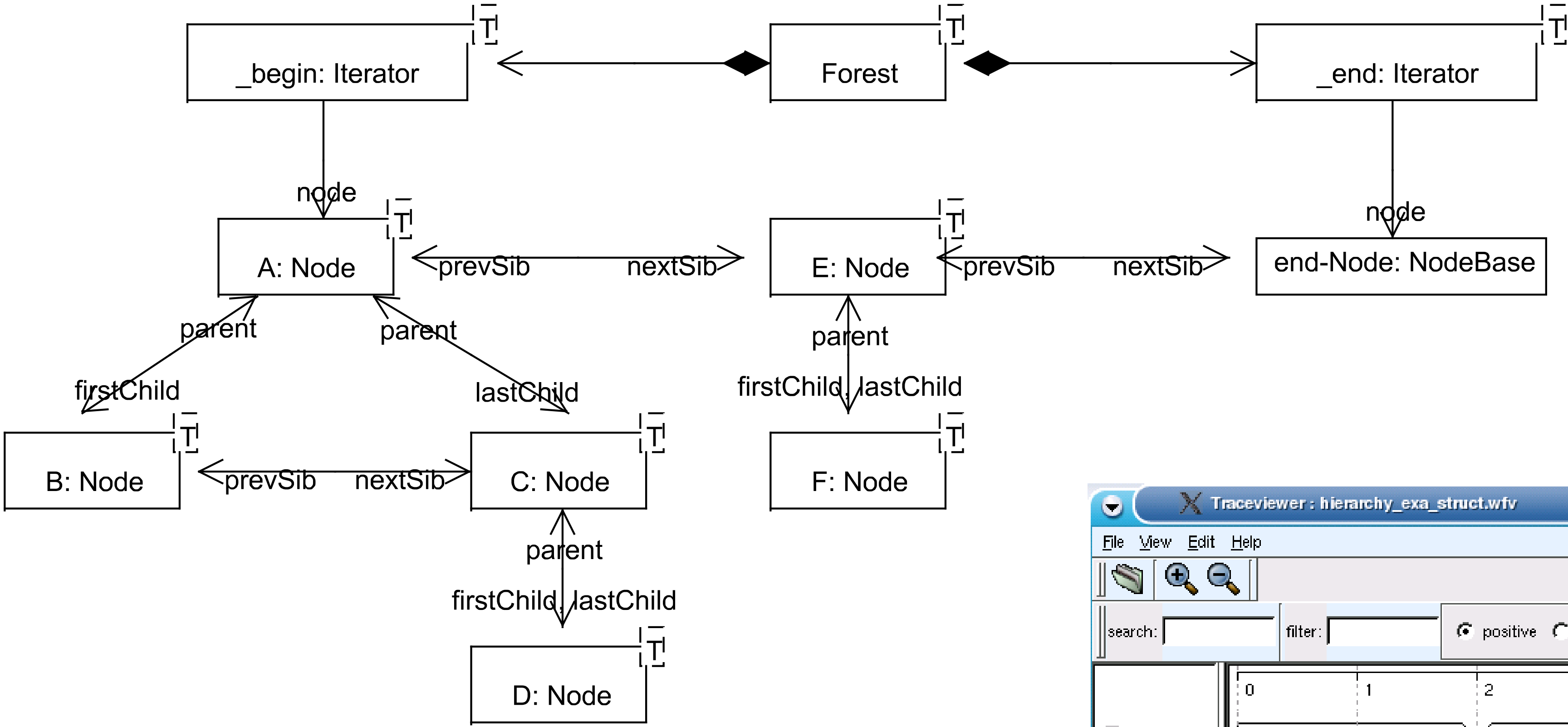
The Other Way Round

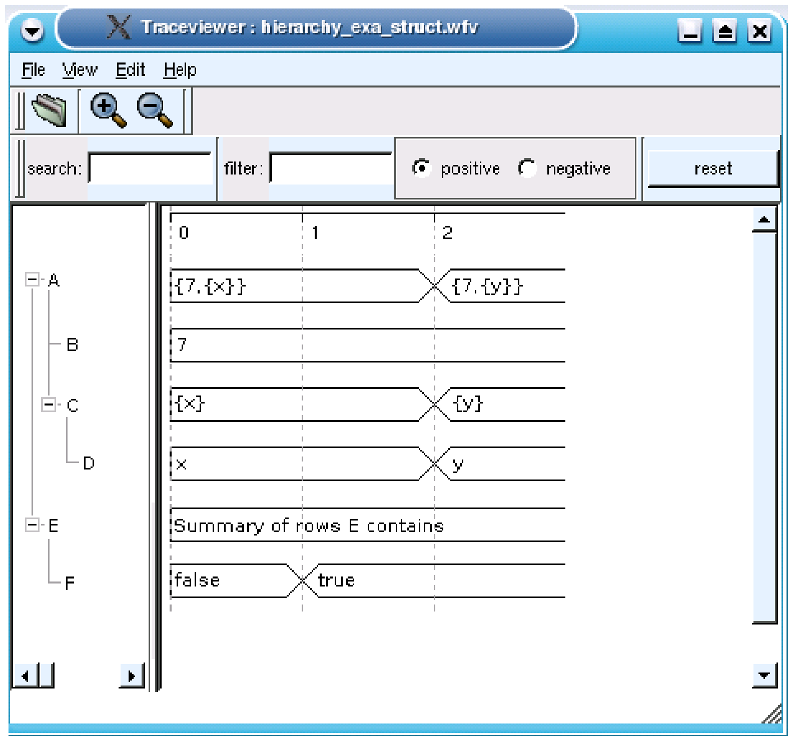
Example: Object Diagrams for Documentation

Example: Data Structure [Schumann et al., 2008]



Example: Illustrative Object Diagram [Schumann et al., 2008]





OCL Consistency

OCL Satisfaction Relation

In the following, $\mathscr{S}$ denotes a signature and $\mathscr{D}$ a structure of $\mathscr{S}$.

Definition (Satisfaction Relation).

Let φ be an OCL constraint over $\mathscr{S}$ and $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$ a system state.

We write

- $\sigma \models \varphi$ if and only if $I[\![\varphi]\!](\sigma, \emptyset) = \mathit{true}$.
- $\sigma \not\models \varphi$ if and only if $I[\![\varphi]\!](\sigma, \emptyset) = \mathit{false}$.

Note: In general we **can't** conclude from $\neg(\sigma \models \varphi)$ to $\sigma \not\models \varphi$ or vice versa.

Object Diagrams and OCL

- Let G be an object diagram of signature $\mathscr{S}$ wrt. structure $\mathscr{D}$. Let $expr$ be an OCL expression over $\mathscr{S}$.

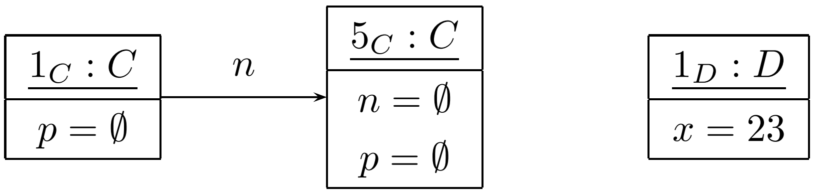
  We say G **satisfies** $expr$, denoted by $G \models expr$, if and only if

  $$\forall\, \sigma \in G^{-1} : \sigma \models expr.$$

- If G is **complete**, we can also talk about "$\not\models$".

  (Otherwise better not to avoid confusion: G^{-1} could comprise different system states in which $expr$ evaluates to *true*, *false*, and $\perp$.)

- **Example**: (complete — what if not complete wrt. object/attribute/both?)



  - context C inv : n -> isEmpty() $\rightsquigarrow$ ~~true~~ false
  - context C inv : $p\,.\,n$ -> isEmpty() $\rightsquigarrow$ $\perp_{Bool}$
  - context D inv : $x \neq 0$ $\rightsquigarrow$ true

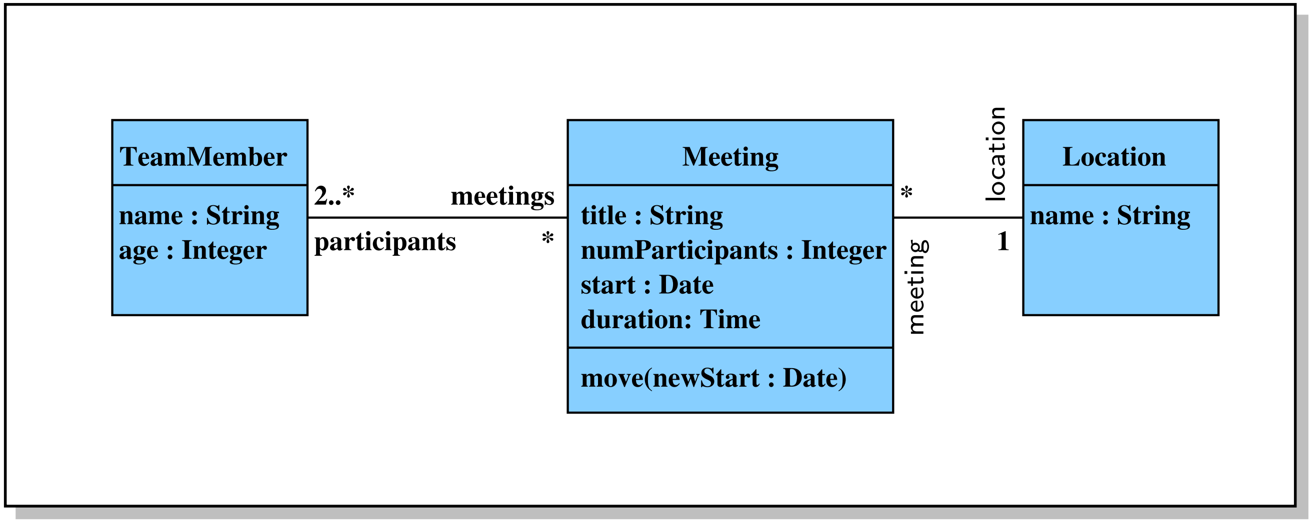
OCL Consistency

Definition (Consistency). A set $Inv = \{\varphi_1, \dots, \varphi_n\}$ of OCL constraints over $\mathscr{S}$ is called consistent (or satisfiable) if and only if there exists a system state of $\mathscr{S}$ wrt. $\mathscr{D}$ which satisfies all of them, i.e. if

$$\exists\, \sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}} : \sigma \models \varphi_1 \wedge \ldots \wedge \sigma \models \varphi_n$$

and inconsistent (or unrealizable) otherwise.

OCL Inconsistency Example



((C) Prof. Dr. P. Thiemann, http://proglang.informatik.uni-freiburg.de/teaching/swt/2008/)

- context $Location$ inv :
  $name =$ 'Lobby' implies $meeting$ -> $isEmpty()$
- context $Meeting$ inv :
  $title =$ 'Reception' implies $location\,.\,name = "Lobby"$
- allInstances$_{Meeting}$ -> exists$(w : Meeting \mid w\,.\,title =$ 'Reception'$)$

Deciding OCL Consistency

- Whether a set of OCL constraints is satisfiable or not is **in general not as obvious** as in the made-up example.

- **Wanted**: A procedure which decides the OCL satisfiability problem.

- **Unfortunately**: in general **undecidable**.

  Otherwise we could, for instance, solve **diophantine equations**

  $$c_1 x_1^{n_1} + \cdots + c_m x_m^{n_m} = d.$$

  constants

  Encoding in OCL:

  $$\mathsf{allInstances}_C \mathrel{\mathsf{->}} \mathsf{exists}(w : C \mid c_1 * w.x_1^{n_1} + \cdots + c_m * w.x_m^{n_m} = d).$$

Deciding OCL Consistency

- Whether a set of OCL constraints is satisfiable or not is **in general not as obvious** as in the made-up example.

- **Wanted**: A procedure which decides the OCL satisfiability problem.

- **Unfortunately**: in general **undecidable**.

  Otherwise we could, for instance, solve **diophantine equations**

  $$c_1 x_1^{n_1} + \cdots + c_m x_m^{n_m} = d.$$

  Encoding in OCL:

  $$\mathsf{allInstances}_C \mathrel{\texttt{->}} \mathsf{exists}(w : C \mid c_1 * w.x_1^{n_1} + \cdots + c_m * w.x_m^{n_m} = d).$$

- **And now**? Options: [Cabot and Clarisó, 2008]
  - Constrain OCL, use a **less rich** fragment of OCL.
  - Revert to **finite domains** — basic types vs. number of objects.

OCL Critique

- **Expressive Power**:
  - "Pure OCL expressions only compute primitive recursive functions, but not recursive functions in general." [Cengarle and Knapp, 2001]
  - **Evolution over Time**: "finally $self.x > 0$"

    Proposals for fixes e.g. [Flake and Müller, 2003]. (Or: sequence diagrams.)
  - **Real-Time**: "Objects respond within 10s"

    Proposals for fixes e.g. [Cengarle and Knapp, 2002]
  - **Reachability**: "After insert operation, node shall be reachable."

    Fix: add transitive closure.
- **Concrete Syntax**

  "The syntax of OCL has been criticized – e.g., by the authors of Catalysis [...] – for being hard to read and write.
  - OCL's expressions are stacked in the style of Smalltalk, which makes it hard to see the scope of quantified variables.
  - Navigations are applied to atoms and not sets of atoms, although there is a collect operation that maps a function over a set.
  - Attributes, [...], are partial functions in OCL, and result in expressions with undefined value." [Jackson, 2002]

References

References

[Cabot and Clarisó, 2008] Cabot, J. and Clarisó, R. (2008). UML-OCL verification in practice. In Chaudron, M. R. V., editor, *MoDELS Workshops*, volume 5421 of *Lecture Notes in Computer Science*. Springer.

[Cengarle and Knapp, 2001] Cengarle, M. V. and Knapp, A. (2001). On the expressive power of pure OCL. Technical Report 0101, Institut für Informatik, Ludwig-Maximilians-Universität München.

[Cengarle and Knapp, 2002] Cengarle, M. V. and Knapp, A. (2002). Towards OCL/RT. In Eriksson, L.-H. and Lindsay, P. A., editors, *FME*, volume 2391 of *Lecture Notes in Computer Science*, pages 390–409. Springer-Verlag.

[Flake and Müller, 2003] Flake, S. and Müller, W. (2003). Formal semantics of static and temporal state-oriented OCL constraints. *Software and Systems Modeling*, 2(3):164–186.

[Jackson, 2002] Jackson, D. (2002). Alloy: A lightweight object modelling notation. *ACM Transactions on Software Engineering and Methodology*, 11(2):256–290.

[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

[OMG, 2007b] OMG (2007b). Unified modeling language: Superstructure, version 2.1.2. Technical Report formal/07-11-02.

[Schumann et al., 2008] Schumann, M., Steinke, J., Deck, A., and Westphal, B. (2008). Traceviewer technical documentation, version 1.0. Technical report, Carl von Ossietzky Universität Oldenburg und OFFIS.