Software Design, Modelling and Analysis in UML

Lecture 10: Core State Machines II

11

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:

- Core State Machines
- UML State Machine syntax
- State machines belong to classes.

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What does this State Machine mean? What happens if I inject this event?
  - Can you please model the following behaviour.
  - What is: Signal, Event, Ether, Transformer, Step, RTC.

- **Content:**
  - Ether, System Configuration, Transformer
  - Run-to-completion Step
  - Putting It All Together

Recall: UML State Machines

Roadmap: Chronologically

(i) What do we (have to) cover?
UML State Machine Diagrams **Syntax**.

(ii) Def.: Signature with **signals**.

(iii) Def.: **Core state machine**.

(iv) Map UML State Machine Diagrams to core state machines.

Semantics:
The Basic Causality Model

(v) Def.: **Ether** (aka. event pool)

(vi) Def.: **System configuration**.

(vii) Def.: **Event**.

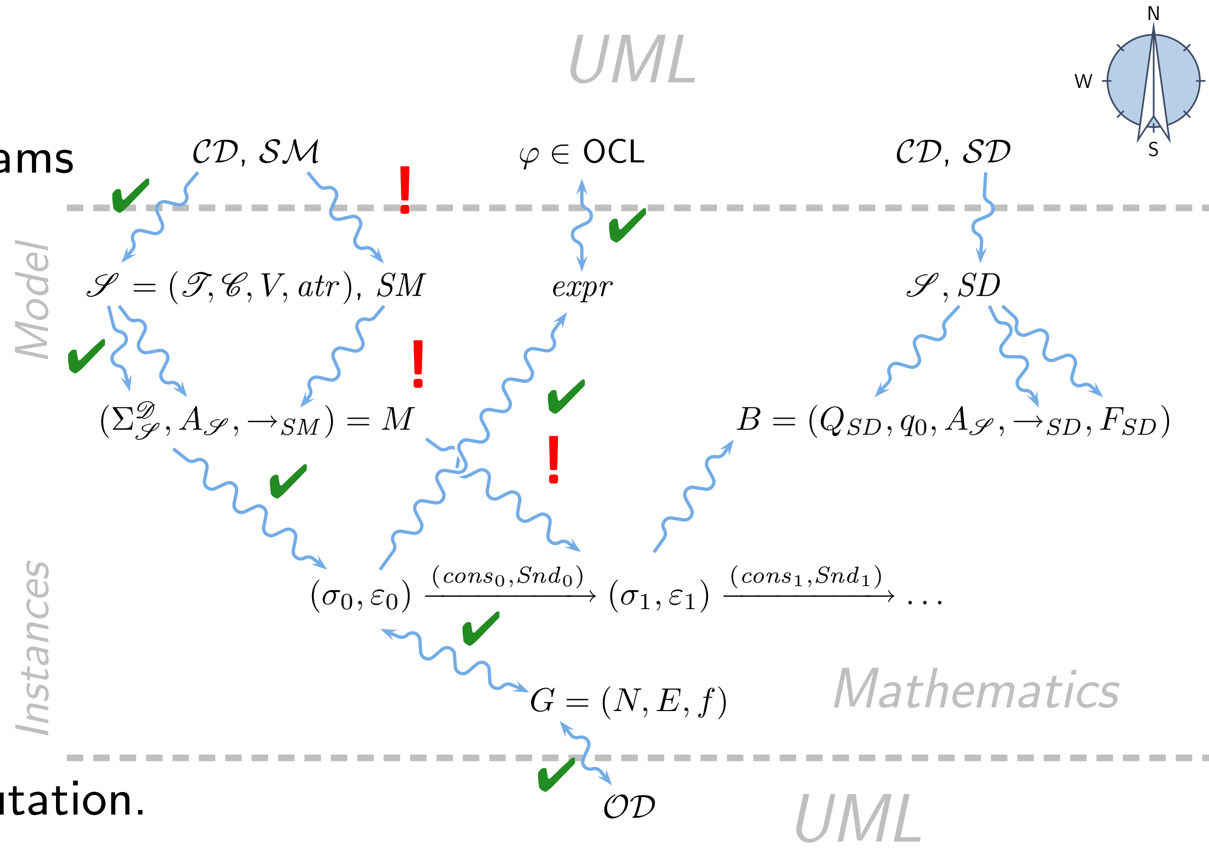
(viii) Def.: **Transformer**.

(ix) Def.: **Transition system**, computation.

(x) Transition relation induced by core state machine.

(xi) Def.: **step**, **run-to-completion step**.

(xii) Later: Hierarchical state machines.



Core State Machine

disjoint union: _ should not already be in $\mathcal{E}$ (otherwise rename first)

Definition.
A core state machine over signature $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \mathscr{E})$ is a tuple

$$M = (S, s_0, \to)$$

where

- S is a non-empty, finite set of **(basic) states**,
- $s_0 \in S$ is an **initial state**,
- and

source state   signals in $\mathscr{S}$ dest. state

$$\to \ \subseteq S \times \underbrace{(\mathscr{E} \,\dot{\cup}\, \{\_\})}_{\text{trigger}} \times \underbrace{Expr_{\mathscr{S}}}_{\text{guard}} \times \underbrace{Act_{\mathscr{S}}}_{\text{action}} \times S$$

  is a labelled transition relation.

We assume a set $Expr_{\mathscr{S}}$ of boolean expressions over $\mathscr{S}$ (for instance OCL, may be something else) and a set $Act_{\mathscr{S}}$ of **actions**.

From UML to Core State Machines: By Example

UML state machine diagram $\mathcal{SM}$:

$$annot ::= \Big[\underbrace{\big[\langle event \rangle [\text{‘,’} \langle event \rangle]^* \big]}_{\text{trigger}} \ \underbrace{[\text{‘[’} \langle guard \rangle \text{‘]’}]}_{\text{guard}} \ \underbrace{[\text{‘/’} \langle action \rangle]}_{\text{action}} \Big]$$

with

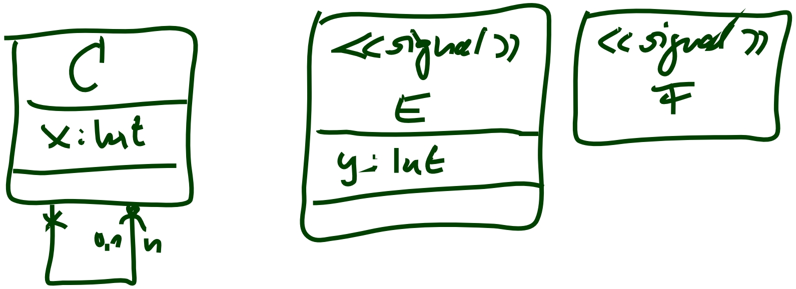
- $event \in \mathscr{E}$, (default: _ if no trigger)
- $guard \in Expr_{\mathscr{S}}$ (default: *true*, assumed to be in $Expr_{\mathscr{S}}$)
- $action \in Act_{\mathscr{S}}$ (default: `skip`, assumed to be in $Act_{\mathscr{S}}$)

maps to

$$M(\mathcal{SM}) = \Big(\underbrace{\{s_1, s_2\}}_{S}, \underbrace{s_1}_{s_0}, \underbrace{(s_1, event, guard, action, s_2)}_{\rightarrow} \Big)$$

initial state

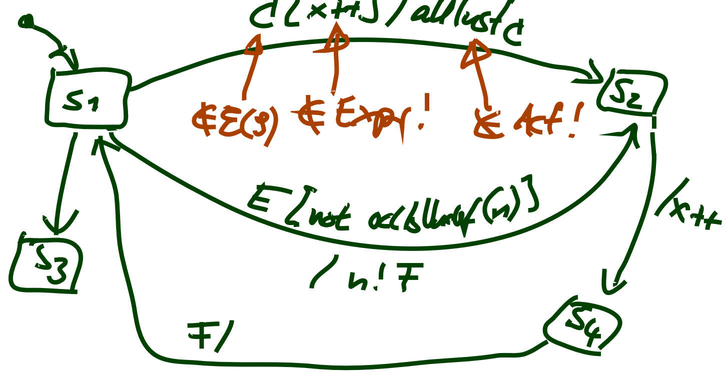
CD:



$Expr_{\mathscr{S}}$: OCL over $\mathscr{S}$

$Act_{\mathscr{S}}$: {skip, x++, n!F}

UML

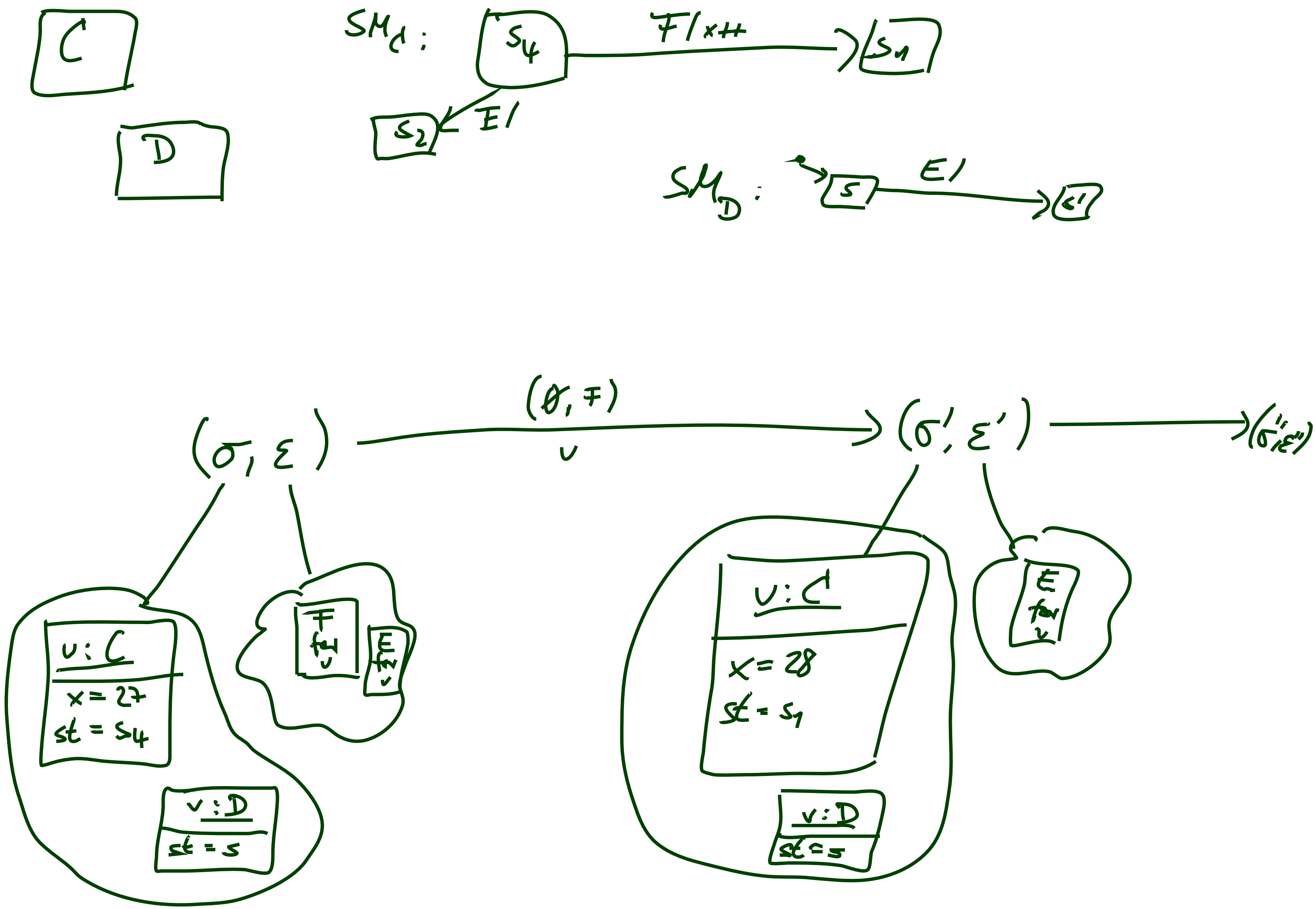


$$\mathscr{S} = \Big(\{Int\}, \{\langle C, \emptyset, 0, 0\rangle, \langle E, \{signal\}, 0, 0\rangle, \langle F, \{signal\}, 0, 0\rangle\}, \{x : Int, y : Int, n : C_{0,1}\}, \{C \mapsto \{x, n\}, E \mapsto \{y\}\} \Big)$$

$$\mathscr{E}(\mathscr{S}) = \{E, F\}$$

$$M = \Big(\{s_1, s_2, s_3, s_4\}, s_1, \{ (s_1, \_, true, skip, s_3), (s_1, E, \text{not oclIsUndef}(n), n!F, s_2), \dots \} \Big)$$

MATH


C
D
SM_C :
s_4
F/x++
s_1
E/
s_2
SM_D :
s
E/
s'
(σ, ε)
(∅, F)
v
(σ', ε')
(σ'', ε'')
v : C
x = 27
st = s_4
v : D
st = s
F
for v
E
for v
v : C
x = 28
st = s_1
v : D
st = s
E
for v

The Basic Causality Model

6.2.3 The Basic Causality Model [OMG, 2007b, 12]

*"**'Causality model'** is a specification of how things happen at run time [...].*

The causality model is quite straightforward:

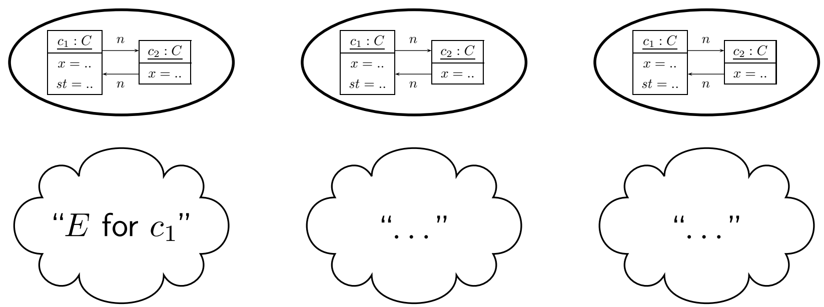
- *Objects respond to **messages** that are generated by objects executing communication actions.*
- *When these messages arrive, the receiving objects eventually respond by executing the behavior that is **matched** to that message.*
- *The dispatching method by which a particular behavior is associated with a given message depends on the higher-level formalism used and is not defined in the UML specification*
  (i.e., it is a semantic variation point).

The causality model also subsumes behaviors invoking each other and passing information to each other through arguments to parameters of the invoked behavior, [...].

This purely 'procedural' or 'process' model can be used by itself or in conjunction with the object-oriented model of the previous example."

15.3.12 StateMachine [OMG, 2007b, 563]

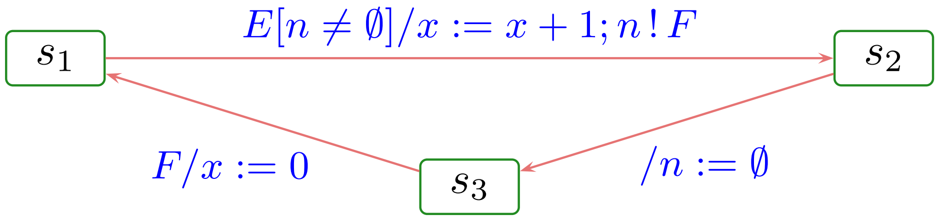
- Event occurrences are detected, dispatched, and then processed by the state machine, one at a time.
- The semantics of event occurrence processing is based on the **run-to-completion assumption**, interpreted as **run-to-completion processing**.
- **Run-to-completion processing** means that an event [...] can only be taken from the pool and dispatched if the processing of the previous [...] is fully completed.
- The processing of a single event occurrence by a state machine is known as a **run-to-completion step**.
- Before commencing on a **run-to-completion step**, a state machine is in a **stable state** configuration with all entry/exit/internal-activities (but not necessarily do-activities) completed.
- The same conditions apply after the **run-to-completion step** is completed.
- Thus, an event occurrence will never be processed [...] in some intermediate and inconsistent situation.
- [IOW,] The **run-to-completion step** is the passage between two stable configurations of the state machine.
- The **run-to-completion assumption** simplifies the transition function of the StM, since concurrency conflicts are avoided during the processing of event, allowing the StM to safely complete its **run-to-completion step**.



15.3.12 StateMachine *[OMG, 2007b, 563]*

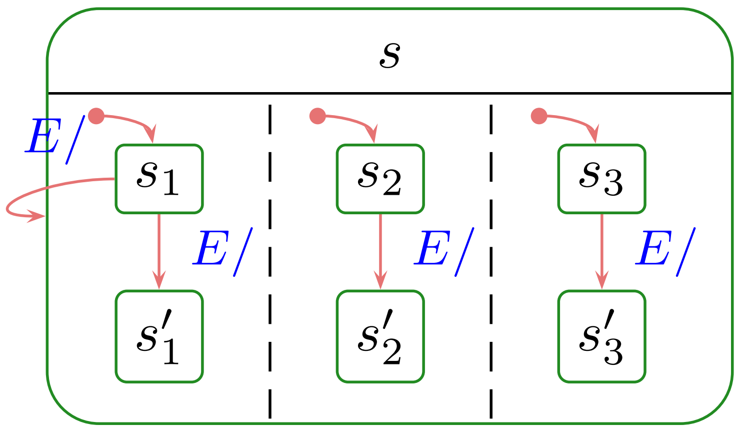
- The order of dequeuing is **not defined**, leaving open the possibility of modeling different priority-based schemes.
- Run-to-completion may be implemented in **various ways**. [...]

And?



- **...**:
  - We have to formally define what **event occurrence** is.
  - We have to define where events **are stored** – what the event pool is.
  - We have to explain how **transitions are chosen** – "matching".
  - We have to explain what the **effect of actions** is – on state and event pool.
  - We have to decide on the **granularity** — micro-steps, steps, run-to-completion steps (aka. super-steps)?
  - We have to formally define a notion of **stability** and RTC-step **completion**.

  - And then: hierarchical state machines.



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System Configuration, Ether, Transformer

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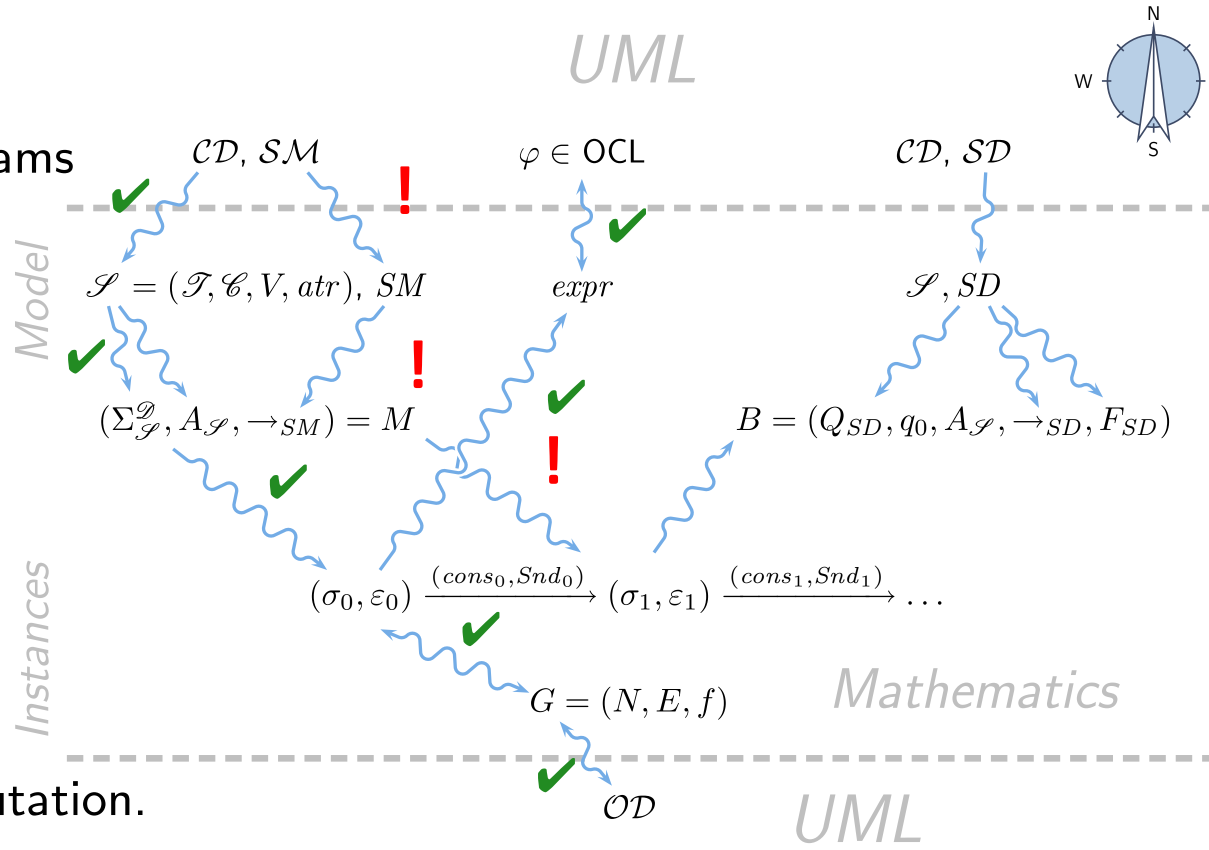


Ether aka. Event Pool

$\mathscr{E}(\mathscr{S}) = \{c \in \mathscr{C} \mid \mathit{signal} \in S_c\}$

Definition. Let $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, \boxed{\mathscr{E}})$ be a signature with signals and $\mathscr{D}$ a structure.

We call a ~~structure~~ tuple $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}$ and $\mathscr{D}$ if and only if it provides

- a **ready** operation which yields a set of events that are ready for a given object, i.e.

  for an event pool

  an object identity

  $$ready : Eth \times \mathscr{D}(\mathscr{C}) \to 2^{\mathscr{D}(\mathscr{E}(\mathscr{S}))}$$

  get a set of signal-instance identities

- a operation to **insert** an event destined for a given object, i.e.

  for a given event pool

  the id of the destination object

  $$\oplus : Eth \times \mathscr{D}(\mathscr{C}) \times \mathscr{D}(\mathscr{E}(\mathscr{S})) \to Eth$$

  ... yield another event pool

  a signal-instance id

- a operation to **remove** an event, i.e.

  $$\ominus : Eth \times \mathscr{D}(\mathscr{E}(\mathscr{S})) \to Eth$$

- an operation to clear the ether for a given object, i.e.

  $$[\,\cdot\,] : Eth \times \mathscr{D}(\mathscr{C}) \to Eth.$$

Ether: Examples

$(Eth, ready, \oplus, \ominus, [\cdot])$
$ready : Eth \times \mathcal{D}(\mathscr{C}) \to 2^{\mathcal{D}(\mathscr{E}(\mathscr{S}))}$
$\oplus : Eth \times \mathcal{D}(\mathscr{S}) \times \mathcal{D}(\mathscr{E}(\mathscr{S})) \to Eth$
$\ominus : Eth \times \mathcal{D}(\mathscr{E}(\mathscr{S})) \to Eth$
$[\cdot] : Eth \times \mathcal{D}(\mathscr{C}) \to Eth$

- A (single, global, shared, reliable) FIFO queue is an ether:
  - our choice: "ready for u" iff in front
  - *Eth*: the set of finite sequences of (u,e)-pairs $u \in \mathcal{D}(\mathscr{C})$, $e \in \mathcal{D}(\mathscr{E}(\mathscr{S}))$
  - *ready*$((u,e).\varepsilon, u) = \{e\}$, $ready((v,e).\varepsilon, u) = \emptyset$, $u \neq v$, $ready(\langle\rangle, u) = \emptyset$ $\quad Eth := (\mathcal{D}(\mathscr{C}) \times \mathcal{D}(\mathscr{E}(\mathscr{S})))^*$
  - $\oplus(\varepsilon, u, e) := \varepsilon.(u,e)$
  - $\ominus((u,e).\varepsilon, u) := \varepsilon$, $\ominus((v,e).\varepsilon, u) := (v,e).\varepsilon$, $u \neq v$, $\ominus(\langle\rangle, u) := \langle\rangle$
  - $[\cdot]$: ...

  our choice: remove only if in front

- Rhapsody: One FIFO queue per (active) object is an ether.
  $Eth = \mathcal{D}(\mathscr{C}) \times (\mathcal{D}(\mathscr{C}) \times \mathcal{D}(\mathscr{E}(\mathscr{S})))^*$
- Lossy queue. (would need $\oplus$ to yield sets of ethers)
- One-place buffer.
- Priority queue.
- Multi-queues (one per sender).
- Trivial example: sink, "black hole".
  $Eth = \{blackhole\}$
  $\oplus(\varepsilon, u, e) = \varepsilon$
  $ready(\varepsilon, u) = \emptyset$
  ...
- . . .

15.3.12 StateMachine [OMG, 2007b, 563]

- The order of dequeuing is **not defined**, leaving open the possibility of modeling different priority-based schemes.
- Run-to-completion may be implemented in **various ways**. [...]

Ether and [OMG, 2007b]

The standard distinguishes (among others)

- **SignalEvent** [OMG, 2007b, 450] and **Reception** [OMG, 2007b, 447].

On **SignalEvents**, it says

"receipt takes place"; for us: event

more conceptual: corresp. discard / dispatch

> *A signal event represents the receipt of an asynchronous signal instance. A signal event may, for example, cause a state machine to trigger a transition.* [OMG, 2007b, 449]
>
> [...]
>
> **Semantic Variation Points**
>
> = messages
>
> *The means by which requests are transported to their target depend on the type of requesting action, the target, the properties of the communication medium, and numerous other factors.*
>
> *In some cases, this is instantaneous and completely reliable while in others it may involve transmission delays of variable duration, loss of requests, reordering, or duplication.*
>
> *(See also the discussion on page 421.)* [OMG, 2007b, 450]

Our **ether** is a general representation of the possible choices.

Often seen minimal requirement: order of sending **by one object** is preserved.
But: we'll later briefly discuss "discarding" of events.

(*) maybe better: no associations to signals, i.e.
$\forall\, (v : C_{0,1}) \in V_0 \bullet C \notin \mathscr{E}(\mathscr{S}_0)$
$\wedge\, \forall\, (v : C_*) \in V_0 \bullet C \notin \mathscr{E}(\mathscr{S}_0)$

Definition. Let $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \cancel{\mathscr{E}_0})$ be a signature with signals, $\mathscr{D}_0$ a structure of $\mathscr{S}_0$, $(Eth, ready, \oplus, \ominus, [\,\cdot\,])$ an ether over $\mathscr{S}_0$ and $\mathscr{D}_0$. Furthermore assume there is one core state machine M_C per class $C \in \mathscr{C}$.

A system configuration over $\mathscr{S}_0$, $\mathscr{D}_0$, and Eth is a pair

a particular system state

$$(\sigma, \varepsilon) \in \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth$$

an event pool situation

type name for the set of C's states of state machine

where

- $\mathscr{S} = (\mathscr{T}_0 \,\dot\cup\, \{S_{M_C} \mid C \in \mathscr{C}\}, \quad \mathscr{C}_0,$

  no signal links to instances

  $V_0 \,\dot\cup\, \{\langle stable : Bool, -, true, \emptyset\rangle\}$

  $\dot\cup\, \{\langle st_C : S_{M_C}, +, s_0, \emptyset\rangle \mid C \in \mathscr{C}\}$

  $\mathscr{E}(\mathscr{S}_0)$

  $\dot\cup\, \{\langle params_E : E_{0,1}, +, \emptyset, \emptyset\rangle \mid E \in \cancel{\mathscr{E}_0}\},$

  the set of states of M_C

  the state machine of C

  $\{C \mapsto atr_0(C)$

  $\mathscr{E}(\mathscr{S}_0)$

  $\cup\, \{stable, st_C\} \cup \{params_E \mid E \in \cancel{\mathscr{E}_0}\} \mid C \in \mathscr{C}\}, \cancel{\mathscr{E}_0})$

- $\mathscr{D} = \mathscr{D}_0 \,\dot\cup\, \{S_{M_C} \mapsto S(M_C) \mid C \in \mathscr{C}\}$, and
- $\sigma(u)(r) \cap \mathscr{D}(\cancel{\mathscr{E}_0}) = \emptyset$ for each $u \in \operatorname{dom}(\sigma)$ and $r \in V_0$. (*)

  $\mathscr{E}(\mathscr{S}_0)$

System Configuration Step-by-Step

- We start with some signature with signals $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \cancel{\mathscr{E}_0})$.
- A **system configuration** is a pair (σ, ε) which comprises a system state σ wrt. $\mathscr{S}$ (not wrt. $\mathscr{S}_0$).
- Such a **system state** σ wrt. $\mathscr{S}$ provides, for each object $u \in \text{dom}(\sigma)$,
  - values for the **explicit attributes** in V_0,
  - values for a number of **implicit attributes**, namely
    - a **stability flag**, i.e. $\sigma(u)(stable)$ is a boolean value,
    - a **current (state machine) state**, i.e. $\sigma(u)(st)$ denotes one of the states of core state machine M_C,
    - a temporary association to access **event parameters** for each class, i.e. $\sigma(u)(params_E)$ is defined for each $E \in \mathscr{E}$.
- For convenience require: there is **no link to an event** except for $params_E$.

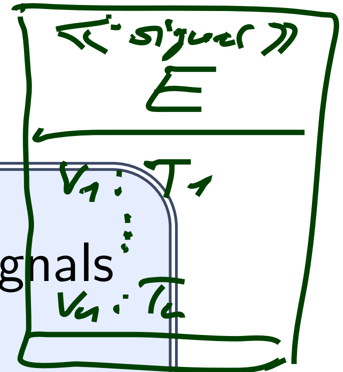
Stability

Definition.
Let (σ, ε) be a system configuration over some $\mathscr{S}_0$, $\mathscr{D}_0$, Eth.

We call an object $u \in \mathrm{dom}(\sigma) \cap \mathscr{D}(\mathscr{C}_0)$ stable in σ if and only if

$$\sigma(u)(stable) = \textit{true}.$$

Events Are Instances of Signals



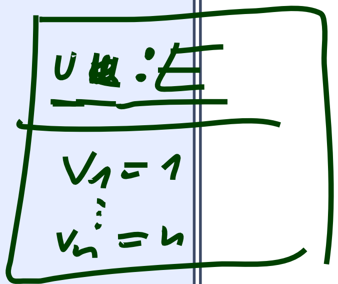
Definition. Let $\mathscr{D}_0$ be a structure of the signature with signals $\mathscr{S}_0 = (\mathscr{T}_0, \mathscr{C}_0, V_0, atr_0, \cancel{\mathscr{E}_0})$ and let $E \in \cancel{\mathscr{E}_0}$ $\mathcal{E}(\mathscr{S}_0)$ be a **signal**.

Let $atr(E) = \{v_1, \dots, v_n\}$. We call

$$e = (E, \{v_1 \mapsto d_1, \dots, v_n \mapsto d_n\}), \in \mathcal{E}(\mathscr{S}_0) \times (V_0 \mapsto \mathscr{D}(\mathscr{T}_0) \cup \mathscr{D}(\mathscr{C}_0))$$

or shorter (if mapping is clear from context)

$$(E, (d_1, \dots, d_n)) \text{ or } (E, \vec{d}),$$



an event (or an instance) of signal E (if type-consistent).

We use $Evs(\mathscr{E}_0, \mathscr{D}_0)$ to denote the set of all events of all signals in $\mathscr{S}_0$ wrt. $\mathscr{D}_0$.

As we always try to maximize confusion...:

- By our existing naming convention, $u \in \mathscr{D}(E)$ is also called **instance** of the (signal) class E in system configuration (σ, ε) if $u \in \mathrm{dom}(\sigma)$.
- The corresponding event is then $(E, \sigma(u))$.

Signals? Events...? Ether...?!

The idea is the following:

- **Signals** are **types** (classes).

- **Instances of signals** (in the standard sense) are kept in the **system state** $\underbrace{\text{component of}}_{\sigma}$ system configurations. (σ, ε).

- **Identities** of signal instances are kept in the **ether**. ε.

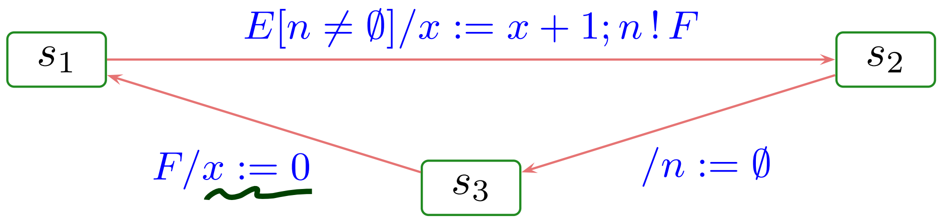
- Each signal instance is in particular an **event** — somehow "a recording that this signal occurred" (without caring for its identity)

- The main difference between **signal instance** and **event**:

  Events don't have an identity.

- Why is this useful? In particular for **reflective** descriptions of behaviour, we are typically not interested in the identity of a signal instance, but only whether it is an "E" or "F", and which parameters it carries.

Where are we?



- **Wanted**: a labelled transition relation

$$(\sigma, \varepsilon) \xrightarrow[u]{(cons, Snd)} (\sigma', \varepsilon')$$

  on system configuration, labelled with the **consumed** and **sent** events, (σ', ε') being the result (or effect) of $\underbrace{\textbf{one object}}_{u}$ taking a transition of **its** state machine. from the current state $\sigma(u)(st_c)$.

- **Have**: system configuration (σ, ε) comprising current state machine state and stability flag for each object, and the ether.

- **Plan**:
  - (i) Introduce **transformer** as the semantics of action annotions. **Intuitively**, (σ', ε') is the effect of applying the transformer of the taken transition.
  - (ii) Explain how to choose transitions depending on ε and when to stop taking transitions — the **run-to-completion "algorithm"**.

Transformer

Definition.

state

Let $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ the set of system ~~configurations~~ over some $\mathscr{S}_0$, $\mathscr{D}_0$, Eth.

We call a partial function

$$t : \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth \rightarrow\!\!\!\!\!+ \; \Sigma_{\mathscr{S}}^{\mathscr{D}} \times Eth$$

a (system configuration) **transformer**.

creation destruction

- In the following, we assume that each application of a transformer t to some system configuration (σ, ε) is associated with a set of **observations**

$$Obs_t(\sigma, \varepsilon) \in 2^{\mathscr{D}(\mathscr{C}) \times Evs(\mathscr{E} \,\dot{\cup}\, \{*,+\}, \mathscr{D}) \times \mathscr{D}(\mathscr{C})}.$$

- An observation

source object, event, destination object

$$(u_{src}, (E, \vec{d}), u_{dst}) \in Obs_t(\sigma, \varepsilon)$$

represents the information that, as a "side effect" of t, an event $(E, \vec{d})$ has been sent from u_{src} to u_{dst}.

Why Transformers?

- **Recall** the (simplified) syntax of transition annotations:

$$annot ::= \left[\quad \langle event\rangle \quad [\ \text{`['}\ \langle guard\rangle\ \text{`]'}\ ] \quad [\ \text{`/'}\ \langle action\rangle] \quad \right]$$

- **Clear**: $\langle event\rangle$ is from $\mathscr{E}$ of the corresponding signature.

- **But:** What are $\langle guard\rangle$ and $\langle action\rangle$?
  - UML can be viewed as being **parameterized** in **expression language** (providing $\langle guard\rangle$) and **action language** (providing $\langle action\rangle$).
  - **Examples**:
    - **Expression Language**:
      - OCL
      - Java, C++, ... expressions
      - ...
    - **Action Language**:
      - UML Action Semantics, "Executable UML"
      - Java, C++, ... statements (plus some event send action)
      - ...

Transformers as Abstract Actions!

In the following, we assume that we're **given**

- an **expression language** $Expr$ for guards, and
- an **action language** Act for actions,

and that we're **given**

- a **semantics** for boolean expressions in form of a partial function

  $$I[\![\,\cdot\,]\!](\,\cdot\,) : Expr \to (\Sigma_{\mathscr{S}}^{\mathscr{D}} \nrightarrow \mathbb{B})$$

  which evaluates expressions in a given system configuration,

  Assuming I to be partial is a way to treat "undefined" during runtime. If I is not defined (for instance because of dangling-reference navigation or division-by-zero), we want to go to a designated "error" system configuration.
- a **transformer** for each action.

Expression/Action Language Examples

We can make the assumptions from the previous slide because **instances exist**:

- for OCL, we have the OCL semantics from Lecture 03. Simply remove the pre-images which map to "$\perp$".
- for Java, the operational semantics of the SWT lecture uniquely defines transformers for sequences of Java statements.

We distinguish the following kinds of transformers:

- **skip**: do nothing — recall: this is the default action
- **send**: modifies ε — interesting, because state machines are built around sending/consuming events
- **create**/**destroy**: modify domain of σ — not specific to state machines, but let's discuss them here as we're at it
- **update**: modify own or other objects' local state — boring

References

References

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[OMG, 2007a] OMG (2007a). Unified modeling language: Infrastructure, version 2.1.2. Technical Report formal/07-11-04.

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