Composite States

- In a sense, composite states are about abbreviation, structuring, and avoiding redundancy.
- Idea: In Tron, for the Player’s State machine, instead of

**Recall: Syntax**

```
write
```

```markdown
```
**Examples**

- For instance, the function $\psi: \mathcal{M} \to \mathcal{P}(\mathcal{M})$ translates to
  
  $\{s, t\} \mapsto \{(s, s), (t, t)\}$

- Naming convention: $\psi(S) = (\text{source}(), \text{target}())$.

**States: st. (Legal) State Configurations**

- The type of $\mathcal{M}$ is from now on a set of states, i.e. $\mathcal{M} = 2^S$
- A set $S \subseteq \mathcal{M}$ is called legal state configuration if and only if
  
  - $\top \in S$, and
  - For each state $s \in S$, $s$ has exactly one (new pseudo-state) child of $s$, i.e.
    
    $\{s \in S \mid \exists \text{ child}(s) \in S \land s \neq \top\} = 1$.

**Towards Transitions: A Partial Order on States**

The substate (or child) relation induces a partial order on states:

- $s \leq s'$, for all $s \in S$, $s'$.
- $s \leq s'$, for all $s' \in \text{child}(s)$.
- transition, reflexive, asymmetric and transitive.
- $s \leq s'$ and $s' \leq s''$ implies $s \leq s''$ or $s'' \leq s'$.

**Composite States: Blessing or Curse?**

- What may happen on $E, E'$?
- Can $E, E'$ kill the object?

**Least Common Ancestor and They**

- The least common ancestor is the function $\lambda: 2^S \to S$ such that
  
  - $\lambda(S)$ is minimal, i.e. if $\top \in S$, then $\lambda(S) \subseteq S$.
  - $\lambda(S) = \top$, for all $S \subseteq S$.

- Note: $\lambda(S)$ exists for all $S \subseteq S$ (last candidate: $\top$)
Least Common Ancestor and Ting

• Two states \( s_1, s_2 \in S \) are called orthogonal, denoted \( s_1 \perp s_2 \), if and only if
  - they are unordered, i.e. \( s_1 \not\leq s_2 \) and \( s_2 \not\leq s_1 \), and
  - they live in different regions of an AND-state, i.e.
    \( \exists \text{ region}(s) = \{ S_1, \ldots, S_n \} : 1 \leq i \neq j \leq n \Rightarrow s_1 \in \text{child}(S_i) \land s_2 \in \text{child}(S_j) \).

References


