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Tutorials for Decision Procedures Exercise sheet 3

Exercise 1: Prenex Normal Form

Transform the following formula into prenex normal form:

$$F : \left(\forall z. \left((\forall x. q(x, z)) \rightarrow p(x, g(y), z) \right) \right) \wedge \neg (\forall z. \neg (\forall x. q(f(x, y), z)))$$

Exercise 2: Correctness of PNF

Show that the formula F from exercise 1 and the formula F' in prenex form that you computed are equivalent by proving $F \leftrightarrow F'$ with a semantic tableaux.

Exercise 3: Semantic Tableaux

Use the semantic tableaux method to prove the validity of the following formulae.

- (a) $(\forall x. (p(x) \rightarrow q(a))) \wedge (\exists x. p(x)) \rightarrow q(a)$
- (b) $(\forall x. p(f(x))) \wedge (\forall y. (q(y) \rightarrow \neg p(f(y)))) \rightarrow \neg q(b)$
- (c) $(\forall x, y. (p(x, y) \vee p(y, x))) \rightarrow \forall z. p(z, z)$
- (d) $\forall y. \exists x. (p(x) \rightarrow p(y))$
- (e) $\exists x. \forall y. (p(x) \rightarrow p(y))$