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Tutorials for Decision Procedures
Exercise sheet 5

Exercise 1: Induction in T_{PA}
Prove the T_{PA}-validity of the following formula using the semantic tableaux.

$$\forall x.\ 0 + x = x$$

Write down each proof step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as T_{PA}-valid. Note, that you may *not* assume commutativity, associativity, etc. Only use the Peano-axioms and the axioms from T_{E}. You need the induction axiom.

Exercise 2: Semantic Argument in $T_{\mathbb{R}}$
Show the $T_{\mathbb{R}}$-validty of the following formula using the semantic argument.

$$\forall x.\ x \cdot x \geq 0$$

Write down every step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as $T_{\mathbb{R}}$-valid. Additionally, you may use the following derived facts without proving them:

$$\forall x.\ 0 \geq x \to -x \geq 0$$
$$\forall x.\ (-x) \cdot (-x) = x \cdot x$$

Exercise 3: Integer Arithmetic
Consider the $T_{\mathbb{Z}}$-formula $F : \exists x.\forall y.\ \neg(y + 1 = x)$.

(a) Convert F into an equisatisfiable $T_{\mathbb{N}}$-formula G.

(b) Prove unsatisfiability of G using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

(c) Prove validity of the $T_{\mathbb{N}}$-formula $\exists x.\forall y.\ \neg(y + 1 = x)$.