ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

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Tutorials for Cyber-Physical Systems - Hybrid Models
Exercise sheet 1

Exercise 1: State reachability
Take the hybrid automaton from Example 9, page 23 of the script; see also the visualization of its behavior on page 25. Use the rules given on page 24.

(a) Show the reachability of a state where $x = 0$ at the time point $t = 1.5$.
(As an aside: we write "a state where $x = 0$" and not "the state where $x = 0$"; why?)

(b) Show the reachability of a state where $x = 2$ at the time point $t = 6$.

(c) Give an example of a trace which has three times a state where $x = 0$.
(As an aside: we write "a trace which ..." and not "the trace which ..."; why?)

Exercise 2: State reachability
Take the hybrid automaton from Example 10, page 25 of the script; see also the visualization of set of reachable states on page 26. Use the rules given on page 24.

(a) Show the reachability of a state with $x = 1, y = 1$.

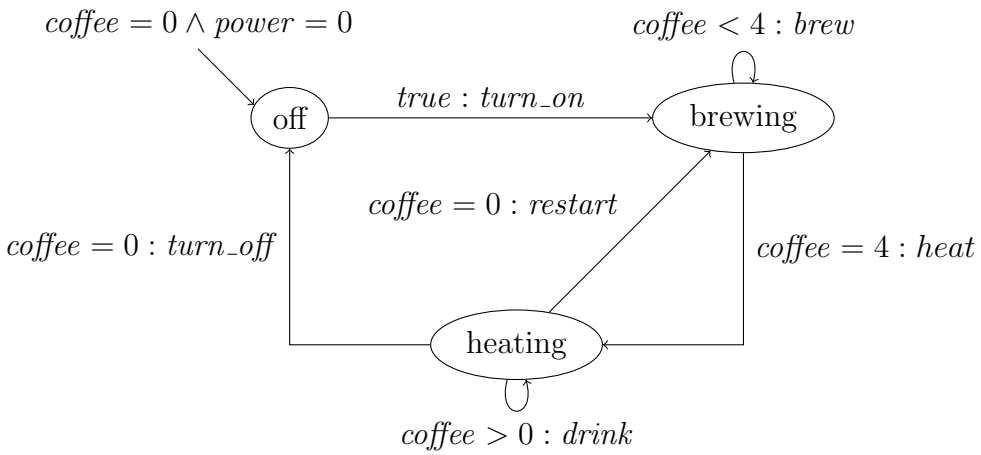
(b) Show the reachability of a state with $x = 2, y = 2$.

Exercise 3: Product automaton
Take the hybrid automaton from the parallel composition of the three hybrid automata in Example 14, page 29 and 30 of the script.

(a) Give the set of states of the corresponding LSTS, in the form of a Cartesian product.

(b) Give an example of a trace of this automaton which starts in the initial location $(far, 0, up)$ and returns to the initial location.

Exercise 4: Coffee Machine ("Just for fun")
The following graph, a variant of a labeled transition system (LTS), describes a coffee machine. The edges are labeled of the form "*guard:label*" where *guard* is a condition on the two variables of the LTS, *coffee* and *power*, and *label* is the label ("action") of the transition.



The function *Effect* specifies the update statement associated with each label.

$$
\begin{aligned}
\mathit{Effect}(\mathit{turn\_on}) &= \mathit{power} := 1 \\
\mathit{Effect}(\mathit{turn\_off}) &= \mathit{power} := 0 \\
\mathit{Effect}(\mathit{brew}) &= \mathit{coffee} := \mathit{coffee} + 1 \\
\mathit{Effect}(\mathit{drink}) &= \mathit{coffee} := \mathit{coffee} - 1 \\
\mathit{Effect}(\mathit{restart}) &= \text{no change of variable values} \\
\mathit{Effect}(\mathit{heat}) &= \text{no change of variable values}
\end{aligned}
$$

(a) Give the set of states of the corresponding labeled state transition system (LSTS), in the form of a Cartesian product.

(b) Draw the corresponding LSTS. Restrict the drawing to the part that is reachable from the reachable states.

(c) Do the following properties about the behavior of the system hold? If not: why not?

(i) Whenver the machine is turned off ($\mathit{power} = 0$) it contains no coffee ($\mathit{coffee} = 0$).

(ii) Whenever there are two cups of coffee ($\mathit{coffee} = 2$), there are either three or four cups of coffee in the next step.

(iii) There are always at most four cups of coffee ($\mathit{coffee} \leq 4$).

(iv) The coffee machine will be eventually turned off.
(English! The meaning of *eventually* is not the same as the German *eventuell* or the French *eventuellement*).

(v) Whenever there is no coffee ($\mathit{coffee} = 0$), there will be coffee after at most three steps.