7 Hashing: chaining

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Possible ways of treating collisions

Treatment of collisions:

- collisions are treated differently in different methods.
- a data set with key *s* is called a colliding element if bucket $B_{h(s)}$ is already taken by another data set.
- what can we do with colliding elements?
  1. chaining: implement the buckets as linked lists. Colliding elements are stored in these lists.
  2. open addressing: colliding elements are stored in other vacant buckets. During storage and lookup, these are found through so-called probing.

Chaining (1)

- The hash table is an array (length *m*) of lists.
  Each bucket is implemented by a list

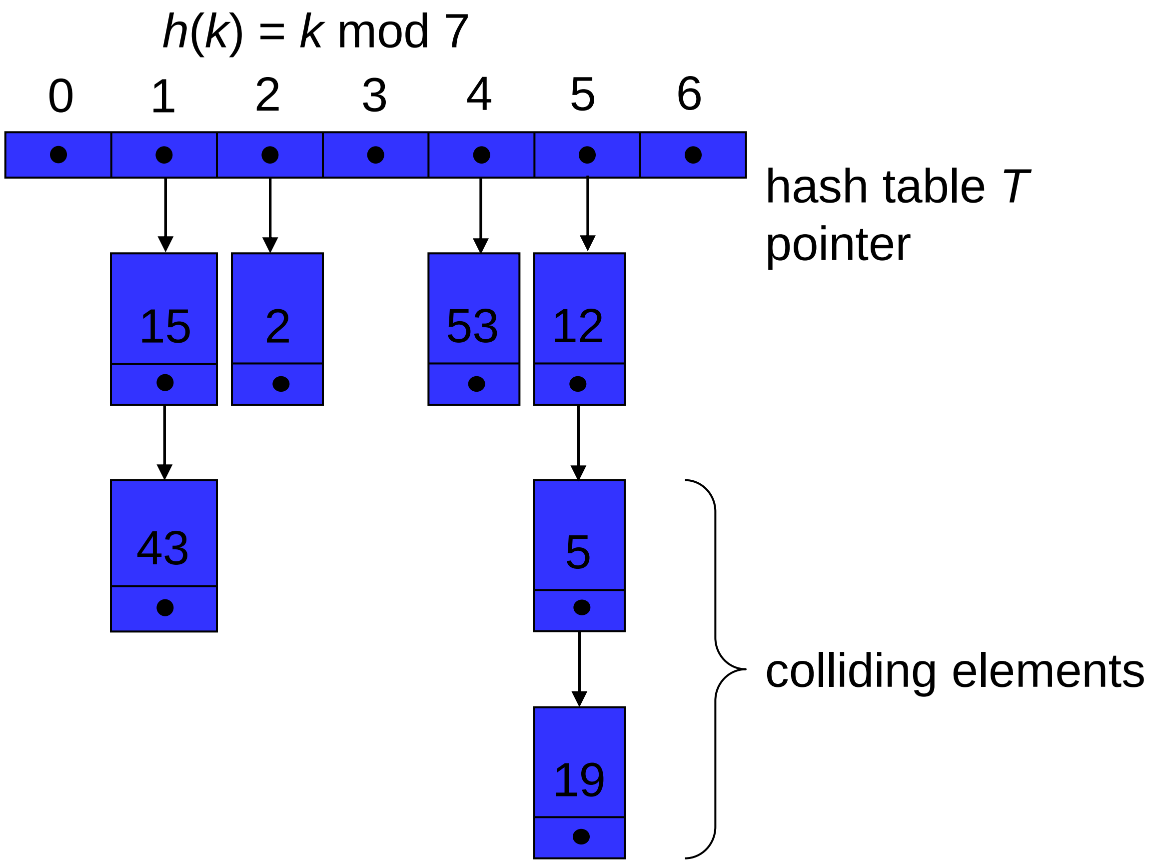
```
class hashTable {
    List[] ht;             // an array of lists
    hashTable (int m){     // Construktor
        ht = new List[m];
        for (int i = 0; i < m; i++)
            ht[i] = new List();  // Construct a list
    }
    ...
}
```

- Two different ways of using lists:
  1. direct chaining:
  hash table only contains list headers; the data sets are stored in the lists

  2. separate chaining:
  hash table contains at most one data set in each bucket as well as a list header. Colliding elements are stored in the list

Haching by chaining

Keys are stored in overflow lists



This type of chaining is also known as direct chaining.

Direct chaining

Search key k

- compute $h(k)$ and overflow list $T[h(k)]$
- look for k in the overflow list

Insert a key k

- search k (fails)
- insert k in the overflow list

Delete a key k

- search k (successfully)
- remove k from the overflow list

$\longrightarrow$ only list operations

Implementation in Java

```
class TableEntry {
        private Object key,value;
    }
    abstract class HashTable {
        private TableEntry[] tableEntry;
        private int capacity;
        // Constructor
        HashTable (int capacity) {
            this.capacity = capacity;
            tableEntry = new TableEntry [capacity];
            for (int i = 0; i <= capacity-1; i++)
                tableEntry[i] = null;
        }
        // the hash function
        protected abstract int h (Object key);
        // insert element with given key and value (if not there already)
        public abstract void insert (Object key Object value);
        // delete element with given key (if there)
        public abstract void delete (Object key);
        // lookup element with given key
        public abstract Object search (Object key);
    } // class hashTable
```

Implementation in Java

```
class ChainedTableEntry extends TableEntry {
        // Constructor
        ChainedTableEntry(Object key, Object value) {
            super(key, value);
            this.next = null;
        }
        private ChainedTableEntry next;
    }
    class ChainedHashTable extends HashTable {
        // the hash function
        public int h(Object key) {
            return key.hashCode() % capacity ;
        }
    // lookup key in the hash table
    public Object search (Object key) {
        ChainedTableEntry p;
        p = (ChainedTableEntry) tableEntry[h(key)];
        // Go through the liste until end reached or key found
        while (p != null && !p.key.equals(key)) {
            p = p.next;
        }
        // Return result
        if (p != null)
            return p.value;
        else return null;
    }
```

Implementation in Java

```
/* Insert an element with given key and value (if not there) */
   public void insert (Object key, Object value) {
       ChainedTableEntry entry = new ChainedTableEntry(key, value);
       // Get table entry for key
       int k = h (key);
       ChainedTableEntry p;
       p = (ChainedTableEntry) tableEntry [k];
       if (p == null){
           tableEntry[k] = entry;
           return ;
       }
       // Lookup key
       while (!p.key.equals(key) && p.next != null) {
           p = p.next;
       }
       // Insert the element (if not there)
       if (!p.key.equals(key))
           p.next = entry;
   }
```

Implementation in Java

```
// Delete element with given key (if there)
public void delete (Object key) {
    int k = h (key);
    ChainedTableEntry p;
    p = (ChainedTableEntry) TableEntry [k];
    TableEntry[k] = recDelete(p, key);
}
// Delete element with key recursively (if there)
public ChainedTableEntry recDelete (ChainedTableEntry p, Object key) {
    /* recDelete returns a pointer to the start of the list that p points to,
       in which key was deleted */
    if (p == null)
        return null;
    if (p.key.equals(key))
        return p.getNext();
    // otherwise:
    p.next = recDelete(p.next, key);
    return p;
}
public void printTable () {...}
} // class ChainedHashTable
```

Test program

```
public class ChainedHashingTest {
        public static void main(String args[]){
            Integer[] t= new Integer[args.length];
            for (int i = 0; i < args.length; i++)
                t[i] = Integer.valueOf(args[i]);
            ChainedHashTable h = new ChainedHashTable(7);
            for (int i = 0; i <= t.length - 1; i++)
                h.insert(t[i], null);
            h.printTable ();
            h.delete(t[0]); h.delete(t[1]);
            h.delete(t[6]); h.printTable();
        }
    }
```

Call:

```
java ChainedHashingTest 12 53 5 15 2 19 43
```

Output:

0: -|	0: -|
1: 15 -> 43 -|	1: 15 -|
2: 2 -|	2: 2 -|
3: -|	3: -|
4: 53 -|	4: -|
5: 12 -> 5 -> 19 -|	5: 5 -> 19 -|
6: -|	6: -|

Analysis of direct chaining

Uniform hashing assumption:

- all hash addresses are chosen with the same probability, i.e.:

  $Pr(h(k_i) = j) = 1/m$

- independent from operation to operation

Average chain length for n entries:

$$n/m = \alpha$$

Definition:

C'_n = expected number of entries inspected during a failed search

C_n = expected number of entries inspected during a successful search

Analysis:

$$\boxed{\begin{aligned} C'_n &= \alpha \\ C_n &\approx 1 + \frac{\alpha}{2} \end{aligned}}$$

Chaining

Advantages:
+ C_n and C'_n are small
+ $\alpha > 1$ possible
+ suitable for secondary memory

Efficiency of lookup

α	C_n (successful)	C'_n (unsuccessful)
0.50	1.250	0.50
0.90	1.450	0.90
0.95	1.457	0.95
1.00	1.500	1.00
2.00	2.000	2.00
3.00	2.500	3.00

Disadvantages:
- Additional space for pointers
- Colliding elements are outside the hash table

Summary

Analysis of hashing with chaining:

- worst case:

  $h(s)$ always yields the same value, all data sets are in a list.

  Behavior as in linear lists.

- average case:

  – Successful lookup & delete:

  complexity (in inspections) ≈ 1 + 0.5 × load factor

  – Failed lookup & insert:

  complexity ≈ load factor

  This holds for direct chaining, with separate chaining the complexity is a bit higher.

- best case:

  lookup is an immediate success: complexity $\in O(1)$.