Theory I, Sheet 1

- The solutions should be submitted in English.
- JUST FOR FUN exercises are not mandatory.
- Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (April 30, 4:00 p.m.).
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 1.1 - Proof by induction

1. Prove by induction that
$$\sum_{i=0}^{n} i = \frac{n}{2} \cdot (n+1)$$
for any $n \in \mathbb{N}$.
2. JUST FOR FUN. Prove by induction that
$$\sum_{i=0}^{n} i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}$$
for any $n \in \mathbb{N}$.

Exercise 1.2 - Complexity

Characterize the relationship between $f(n)$ and $g(n)$ in the following examples using the $\mathcal{O}$-, Θ- or Ω-notation. Do we have case a, b or c where: a) $g \in \mathcal{O}(f)$, b) $g \in \Theta(f)$, c) $\Omega(f)$?.

Example: $f(n) = n^2$ $g(n) = n \log n$. Solution: $\mathcal{O}(f) \ni g$.

1. $f(n) = n^{0.99998}$ $g(n) = \sqrt{n}$
2. $f(n) = 2^{\log^2(n)}$ $g(n) = \sum_{k=1}^{n^2} \frac{n}{2^k}$
3. $f(n) = \sqrt{n}$ $g(n) = 1000n$

Exercise 1.3 - Complexity

JUST FOR FUN. In order to solve a certain problem, five different algorithms $A_1, \ldots, A_5$ were developed. Algorithm A_i needs $T_i(n)$ time steps to solve the problem for an instance of size n.

1. $T_1(n) = 1000n$
2. $T_2(n) = 500n \log_2(n)$
3. $T_3(n) = n\sqrt{n}$
4. $T_4(n) = 10n^3$
5. $T_5(n) = 2^n$

The algorithms will be executed on a Pentium 1GHz processor. For simplicity, we assume that the processor executes exactly 10^9 computations per second.
Compute for any i the input size n, for which the problem can be solved by algorithm A_i within 1h.

Exercise 1.4 - Complexity classes

JUST FOR FUN. Order the following classes according to set inclusion: DLOG, PSPACE, PTIME, NP, coNP, NLOG. Example: NLOG $\subseteq$ PTIME.