Theory I, Sheet 2

- The solutions should be submitted in English.
- JUST FOR FUN exercises are not mandatory.
- Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (May 07, 4:00 p.m.).
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 2.1 - Proof by induction

Consider the definition of a binary tree. Either a tree is

1. a leaf written $\Box$,
2. or an inner node with two children t_1 and t_2, which are both binary trees, written $N(t_1, t_2)$.

The number of inner nodes of a binary tree $I(t)$ is given by:

$$I(t) = \begin{cases} 0 & \text{if } t = \Box \\ I(t_1) + I(t_2) + 1 & \text{if } t = N(t_1, t_2) \end{cases}$$

The number of leafs of a tree t is given by:

$$L(t) = \begin{cases} 1 & \text{if } t = \Box \\ L(t_1) + L(t_2) & \text{if } t = N(t_1, t_2) \end{cases}$$

Prove that the difference between the number of leaves and the number of internal nodes in a binary tree is 1, i.e., $I(t) - L(t) = 1$.

Exercise 2.2 - Internal Path Length

The internal path length $l(t)$ of a search tree t is defined as follows:

$$l(t) = \begin{cases} 0 & \text{if } t \text{ is empty} \\ l(t_l) + l(t_r) + size(t) & \text{otherwise} \end{cases}$$

where t_l, t_r are respectively the left and right subtrees of t and $size(t)$ denotes the number of internal nodes of t. Now, let $N(t)$ denote the internal nodes of t. Using induction show that:

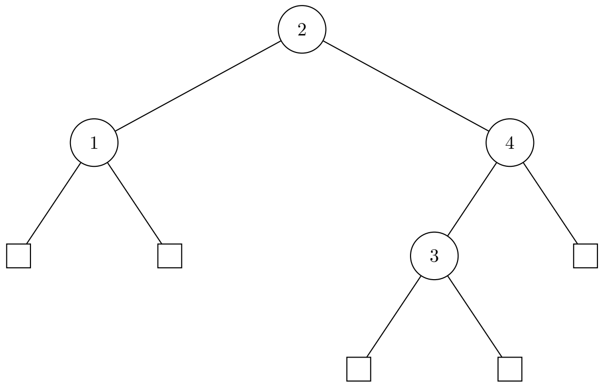
$$l(t) = \sum_{p \in N(t)} (depth(p) + 1)$$

where $depth(p)$ is the distance of node p from the root of t.

Hint: For the base case consider the empty tree. Then, for the inductive step, assume that for a given tree t the desired property holds for subtrees t_l and t_r. Notice that $depth(p)$ has different values depending on the given tree. Assining different names might be helpful (e.g. $depth_t, depth_{t_l}, depth_{t_r}$).

Exercise 2.3 - Trees

Consider the following Binary Search Tree:



1. Which sequences (permutations) of the keys 1,2,3,4 will result in this shape, if keys are inserted sequentially in an empty tree?
2. Draw all structurally different binary trees with four internal nodes. (Do not draw the leaf nodes).
3. JUST FOR FUN. Derive the formula for the number B_N of structurally different binary trees with N internal nodes. Explain your solution.